A STUDY OF DYNAMICS OF TRAIN MOTION WITH SPECIAL REFERENCE TO THE THEORY AND THE APPLICATION OF SPEED-TIME, DISTANCE-TIME, AND SPEED-DISTANCE CURVES

BY

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY SENTARO SEKINE

ENTITLED A STUDY OF THE DYNAMICS OF TRAIN MOTION WITH SPECIAL REFERENCE TO THE THEORY AND THE APPLICATIONS OF SPEED-TIME, DISTANCE-TIME AND SPEED-DISTANCE CURVES.

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

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A STUDY OF THE DYNAMICS OF TRAIN MOTION
WITH SPECIAL REFERENCE TO THE THEORY AND THE APPLICATIONS OF SPEED-TIME, DISTANCE-TIME AND SPEED-DISTANCE CURVES.

I. INTRODUCTION.

1. The Problem of Railway Engineers.
2. The Dynamics of Train Motion and Its Relation to Railway Problems.
3. The Object of this Paper.

In the early days of railroading and also in the period of rapid trunk line expansion, the questions of speed, safety and economical train operation in the present sense, were rather secondary problems to railway officials and to the general public as well. But, today, when the demand for safety, speed and low rates is so acute under the economic conditions and the government regulations which prevail, these problems have become so important that the fate of a railroad often depends upon its ability successfully to solve them. Most of the effort of railway engineers is directed primarily toward the movement of trains economically and safely. Insofar as both these objects are not attained, this effort is ineffective. Such solution depends in no small measure upon an accurate knowledge and appreciation of the dynamics of train motion.

The Dynamics of Train Motion is a certain special branch of applied mechanics, in which we study the dynamic principles which govern the motion of railway trains, and especially the nature and magnitude of the forces acting upon the train and also the means of producing the complete diagram of the motion. An accurate knowledge of the forces - tractive effort, train resistance and braking force - and of the production or elimination of these forces, has, no doubt, a very important bearing on the design, construction, maintenance and economical operation of railways. Further, the motion diagrams or "speed curves" - speed-time, distance-time, and speed-distance curves - which, as will be seen later, serve as the connecting links between the dynamic and economic phases of railway train operation, are the most important means available in the scientific solution of such problems as economic tonnage ratings, selection of proper and most economical locomotives for a given division, determination of energy or fuel and water consumption, construction of proper train schedules, railway location and re-location, etc., which make up the general problem - safe and economical train operation.

The objects of this thesis are:

1. To study the dynamic principles of railway train motion.

2. To investigate the character of the forces acting upon the train in motion.

3. To develop the method of producing motion diagrams.

4. To illustrate the application of the motion diagrams to the solution of the railway problems mentioned above.

II. REVIEW OF SOME FUNDAMENTAL PRINCIPLES OF MECHANICS.

A. The First Law of Motion and the Motion of Railway Trains.

A statement of Newton's first law of motion reads: "Every body persists in its state of rest or of uniform motion in a straight line, unless it be compelled by some force to change that state". Due to this law and as our every-day experience shows, in order to start a train from rest an application of a certain force is necessary; and to maintain the uniform motion of the train the continuous application of a certain force is also necessary, since a train in motion is constantly opposed by numerous resisting forces. Further, to reduce the speed or to stop a train a sufficient amount of force which acts against the motion of the train must be applied. Thus, the most important factors in the motion of railway trains are: First, the force which starts a train from rest and accelerates and then maintainsits proper speed; second, the resisting forces acting against the starting and the motion of the train; and third, the force which reduces an excessive speed or stops the train at a desired point within a given time. These three classes of forces are usually called tractive effort, train resistance and braking force, respectively. When the values of these forces are accurately known, the speed or the motion of a train can be computed by a very simple formula derived in the next article and the study of train motion is no longer a difficult problem. The accurate determination, however, of tractive effort, train resistance, etc. is difficult. These problems nevertheless being of great economic importance in railway operation as well as of great interest for railway engineers, have been studied by many prominent engineers and investigators. As a consequence, fairly accurate information on these subjects has been accumulated and a sufficiently precise predetermination of train motion is now possible.

B. The Second Law of Motion and Equation of Motion.

The second law of motion states that the rate of change of the momentum of a body is proportional to the force acting on the body and is in the direction of the force. Reducing this to a mathematical expression, we have

$$F \propto \frac{mv_2 - mv_1}{t}$$

where F is the force acting on a mass m and changes its velocity from v_1 to v_2 within a time interval, t. Then, introducing a certain constant k and expressing by a the acceleration, we have

$$F = k\left(\frac{mv_2 - mv_1}{t}\right).$$

But, $(v_2 - v_1) \div t = a$, hence

$$F = kma.$$

The numerical value of k in the above equation depends on the units employed for expressing the values of force, mass and acceleration and when we use the <u>weight</u> of a pound as the unit of force, 32.2 pounds as the unit of mass, and one foot per second per second as the unit of acceleration, k becomes unity and

$$F = ma. \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (1).$$

Similarly, if W is the weight of a body and g the acceleration due to gravitation,

$$W = mg,$$

or

$$m = W/g$$

and (1) becomes

$$F = \frac{W}{g}a. \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (2)$$

The equation (1) or (2), though very simple in its form, is the fundamental formula or equation of motion and has extensive application in the study of the motion of railway trains. It may be noted here that in this formula the point of application of F is understood to be at the mass center and F is the component, along the direction of motion of the train, of the effective or resultant force; but in any actual case of train motion accurate determination of the mass center and the point of application of the effective force is practically impossible. This formula, however, is successfully used by assuming that the effective force is acting on the mass center, the error due to this assumption being automatically corrected by including in the train resistance the resistance due to oscillation of the train, which is, no doubt, caused by the excentricity of application of the effective force.

C. Distance, Speed, and Acceleration.

1. Displacement and distance. - A displacement, that is, a change of position of a body, has no reference to time involved and its specification consists in the statement of the length and the direction of the motion by which the body changes its position. Thus, distance or length is implied by the term displacement and it represents the magnitude of displacement of a body.

2. Velocity and Speed. - Velocity is defined as the time rate of displacement. Its specification is, then, necessarily the statement of the distance, time, and direction. The magnitude of velocity, or speed may be expressed, according to the definition as

$$v = \frac{\Delta s}{\Delta t} \qquad \text{or} \qquad a = \frac{ds}{dt} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

in which v is the magnitude of velocity, s the magnitude of displacement and t the time.

3. Acceleration. - The time rate of change of velocity is called acceleration. It is specified by its magnitude in distance and in the time element, and by the direction of the force which acting upon the body accelerates (or retards) it. The mathematical expression of acceleration can be readily deduced from the definition as follows:

$$a = \frac{\Delta v}{\Delta t} \qquad \text{or} \qquad a = \frac{dv}{dt} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

But, $v = ds/dt$, hence

$$a = \frac{d^2 s}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

4. Acceleration of railway trains. - The equations (3) (4) and (5) may be rewritten respectively as follows:

$$v = \int a\,dt$$

$$s = \int v\,dt$$

and

$$s = \iint a\,dt^2.$$

It may be observed here that if the acceleration, a, be a constant or some simple function of time t, these equations can be easily integrated and the speed and distance at any instant can be computed. In the case of railway trains, however, the acceleration or the effective force which accelerates the train is not a constant nor any function of time, but a function of speed, and as will be seen later, not one which can be determined easily.

D. Work and Energy.

1. Work. - When a body is displaced through a certain distance by a force acting on it, work is said to be performed by the force, and its measure is the product of the force and the distance, that is

$$\text{Work done} = FS,$$

where F is the component of the force and S the distance both measured along the path of the body. If F is a variable, we have

$$\text{Work done} = \int F ds,$$

and it is computable when the relation, $F = f(s)$ is known.

2. Energy. - When a body, by virtue of its position or velocity, is capable of doing work in undergoing a change of that state, it is said to possess energy. Potential energy is due to the position of a body relative to some standard of reference, and is measured by the work the body can do while changing its position relative to the standard reference. For instance, if the first position of a body, whose weight is W, is at the elevation h_1 and its second position h_2, then the potential energy of the body at the first and the second is Wh_1 and Wh_2 respectively, and the change in its potential energy is $W(h_1 - h_2)$. A railway train has always a great amount of weight and it changes its elevation, hence the potential energy becomes a very important factor in the motion of railway trains. Kinetic energy is the energy due to the velocity or motion of a body, and is measured by $(1/2)mv^2$, where m denotes the mass of the body and v the velocity. Since a railway train

has a great mass and often very great velocity, the kinetic energy becomes also a very important factor in the train motion.

a). Kinetic energy due to translation. - If a body in translation with a velocity v encounter a constant resisting force F, it will reduce its velocity gradually, but will move, before coming to rest, through a distance, say s. Then the work done by this force or the work done by the body due to its kinetic energy,

$$K_t = Fs.$$

But, $s = (1/2)at^2$, $F = ma$, and $v = at$,

$$K_t = ma\left(\frac{1}{2}at^2\right),$$

$$= ma\left(\frac{v^2}{2a}\right)$$

$$= \frac{1}{2}mv^2.$$

Hence, the kinetic energy due to the translation of a body whose mass is m and velocity v is expressed directly by $1/2\ mv^2$. It may also be easily proved that the change of kinetic energy due to the translation of a body whose mass is m and its initial and final velocities v_1 and v_2 respectively, is equal to

$$\frac{1}{2}m(v_1^2 - v_2^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Formula (6) holds good also when the force acting on the body is not constant. The proof is simple and may be found in any text book in physics.

b). Kinetic energy due to rotation. - Let us consider an elementary mass dm rotating around the axis o with the angular acceleration α, and angular velocity ω, and assume that

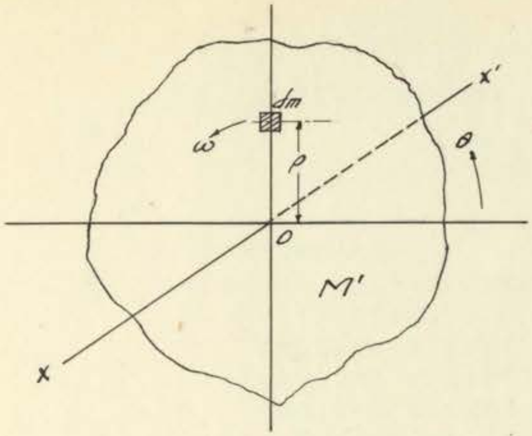


Fig. 1.

the axis coincides with the mass center of the rotating body. Then the kinetic energy of dm,

$$dK_r = \frac{1}{2}\, dmv^2$$

but $v = \rho\omega$, so

$$dK_r = \frac{1}{2} dm\, \rho^2\omega^2$$

and by integration we get the total kinetic energy due to the rotation of the body,

$$K_r = \frac{1}{2}\int \omega^2\rho^2 dm$$

$$= \frac{1}{2}\omega^2 \iiint \rho^3 d\theta\, d\rho\, dx.$$

But the value $\int \rho^3 dm$ or $\iiint \rho^3 d\theta\, d\rho\, dx$ is the second moment or moment of inertia I, then

$$K_r = \frac{1}{2}\,\omega^2 I,$$

$$= \frac{1}{2}\omega^2 k^2 M'.$$

since the moment of inertia is equal to the square of the radius of gyratioh k, times the total mass M. Similarly we have for the change in the kinetic inertia due to change in the angular velocity from ω_1 to ω_2

$$K_r = \frac{1}{2} M' k^2 (\omega_1^2 - \omega_2^2) \quad \text{.................. (7).}$$

III. MOTIVE FORCES OF TRAIN MOTION.

A. Motive Forces.

By the "motive forces of train motion", we mean any dynamic force or forces which, acting on a railway train, effect the motion of the train in a desired direction. The proper function of a motive force is to overcome constantly the enormous amount of the train resistances, both inherent and incidental, and to accelerate or keep uniform the motion of a train. The tractive effort or drawbar pull of motive powers is intended solely to perform this important function; while the kinetic energy or inertia force of a train and the potential energy or gravitational force also act as motive forces according to certain physical laws, no matter whether they are desirable or not. Among these motive forces, that of the motive powers is of paramount importance, but we should not lose sight of these secondary but uncontrollable forces, as they have a great influence on the motion of railway trains.

B. Forces of Motive Powers.

1. Motive powers. - In railroad parlance by a "motive power" is meant any contrivance which develops a dynamic force for the traction of itself and of a train of cars following it. Practically speaking, the motive power used in railway transportation is only the steam locomotive, although the electric locomotive, which is of rather recent introduction, is gaining a footing in railway traction. A motive power which is under development is the Diesel engine locomotive, which is prophesied to become the most economical railway motive power in the future. Its success at the present time is, however, somewhat doubtful. For certain purposes, some cars or trains of cars have their motive power installed on themselves. The street cars, motor car trains, storage battery cars, gas engine cars, gas-electric cars and some others belong to this class. Each of these has certain advantages and may be extensively employed in fields for which its merits are recognized.

2. Tractive effort of steam locomotives. - A steam locomotive is practically a constant power prime mover, hence its tractive force is not a constant value, but varies inversely with speed. In fact, the tractive force is a function not only of speed but of many other variables. For its practical application, however, it is necessary and sufficient to express the tractive force in term of a single variable, speed,

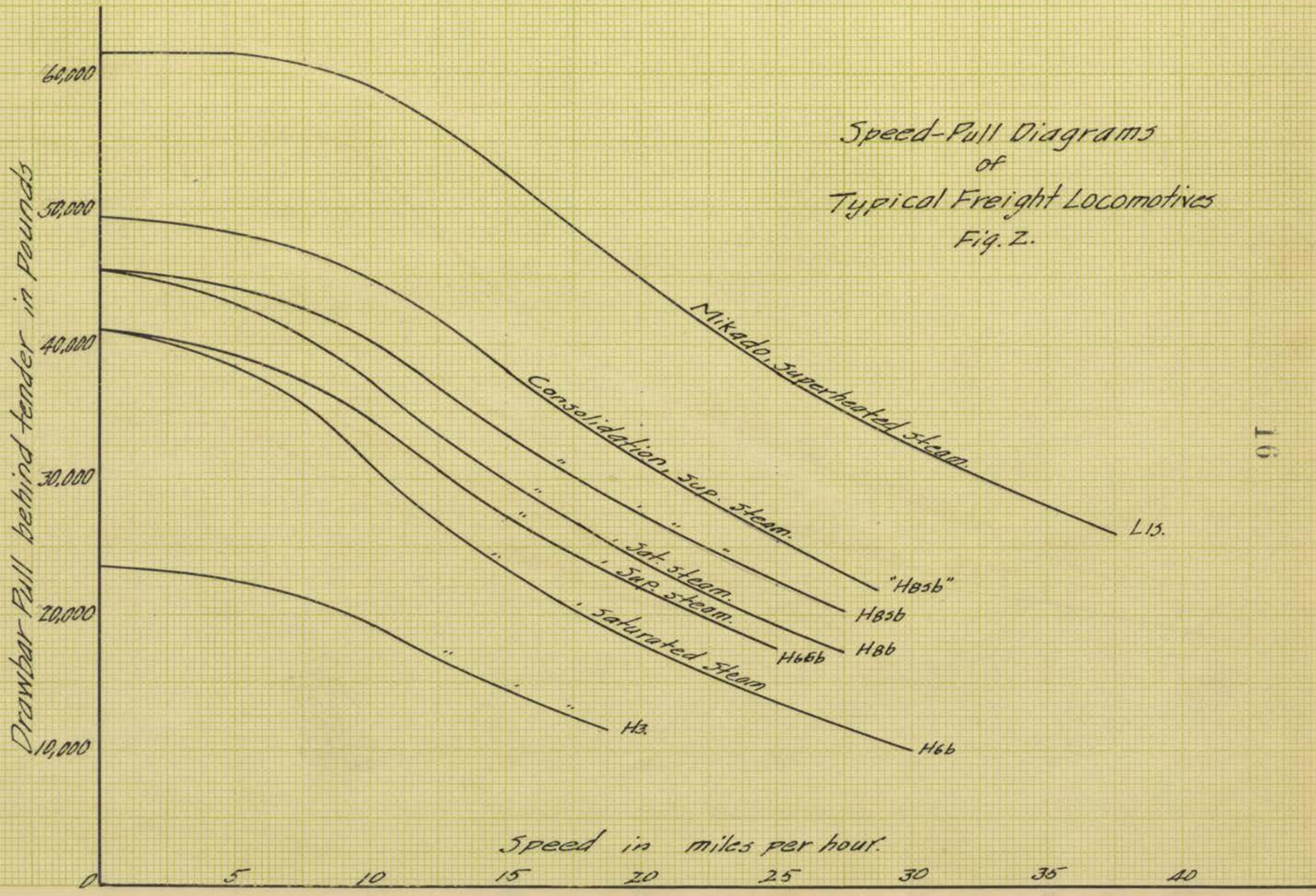
that is by defining its speed-tractive effort or speed-pull relation.

For the determination or estimation of this speed-pull relation of a steam locomotive, there are at least four different methods. They are: (1) the method in which the relation is computed from speed and dynamometer records obtained in a locomotive laboratory test, (2) the relation is computed from the drawbar pull and speed records of a dynamometer car test, (3) the relation is computed from steam indicator diagrams and speed records of a "road test", and (4) the relation is estimated by means of a formula. The first two methods are equally direct and accurate, and are regarded as the standard methods for the determination of the speed-pull relation. The third or the road test method, being the only one known for this sort of investigation before the introduction of the dynamometer car, has been popularly employed and is still in use when a dynamometer car or a locomotive testing plant is not available; although tests by this method are made under many difficulties and the result obtained is naturally not so reliable as that of the first two methods. The fourth method mentioned above is not a direct method but the relation is estimated by a formula which is a generalization of known speed-pull relations of other locomotives, determined by reliable direct or actual tests. At present, however, we have not enough experimental data to make a complete generalization, so that the formula or method must be largely empirical, and consequently the relation estimated by this method is not so reliable as those determined by actual tests. Nevertheless, the estimation can be made within ordinarily desired limits of accuracy, and, moreover, the time and labor required is extremely small compared with other methods. The fair accuracy and simplicity of this method makes it the most convenient and satisfactory, but it is good only if an accurate and reliable formula be established.

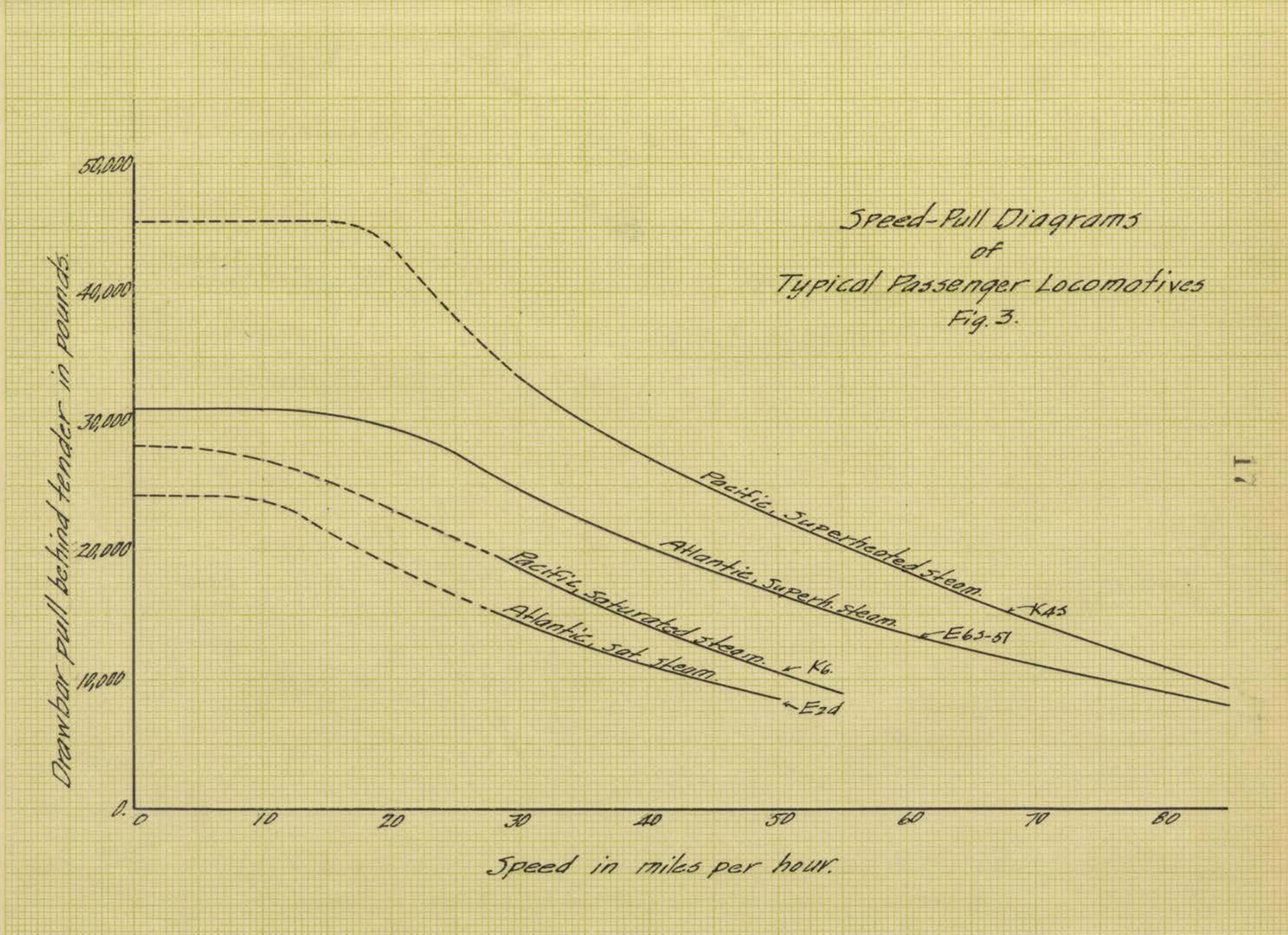
Further study of the subject of steam locomotive tractive effort will be postponed to the next chapter. Here only the speed-pull diagrams of typical locomotives, reproduced from the most reliable tests will be exhibited. (See Figs. 2 and 3).

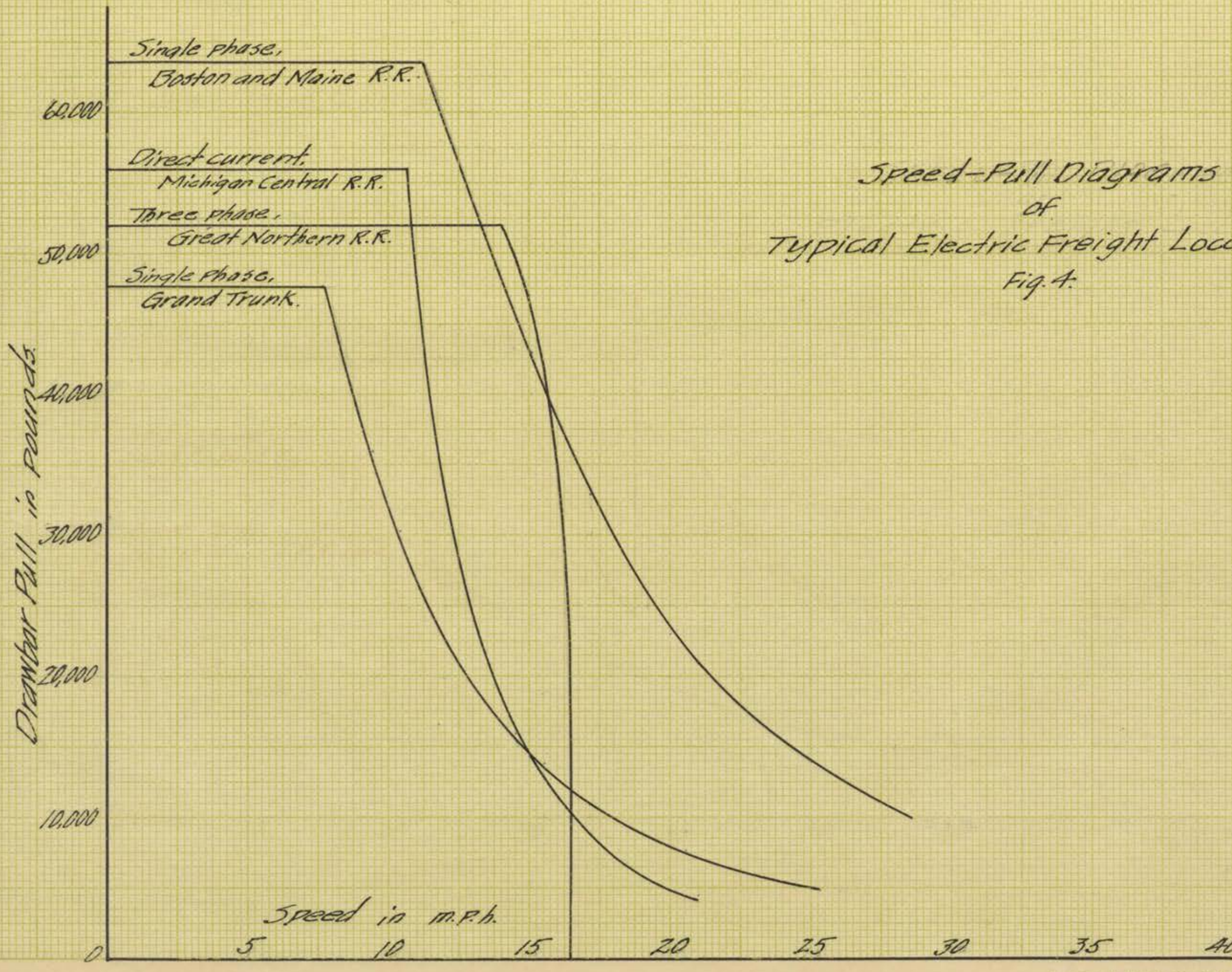
3. Tractive force of electric locomotives. - The speed-pull relation of electric locomotives could properly be determined by means of a dynamometer car or a locomotive testing plant as in the case of steam locomotives. On account of the simplicity of the power transmission mechanism between the motors and the drawbar, and the regularity of performance of the motors, the speed-pull relation of an electric locomotive, however, is very easily and accurately determined when the speed-torque relation of the motors is obtained by an ordinary laboratory test, which is, no doubt, less expensive than the other methods. Figs. 4 and 5 show the speed-pull relation of typical electric locomotives.

As mentioned before, unlike a steam locomotive, the intrinsic function of an electric locomotive is very simple - merely the transformation of high grade electric energy to high grade mechanical energy - and its performance is not materially disturbed by the surrounding or operating conditions.

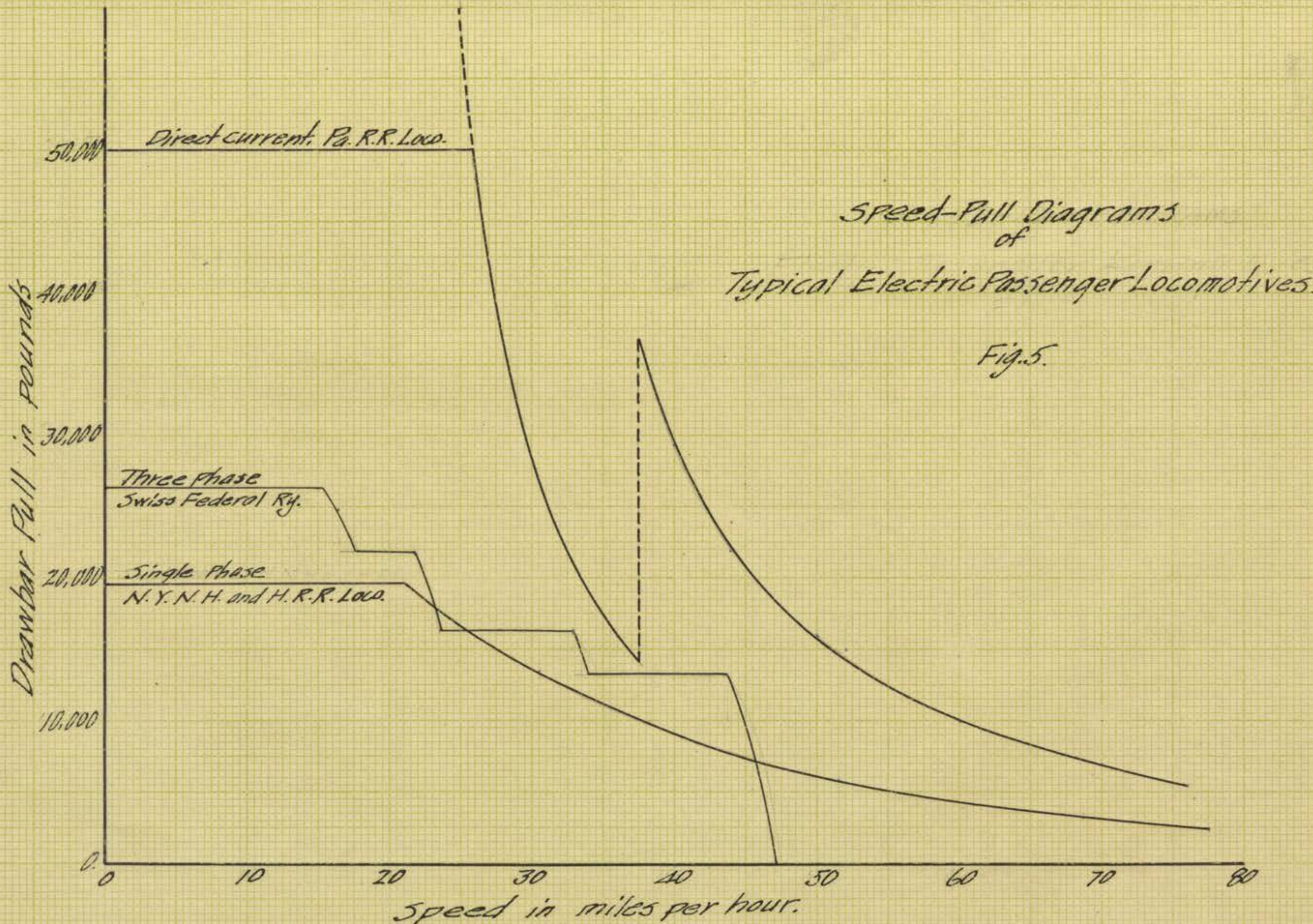


Speed-Pull Diagrams
of
Typical Freight Locomotives
Fig. 2.


Speed-Pull Diagrams
of
Typical Passenger Locomotives
Fig. 3.
Drawbar pull behind tender in pounds.
50,000
40,000
30,000
20,000
10,000
0.
0
10
20
30
40
50
60
70
80
Speed in miles per hour.
Pacific, Superheated steam.
K4s
Atlantic, Superh. steam.
E6s-51
Pacific, Saturated steam.
K6.
Atlantic, sat. steam.
E2d



Speed-Pull Diagrams
of
Typical Electric Freight Locomotives
Fig. 4.



Speed-Pull Diagrams of Typical Electric Passenger Locomotives.

Fig. 5.

Consequently, its energy consumption, work done, capacity, etc., which have an important relation to the central and substation design and operation and also to some larger railway economic problems, can be very definitely estimated. This point will be more fully discussed in ~~the~~ chapter X.

4. Tractive effort of other motive powers. - The speed-pull relation of electric cars is similar to that of electric locomotives, whether they receive energy from a power station, a storage battery, or are directly connected (electrically) to gas engines; and it depends only upon the system and design of the motors. Fig. 6 exhibits the speed-current-torque relations of typical electric traction motors which are commonly used on street cars and motor car trains. No experimental data showing the speed-pull relations of the Diesel engine locomotive, gasoline engine cars, and compressed air locomotives are available. Fig. 7 shows the general character of the speed-pull relation of a gas-electric locomotive.

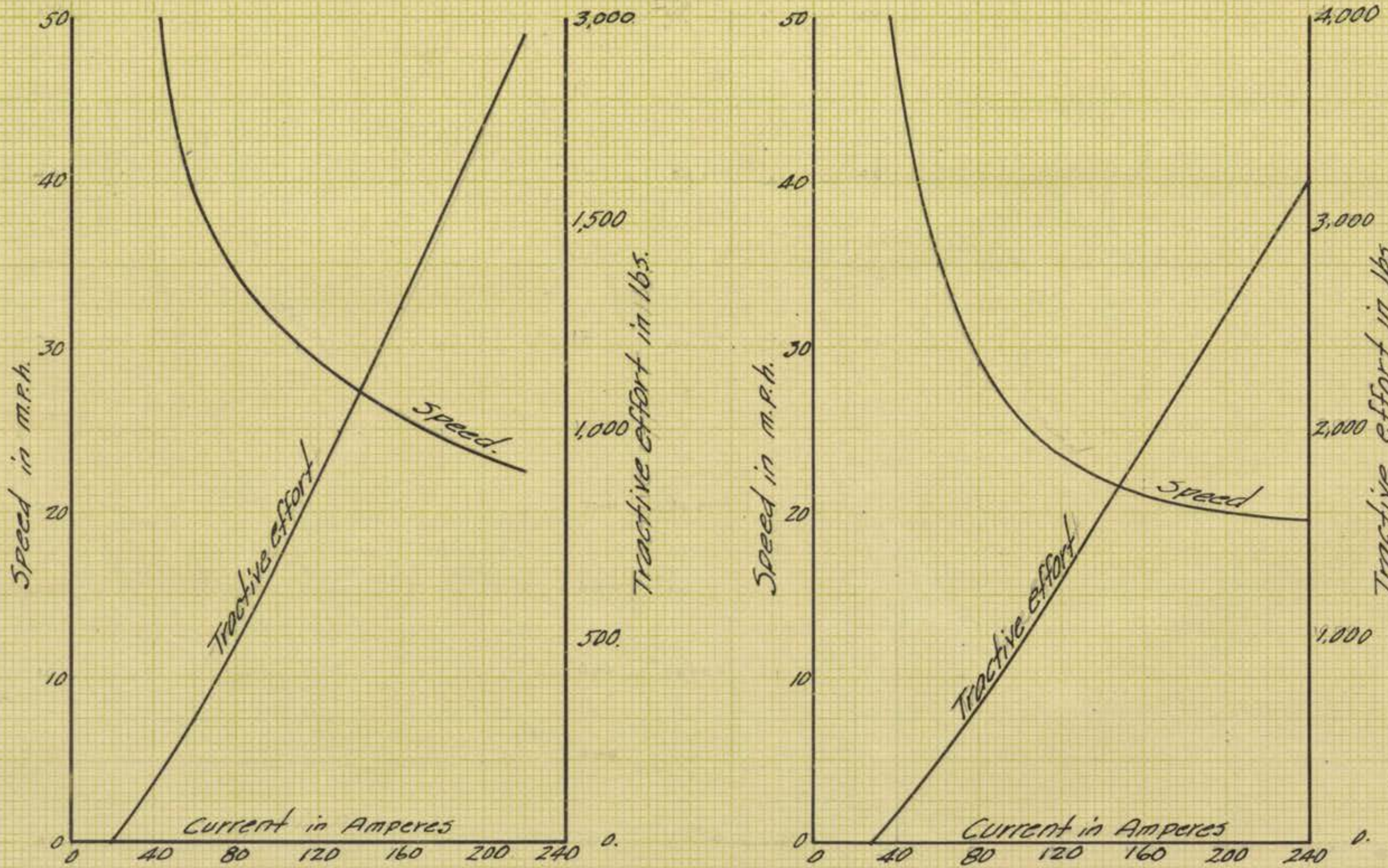
C. Kinetic Energy or Inertia Force of Trains in Motion.

1. Inertia force due to translation. - It was shown in the preceding chapter, that the kinetic energy of a body due to translation could be expressed by the formula,

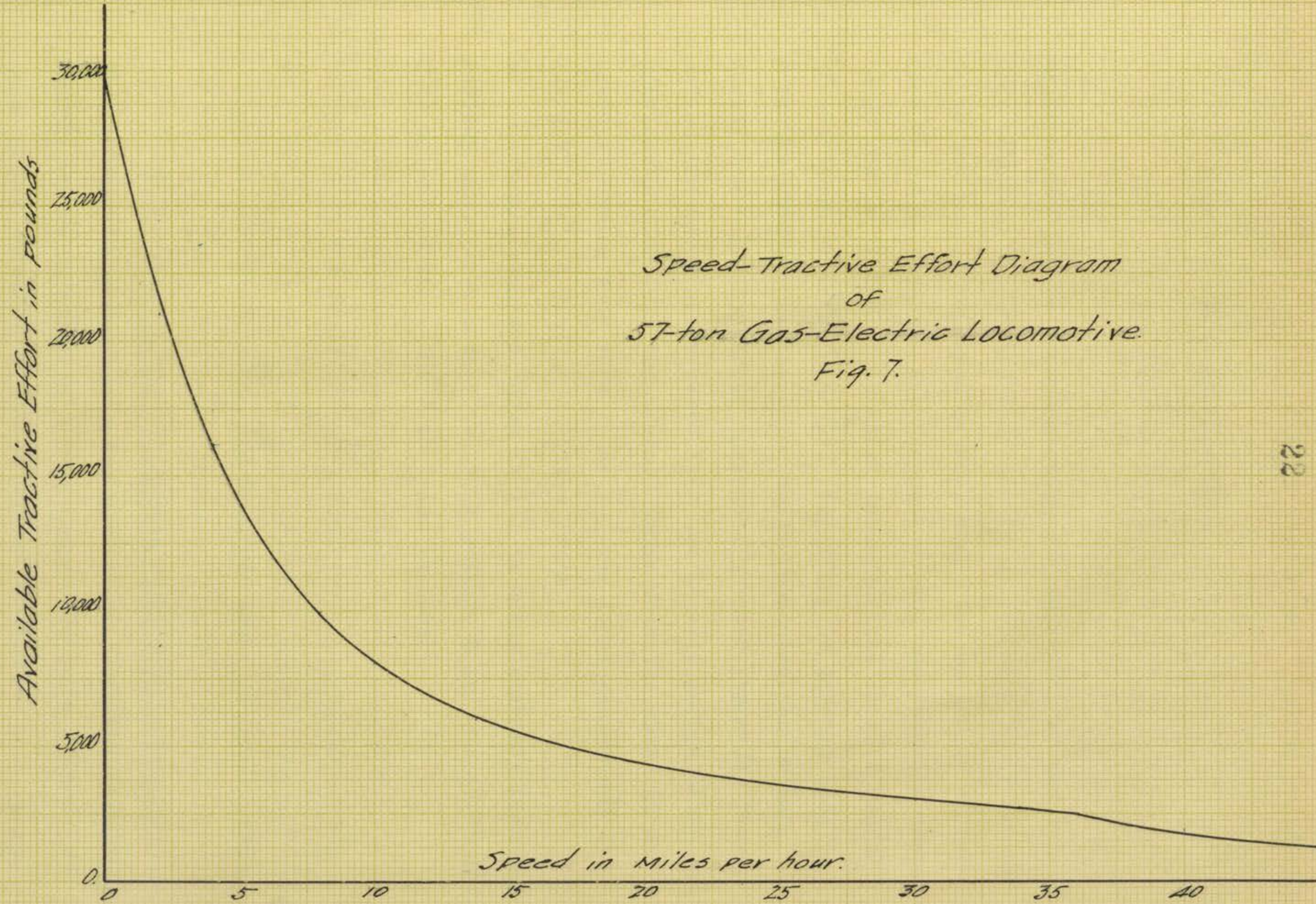
$$K_t = \frac{1}{2}M(V_1^2 - V_2^2)$$

and that, by the principle of the conservation of energy, it is equivalent to the work done, FS.

Characteristic Curves of Electric Railway Motors.



75 H.P. 500 volts Motor. Fig. 6. 100 H.P. 600 volts Motor.



Speed-Tractive Effort Diagram
of
57-ton Gas-Electric Locomotive.
Fig. 7.

Then $$FS = \frac{1}{2}M(V_1^2 - V_2^2)$$

or $$F = \frac{1}{2}M(\frac{V_1^2 - V_2^2}{S}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Equation (1) may be interpreted as follows: (1) if the initial velocity V_1 is less than the final velocity V_2, F is the force constantly required to accelerate the motion of a train from V_1 to V_2 when the displacement during the velocity change is S, and it is the force which increases the kinetic energy of the train by $1/2M(V_2^2 - V_1^2)$; (2) if V_1 is greater than V_2, F is the kinetic inertia force which maintains the motion during the velocity change against a greater resisting force, and is the force transformed from the kinetic energy which was previously stored in the train. Thus, when the speed of a train is coming down from V_1 to V_2, the force equivalent to $1/2\ M(V_1^2 - V_2^2)/S$ acts as a motive force.

Expressing the terms in (1) in units commonly used in railway engineering, that is, F in pounds, M in tons (2000 lb.), V in miles per hour, and S in feet, we have

$$F = 66.7W(\frac{V_1^2 - V_2^2}{S}) \text{ lbs.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or

$$F = 66.7(\frac{V_1^2 - V_2^2}{S}) \text{ lbs. per ton} \quad \ldots\ldots\ldots (2')$$

2. Kinetic energy or inertia force due to rotation. - In the preceding chapter it was also shown that the kinetic energy due to rotation of a body around the axis through its mass center could be expressed by the formula,

$$K_r = \frac{1}{2}M'k^2(\underline{W}_1^2 - \underline{W}_2^2)$$

and that it was equivalent to the work done, FS or F r, if F is the tangential force whose point of application is at r from the axis of rotation, then

$$FS = \frac{1}{2}M'k^2(\underline{W}_1^2 - \underline{W}_2^2)$$

$$F = \frac{1}{2}M'\frac{k^2}{r^2}r^2(\frac{\underline{W}_1^2 - \underline{W}_2^2}{S})$$

$$= \frac{1}{2}M'\frac{k^2}{r^2}(\frac{V_1^2 - V_2^2}{S}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

or in units used in railway engineering,

$$F = 66.7\ W'\frac{k^2}{r^2}(\frac{V_1^2 - V_2^2}{S})\ \text{lbs.} \quad \ldots\ldots\ldots\ldots \quad (4)$$

or

$$F = 66.7\frac{k^2}{r^2}(\frac{V_1^2 - V_2^2}{S})\ \text{lbs. per ton} \quad \ldots\ldots\ldots \quad (4')$$

3. Total kinetic inertia force of a train. - When a railway train is in motion its entire mass is in translation, but certain parts, such as wheels, axles, etc. are in rotation as well as in translation, and have additional kinetic energy due to their rotation, which amounts, roughly speaking, to about five percent of the energy due to translation in ordinary trains and about ten percent in street cars and motor car trains.

Let us consider the case of steam or electric locomotive trains (excluding the locomotive and tender) and suppose the weight of the train is W and the weight of the parts which rotate is W'. Then the total kinetic inertia force of the train is equal to the sum expressed by the equations (2) and (3), or

$$F = 66.7W\left(\frac{V_1^2 - V_2^2}{S}\right) + 66.7W'\frac{k^2}{r^2}\left(\frac{V_1^2 - V_2^2}{S}\right) \text{ lbs.}$$

But, the linear velocity and the displacement of a point on the periphery or the tread of the car wheels are necessarily equal to the velocity and the displacement of the train respectively, so we have

$$F = \left(66.7W + 66.7W'\frac{k^2}{r^2}\right)\frac{V_1^2 - V_2^2}{S} \quad \text{lbs.}$$

or $$F = \left(66.7 + 66.7\frac{W'}{W}\frac{k^2}{r^2}\right)\frac{V_1^2 - V_2^2}{S} \text{ lbs. per ton} \ldots (5)$$

The average weight of a pair of wheels and their axle is about 1950 lb.,* then

$$W' = 1950\ nN \text{ lb.},$$

in which N is the number of cars in the train and n the average number of axles per car. If n = 4,

$$W' = 3.9N \text{ tons.}$$

The average value of k/r for a pair of wheels and their axle

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* E. C. Schmidt: "Freight Train Resistance," Engineering Experiment Station, University of Illinois, Bulletin No.43, p.89. 1910.

of various design is found to be about 0.64,* and consequently

$$F = (66.7 + 66.7 \times 3.9 \times 0.64^2 \times \frac{N}{W})\frac{V_1^2 - V_2^2}{S} \text{ lbs. per ton,}$$

or

$$F = (66.7 + 106.7\frac{N}{W})\frac{V_1^2 - V_2^2}{S} \text{ lbs. per ton} \quad \ldots (6)$$

This formula may be expressed in terms of a constant acceleration. Since $(v_1^2 - v_2^2) = 2\int_{V_2}^{V_1} v\,dv$ and $2\int v\,dv = v^2$, equation (6) becomes

$$F = (66.7 + 106.7\frac{N}{W})\frac{V^2}{S}$$

but since $V = at\ (3600 \div 5280)^2$ and $S = 1/2\ at^2$, and $a = (5280 \div 3600)A$ miles per hour per second, we get from above equation

$$F = (91.08 + 145.5\frac{N}{W})\ A. \text{ lbs. per ton} \quad \ldots\ldots (7)$$

Formulas (6) and (7) are accurate and may be used in the computations of train resistance investigations,* etc. In certain cases, however, the following approximate formulas** will give

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* E. C. Schmidt, "Freight Train Resistance", Bul. 43, p. 90.

** G. R. Henderson, "Locomotive Operation", 2nd ed., p. 2-10;
A.M.Wellington, in his "Economic Theory of Railway Location", states: "Estimating ordinary car wheels to weigh 2.25 tons per 8-wheeled car, or 561 pounds per wheel, the ratio of the weight of the wheels to the total weight will be about:

	In a Passenger or Loaded Freight cars.	In an Empty Freight Cars	In Locomotive and Tender.
Weighing	22.5 tons	9. tons	--
Percent of weight of wheels	10.0 p.c.	25. p.c.	10 - 12.5
Making an addition to the total energy of the train of about	5.0 p.c.	12.5 p.c.	6 - 7.5 p.c.

From this data, he concludes that 6 percent to be the average percentage of rotating energy to the total.

sufficiently accurage values of F:

$$F = 70\left(\frac{V_1^2 - V_2^2}{S}\right) \text{ lbs. per ton} \quad \ldots\ldots\ldots\ldots \quad (6')$$

$$F = 95.5A \text{ lbs. per ton} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7')$$

and for electric cars*

$$F = 100A \text{ lbs. per ton} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7'')$$

D. Potential Energy or Gravitational Force of Trains.

A railway train at a certain elevation does work, by the action of gravitational force or due to its potential energy, when it is allowed to come down to a lower elevation, and vice versa. The change of potential energy or the work done on the train is measured by

$$W(h_1 - h_2),$$

where W denotes the weight of a train, and $(h_1 - h_2)$ the change in its elevation. In this formula the direction of W is always vertical, but its component along the direction of motion

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* A. M. Buck: "The Electric Railway", p. 16.

A. H. Armstrong: "Electric Traction", Standard Handbook for Electrical Engineers, 3rd ed., p. 971-974. He gives:

Percent of Total Tractive Effort Consumed in Rotating Parts.

Electric locomotive and heavy freight train	....	5.%
" " " high speed passenger train	...	7.%
" high speed motor cars	....	7.%
" low " " "	....	10.%

of the train, say F, is readily found thus:

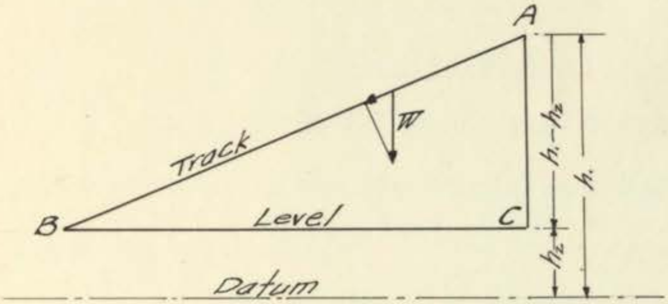

Fig. 8.

Fx(Displacement along the track, AB) = $W(h_1 - h_2)$,

then

$$F = \frac{(h_1 - h_2)}{AB} W,$$

$$F = \frac{AC}{AB} W.$$

But, when the angle ABC is very small, as is the case on most railroad tracks, the length AB is practically equal to BC, then

$$F = \frac{AC}{BC} W.$$

Expressing F in lbs., W in tons, and the gradient AC/BC as G, in percent, we get from above equation

$$F = \frac{G}{100} \times 2000W$$

$$= 20\ WG \text{ lbs.},$$

or $$= 20G \text{ lbs. per ton} \quad \ldots\ldots\ldots\ldots \quad (8).$$

If, however, the gradient is expressed in G' feet per mile, we have

$$F = \frac{G}{5280} \times 200\ W,$$

$$= 0.379WG' \text{ lbs.},$$

or $$= 0.379\ G' \text{ lbs. per ton} \quad \ldots\ldots\ldots \quad (8')$$

The force F of formulas (8) and (8') acts as motive force when the train is on a decending grade, but acts as a resisting force when it is on an ascending grade, and in the latter case it is called "grade resistance".

E. Force of Wind.

A wind blowing in the direction of motion of a train, acts also as a motive force and its magnitude may be computed from the formula,

$$F_w = 0.0025A(V_w^2)$$

in which V_w is the component of the wind velocity parallel to the direction of motion of the train in miles per hour, and A the exposed area in square feet of the rear end of the train. In ordinary operating conditions the value of F_w lbs. per ton is insignificant compared with other motive forces, and besides its aid is so uncertain that we can never rely upon it.

IV. TRACTIVE EFFORT OF STEAM LOCOMOTIVES.

A. Processes in Steam Locomotives.
B. A Study of Tractive Effort Formulas.
   1. Fundamental formulas.
      a). $T = cpld^2/D$.
      b). $T = 375(H.P.)/V$
   2. Formula based on $T = cpld^2/D$.
      a). Speed factor methods.
      b). Improved speed factor methods.
         1). New Baldwin method.
         2). Kiesel's formula
         3). Strahl's formula
         4). Young's method.
   3. Formula based on $T = 375(H.P.)/V$.
      a). Goss' formula and Houston's modification.
      b). Troske's formula.
      c). Williamson's formula.
      d).Houston's formula.
      e). Shutleff's or A.R.E.A. method.
   4. Comparison of the results of the formula with test curves.

A. Processes in Steam Locomotives.

The steam locomotive is a complete power plant in which the chemical energy stored by nature in fuels - usually coal - is transformed, by the combustion of the coal in the furnace, into thermal energy which is absorbed by the boiler water through the heating surfaces; and the steam thus generated is transferred to the engine cylinders after being perhaps superheated,where the thermal energy is transformed into mechanical energy and then transmitted to the drawbar through a certain mechanism. Any factor or element involved in these long and complicated processes of transformation and transmission, therefore, must have some influence on the final value of the tractive effort or drawbar pull of a steam locomotive.

B. A Study of Tractive Effort Formulas.

1. Fundamental formulas. - Let T denote the tractive effort, that is, the force exerted by the steam in the cylinders of a locomotive measured at the periphery or tread of the driving wheels, in pounds; D the diameter of the drivers, in inches; p the boiler pressure in lbs. per sq. in.; l the length of stroke of the piston, in inches; and c a certain constant. Then, since the work done by the steam in the two cylinders (simple, two-cylinder locomotives) during each two strokes of a piston is equal to the work done by the driving wheels during one revolution, we have

$$T\pi D = 2(cp \frac{\pi d^2}{4} (2l))$$

$$T = c\frac{pld^2}{D} \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(I)}$$

Another fundamental formula or relation is derived from the definition of horsepower, H.P., that is, one horsepower is 550 foot-pounds per second. Then, the horsepower of a locomotive, whose tractive effort is T in pounds and whose velocity is V in miles per hour, is

$$H.P. = \frac{5280}{3600} T \frac{1}{550} ,$$

or $$T = \frac{375(H.P.)}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

These two formulas, as they stand now, are perfectly rational being derived from rigid definitions and laws of mechanics. They are, however, of little practical value unless the arbitrary constant c in (I) or (H.P.) in (II) is expressed in terms of variables of which c or (H.P.) is a function. In fact, the different formulas and methods for estimating or predicting the tractive effort of a steam locomotive differ only in this regard.

2. Formulas based on the relation, $T = cpld^2/D$. - Without any exception, all the tractive effort formulas published before 1900, are in the form of $T = cpld^2/D$. To this class belong the following formulas:

1. Henderson (Master Mechanics') formula.
2. Henderson's Formula (Locomotive Operation).
3. Henderson's Hyperbolic Formula.
4. Old Schenectady Locomotive Works Formula.
5. University of Illinois Formula.
6. Adams and Isaac's Formula.
7. Cole's Formula (American Locomotive Company).

These formulas differ only in the value of c, which, being assumed to vary only with speed, is called the "speed factor" of T, $(T = pld^2/D)$. The speed factors were obtained (1) from actual tests, usually with a single locomotive although in some cases with more, (2) by combining the results arrived at by previous investigators, and deducing from them an average value for different speeds. For many years some of these formulas were popularly employed, probably on account of their simplicity in calculation, but partly for lack of better ones. The speed factor methods are rather unpopular today. This, however, is not the fault of the fundamental formula they are based upon, but of substituting the speed factor for the arbitrary constant c, which is a function not only of speed but also many other important factors.

With this fact in view the two following improved formulas were recently developed and published:

8. New Baldwin Formula*.

9. Kiesel's Formula**.

In the new Baldwin formula, the arbitrary constant, c is considered as a function of speed and also of the ratio of rated tractive effort to heating surface, while in Kiesel's formula c is a function of heating surface, evaporation ratio and weight per cubic foot of steam under the condition of initial pressure. These two formulas, being the best developed (except one or two later to be discussed) among the formulas of this class, are generally found to give more satisfactory results than any of the other formulas mentioned above.

In 1913, a prominent German railway engineer published*** a tractive effort formula, which is claimed to agree well with

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* Baldwin Locomotive Works: Locomotive Data, p. 14, (1914)
** Wood: "Locomotive Operation and Train Control", p. 26.
*** Strahl: "Verfahren zur Bestimmung der Belastungsgrenzen der Dampflokomotiven." Zeit. d. Ver. deut. Ing. vol. 57 p. 251, (1913).

the performance of European locomotives. He assumed that the variation of tractive effort of any locomotive with speed could be expressed by the following empirical formula (simply an algebraic expression for speed factor);

$$\frac{Z_i}{Z_i'} = 0.6(2 - x) + \frac{0.4}{x}$$

in which Z_i is the cylinder tractive effort at any speed v, Z_i' the same at v', the speed at which the water rate is least, or in other words, the speed at which the locomotive develops the maximum horsepower, since he assumes that the total evaporation per hour of a locomotive is constant; and x represents the ratio, v:v'. The evaporation in a locomotive boiler per hour is estimated by the following formula:

$$\frac{Q}{R} = \frac{a}{1 + (bR/H)}$$

where Q represents the total evaporation in kg. per hr., R the area of grate surface in sq. meters, H the heating surface in sq. meters, b is a constant found to be 7 for any locomotive, and a an arbitrary constant which has the following values:

a = 3800, for superheated steam locomotives,

a = 4000, for two-cylinder compound engine saturated steam locomotives,

a = 4250, for any other saturated steam locomotive.

He gives the following mean values of steam consumption per metric horsepower-hour at the maximum power of locomotives of different types:

For saturated steam simple engine locomotives	11.5 kg.
" " " 2-cylinder compound engine locomotives	9.75 kg.
For saturated steam 4-cylinder compound engine locomotives	9.5 kg.
For superheated steam two or four cylinder simple	6.75 kg.
" " " four cylinder compound	6.2 kg.

With this data and the above formula the normal maximum horse-power could be determined. Next he gives the following values as the mean effective pressure at the maximum power of locomotives:

p_m = 3.6 kg. per sq. c.m. for two or four cylinder simple locomotives.

p_m = 3.4 kg. per sq. c.m. for any compound locomotive.

With this data and the ordinary formula, $Z_i^!$ can be determined; and knowing this value and the maximum horse power of the locomotive, the speed v' is calculated. Then, the cylinder tractive effort at any speed can be computed from the formula first mentioned.

Mr. E. G. Young*, in 1916, presented a new method which in many respects made a radical departure from any previous method, although it was fundamentally based upon the formula, $T = cpld^2/D$. After a painstaking preparatory study he found a certain relation between the ratios T.E:t and To:E. for different piston speeds, where T.E. is the "cylinder tractive effort", T_o the theoretical tractive effort or pld^2/D, and E the equivalent hourly evaporation, and t the T.E. at

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* Thesis for M.S. in Ry. M.E., University of Illinois, 1916.

600 feet per minute piston speed, found from other relations between t/To and To/E. Although it was not stated, the arbitrary constant, c is regarded by him as a function of t, To, E, and V, or

$$c = f(t, To, E, V).$$

For some reason the function was not given in any mathematical equation, but in the form of diagrams. The value of E is to be estimated separately by means of other diagrams representing the relation between the equivalent hourly evaporation per pound of 14500 B.t.u. coal fired and the length of the boiler tubes for various firing rates referred to the "equvalent grate area". To make allowance for variations in evaporating power of different grades of coal, he multiplies the value of the equivalent evaporation per hour for 14500 B.t.u. coal by the "coal factor" obtained from a certain formula. As previously stated, Mr. Young, in attacking his complicated problem and developing the new method, employed several new and ingenious schemes, especially the method of finding the relation of T.E. to t, To, and E by inter-relating the ratios T.E.:T and To:E. The reader of his thesis may notice that he sometimes made rather questionable assumptions or adopted some very boldly averaged mean values in very important places. This was necessary in order to express simply matters essentially very complex, and when we see the close agreement of his estimated values and the experimental data, we do not only find his method justified, but esteem his good judgement.

3. <u>Formulas based on the relation, T = 375(H.P.)/V.</u> - In 1901, appeared the first tractive effort formula* in the form of T = 375(H.P.)/V. It is given as

$$T = 116 \frac{H}{V} \quad .... \quad (12)$$

The author of this formula, with detailed data of laboratory tests of locomotive performance, and keeping in mind all the factors which have any influence on the horsepower of a steam locomotive, but eliminating insignificant elements, arrived at the conclusion that one square foot of heating surface** is equivalent to 0.43 horsepower. Substituting this value for H.P. in equation II, he obtained the above formula (12). The curve represented by this formula is a hyperbola and it closely represents the tractive effort at any speed except very low speeds. On account of the limited adhesion of the drivers, the curve representing the tractive effort at low speeds must be replaced by a curve or a straight line representing the adhesion until it meets the hyperbola. This formula with the numerical constant as shown above was originally given to represent the speed-tractive effort (cylinder) relation of the locomotive, "Schenectady II", and also that of locomotives currently used about 1900. By proper adjustment of the numerical constant, however, this formula gives quite satisfactory results for any steam locomotive at any date. Mr. H. A. Houston# in order to make this formula readily applicable to the

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* W.F.M. Goss: "Locomotive Performance", p. 413.
** See: Footnote on p. 52, "Locomotive Performance".
# Thesis for M.S. in Ry. M.E., Univ. of Illinois, 1913.

locomotives of later design and greater capacity, adopted 143 instead of 116.

Troske* published, in 1907, his formula:

$$T = 270 \frac{aH}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

in which T is the tractive force of a locomotive based upon boiler capacity, H the heating surface, V the speed, and a an arbitrary constant which varies with the speed, heating surface, thermal value of fuel, dimensions of exhaust nozzle and stack, method of using steam (that is, simple or compound engines) and kind of steam, i.e. superheated or saturated steam.

Mr. A. S. Williamson published a very interesting article as the result of his study of this subject.** From the experimental data of two consolidation locomotives tested at St. Louis, and the Purdue Tests, he produced the expression for water rate of simple saturated steam locomotives, $S = 1500/R + .09 R$, in which S represents the steam consumption in lbs. per indicated horsepower hour, and R the number of revolutions of driving wheels per minute. After study of the results of Shurtleff's investigation made on the evaporation in locomotive boilers, he assumed that the evaporation was equal to 0.68 pound of water for each 1000 B.t.u. in the coal fired, for each square foot of heating surface. That is, if E denotes the equivalent evaporation, B the amount of B.t.u. in the coal fired per hour, and H the heating surface (fire side), $E = 0.68BH$.

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* Allegemeine Eisenbahnkunde, vol. 2., p. 124.

** Railway Age Gazette, March 22, 1912, p. 685.

Then, since H.P. = E/S,

$$T = 375\,\frac{(H.P.)}{V} = \frac{375E}{VS},$$

and substituting 0.68BH for E, he got

$$T = \frac{375 \times 0.68BH}{VS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which, as mentioned before, S = 1500/R+ 0.09R.

In 1913, Mr. Houston* presented a method of estimating the maximum tractive effort of a steam locomotive. After a study of the results of the St. Louis, Purdue, and Altoona tests, he produced a table giving the equivalent evaporation per square foot of heating surface per hour or "fair value of a maximum rate of equivalent evaporation for any locomotive" corresponding to different grades of coal. Knowing the thermal value of the coal to be fired and the heating surface of the locomotive, the equivalent hourly evaporation of any locomotive, E can be estimated by this table. Similarly, he found the formula for water rate of any simple locomotive, $S = 34.0 - 0.1R + 0.00025R^2$, where S is the steam consumption per indicated horsepower hour, and R the number of revolutions per minute. Then assuming the factor of evaporation as 1.2069, he got,

$$H.P. = \frac{EH}{FS} \quad \text{or} \quad \frac{EH}{1.2069S},$$

and substituting this value for H.P. in the fundamental relation II, he finally obtained

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* Thesis for M.S. in Ry. M.E., University of Illinois, 1913.

$$T = \frac{310.7\ EH}{VS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where $S = 34.0 - 0.1R + 0.00025R^2$.

A. K. Shurtleff*, in 1910, as the chairman of the Committee on Economics of Location of the American Railway Engineering Association, presented a method, which being modified slightly and adopted by the association is known as the A.R.E.A. Method.** In this method a rate of firing of 4000 pounds of dry coal per hour is assumed and there are given three important tables of data: (1) Actual evaporation per pound of dry coal fired for different firing rates - dry coal fired per sq. ft. of heating surface, lbs. per hr. - and for different thermal values of dry coal; (2) Steam used per foot of stroke, at full cut-off, for various steam pressures and for different cylinder diameters; (3) Steam consumption per indicated horsepower-hour for various speeds in "M" units. The procedure in computing the speed-pull relation of a locomotive by this method is as follows: Dividing the assumed 4000 lbs. of coal by the heating surface of the locomotive under consideration, the firing rate is found. Knowing this firing rate and the thermal value of the dry coal, from the first table, the actual evaporation per pound of dry coal is found; and multiplying by 4000 (lbs.), the actual hourly evaporation of the locomotive is found. With this evaporation and the value found in the second table, which gives steam used per foot of stroke, the piston speed in feet per hour which can be attained before the boiler pressure

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* Proc. A.R.E.A. vol. 12, Pt. 1, p. 631, 709, and also Manual of A.R.E.A 1911 ed., p. 427.

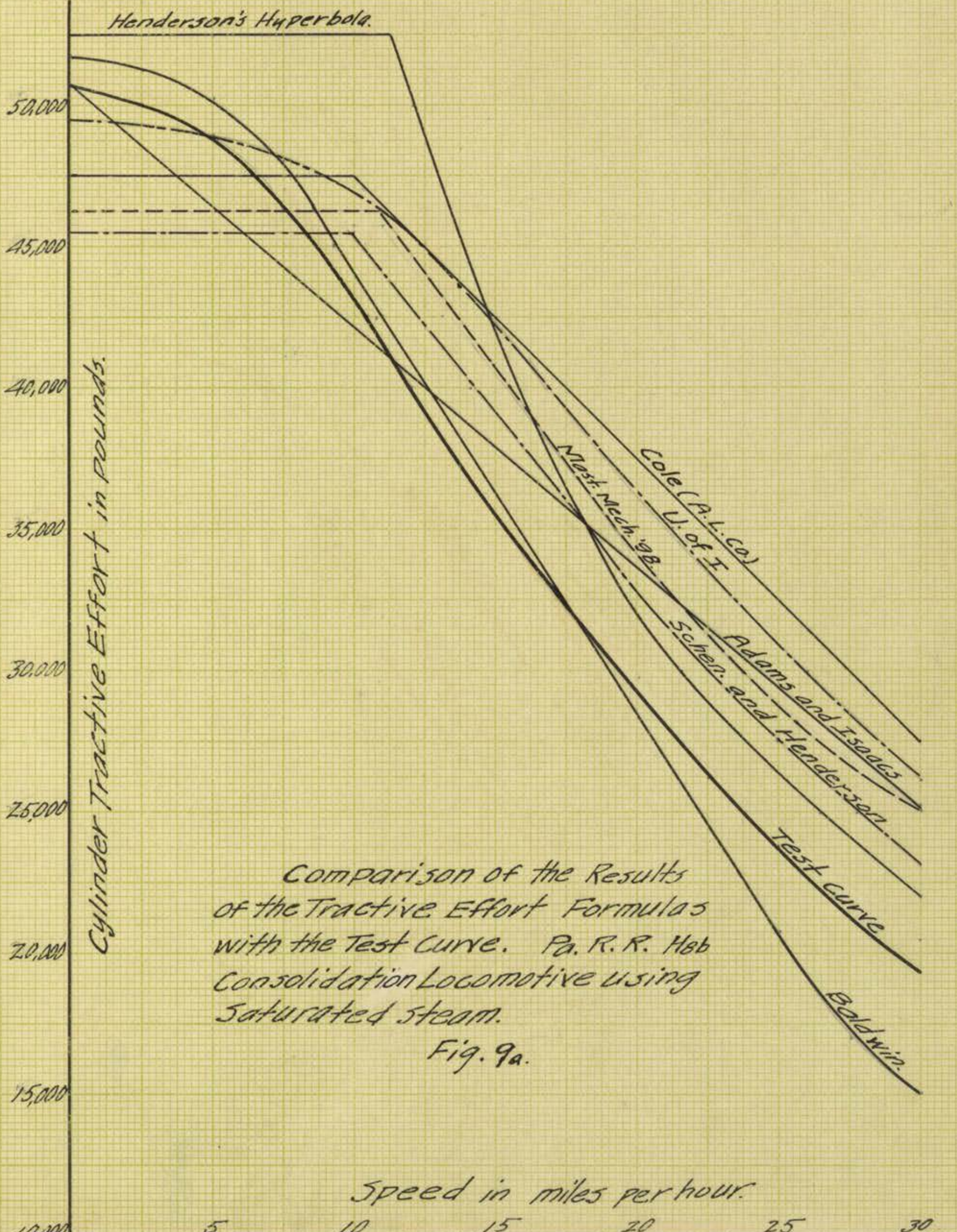
** Proc. A.R.E.A. v.15, p.138. Manual A.R.E.A. 1915 ed., p. 526.

will begin to fall or where the cut-off must be reduced in order to keep the boiler pressure normal, may be found. This transition speed is called "M". Dividing the value of the total hourly evaporation by the values given in the third table, the horsepower at different multiples of M can be found. Then, transforming M into speed in miles per hour the tractive effort at various speeds may be computed readily by means of the formula,

$T = 375(H.P.)/V$.

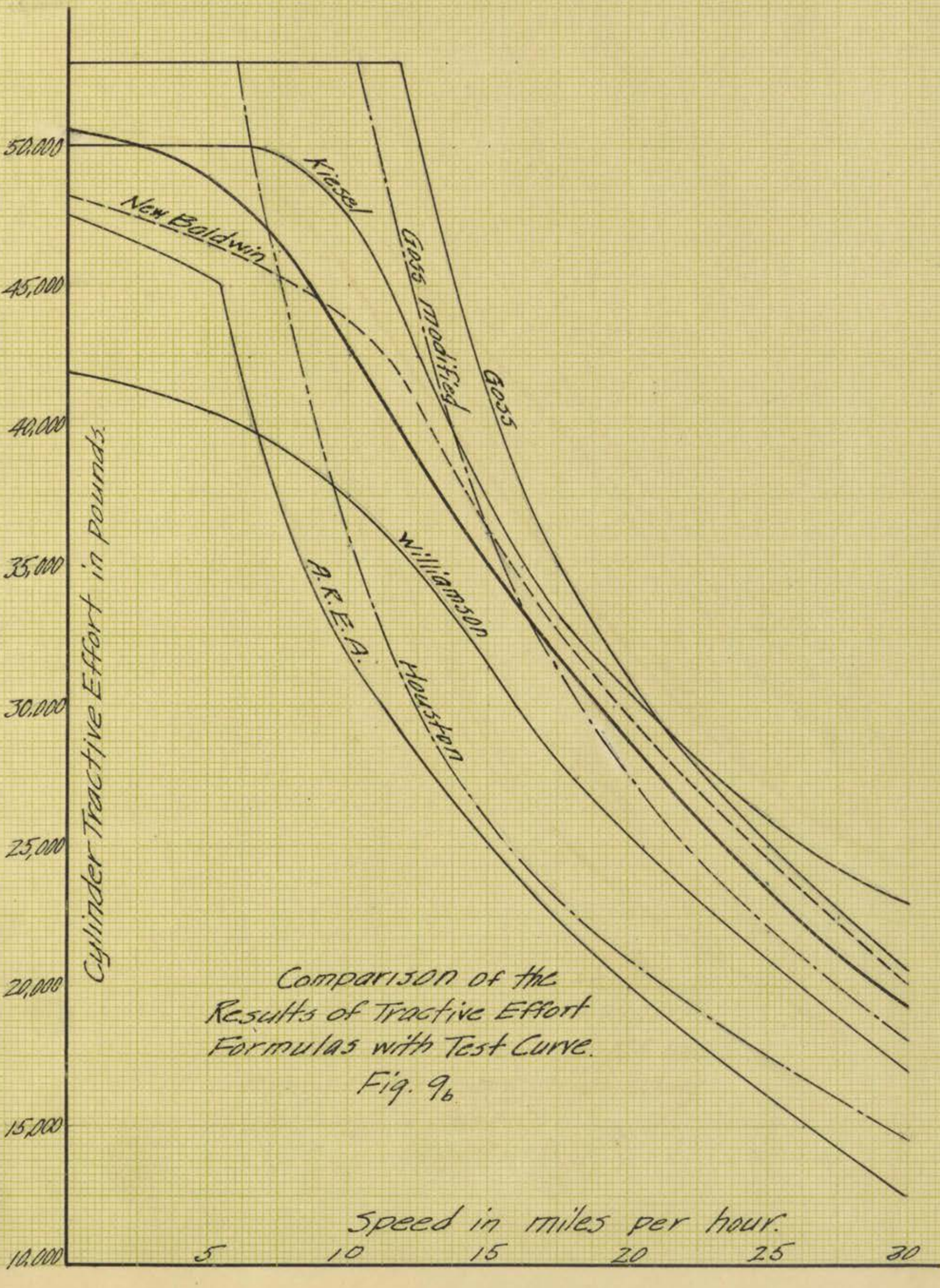
The process just described applies only to locomotives using saturated steam. For locomotives using superheated steam, the speed-tractive effort relation is computed by first finding the relation assuming the locomotive to use saturated steam and then multiplying the values given in the fourth table, which has been deduced from the principle that tractive efforts of saturated and superheated steam locomotives of like general dimensions have definite ratios at various speeds.

4. <u>Comparison of the results of the formulas with test Curves</u>. - Curves derived by means of these various formulas are presented in Figs. 9a, 9b, 9c, which permit comparisons to be drawn among the different methods.

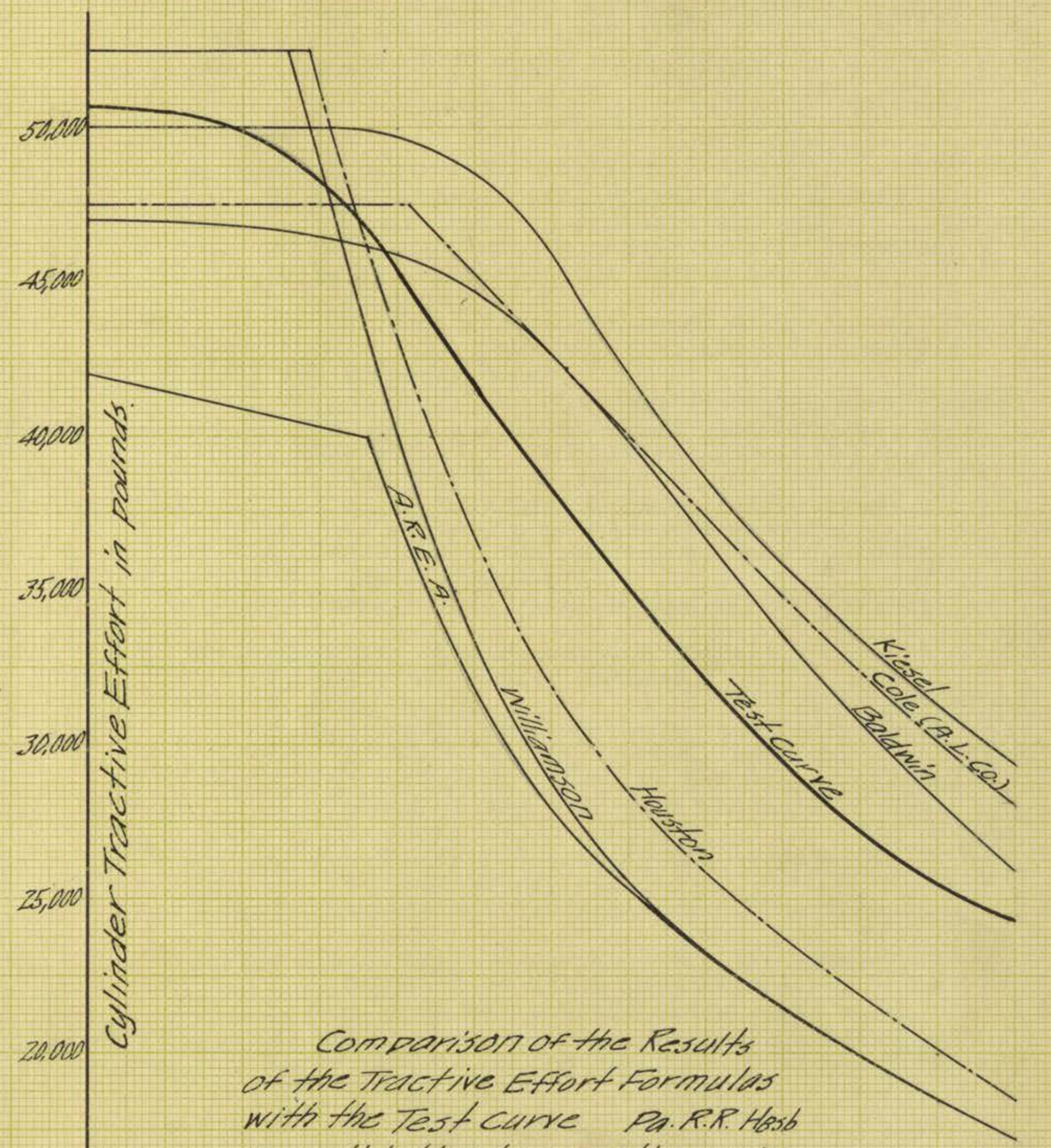


Comparison of the Results of the Tractive Effort Formulas with the Test Curve. Pa. R.R. H8b Consolidation Locomotive using Saturated steam.

Fig. 9a.



Comparison of the Results of Tractive Effort Formulas with Test Curve.
Fig. 9b



Comparison of the Results of the Tractive Effort Formulas with the Test Curve Pa. R.R. H8sb Consolidation Locomotive using Superheated Steam.

Fig. 9c.

V. DERIVATION OF FORMULAS FOR THE SPEED-TRACTIVE EFFORT RELATIONS OF STEAM LOCOMOTIVES.

A. Speed-Drawbar Pull Relations.

1. <u>Speed-pull relation on a semi-logarithmic co-ordinate system</u>. - In spite of the fact that, as mentioned before,

the experimental data of steam locomotive performance are not adequate for a complete generalization or for the deduction of a general law governing the speed-pull relation of the locomotive, the representation of the available data on a semi-logarithmic co-ordinate system* reveals the fact that this relation can be represented remarkably well by two straight lines on that co-ordinate system. This fact is fully demonstrated by the diagrams on Figs. 9 - 24 inclusive. The points plotted are taken from the original data published in the Locomotive Test Plant bulletins** of the Pennsylvania Railroad Co., and the straight lines are those drawn through each set of points for individual locomotives named. Some discordance between the lines and the points may be found in the diagrams, but a careful study of the curves in the bulletins, and comparison of the curves and the straight lines on our diagrams will justify the position of our lines. Accepting, then, this generalization as correct, we will next derive the mathematical expression of the law it implies.

2. <u>Derivation of the characteristic formulas</u>. - A straight line on any co-ordinate system is completely defined by two points on the line, or by a point and the slope of the line. Then, the speed-pull relation of a locomotive, too, can be completely represented on a semi-logarithmic co-ordinate

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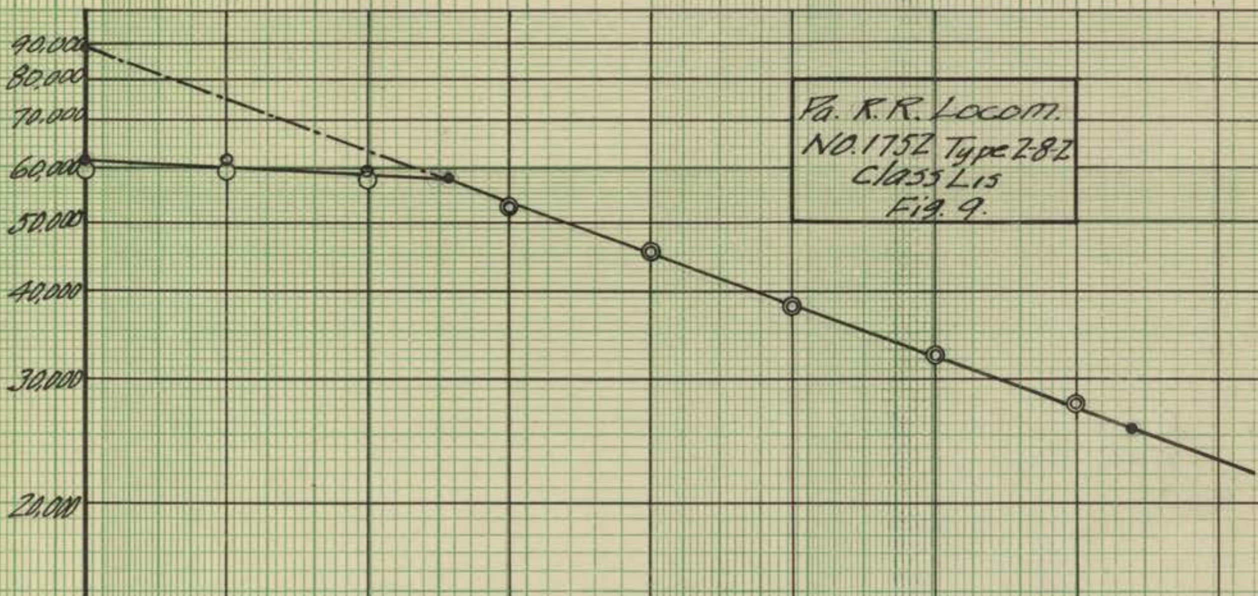
* For theory of sem-logarithmic co-ordinate system, see Mechanical Engineers' Handbook, L. S. Marks, editor-in-chief, p. 177.

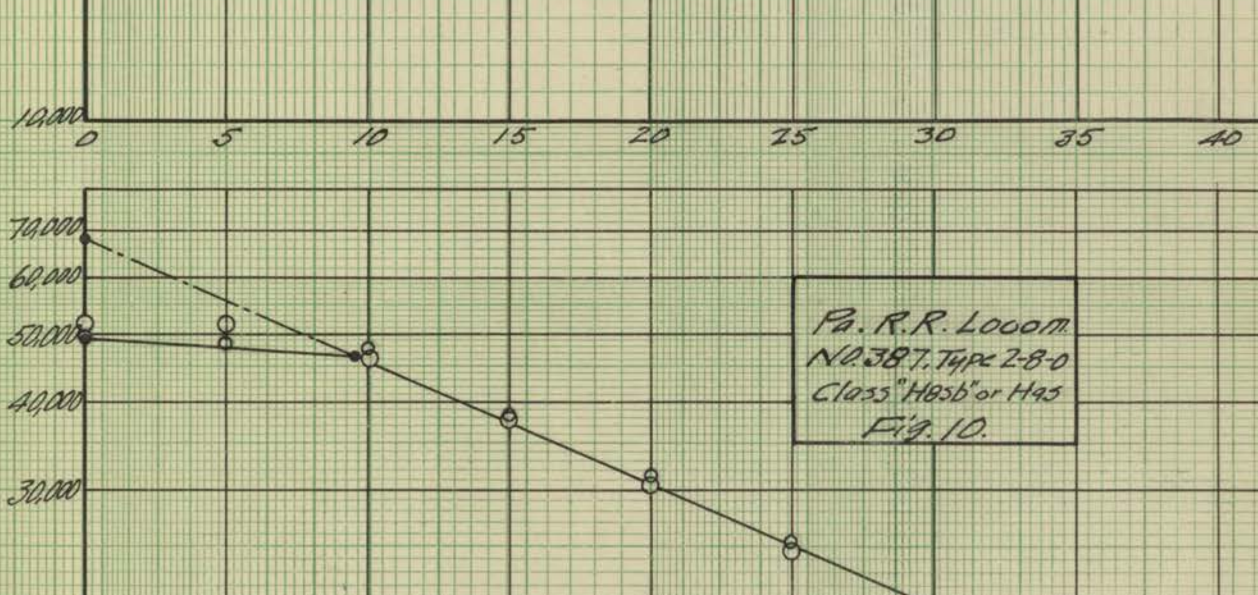
** The data are shown in the Appendix, Tables Ia, Ib, Ic, and Id, and also represented on cartesian co-ordinates in Fig. 2 and 3.

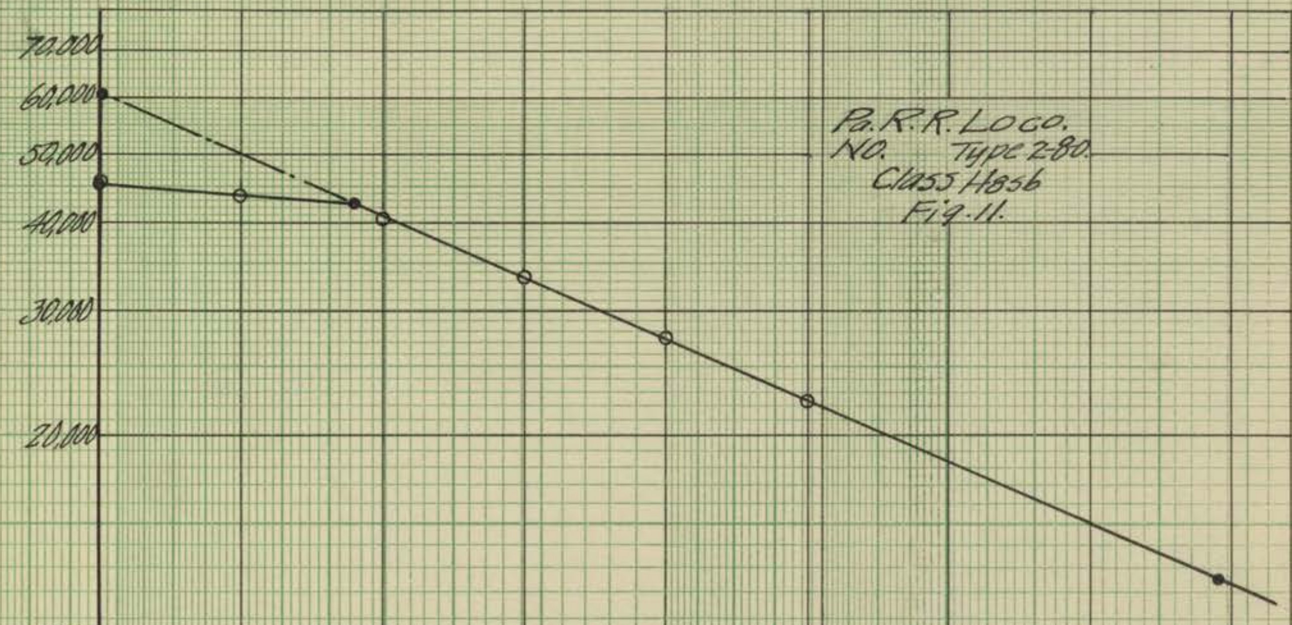
SPEED-PULL RELATIONS ON SEMI-LOGARITHMIC COORDINATE SYSTEM.

FIG. 9 – 24.

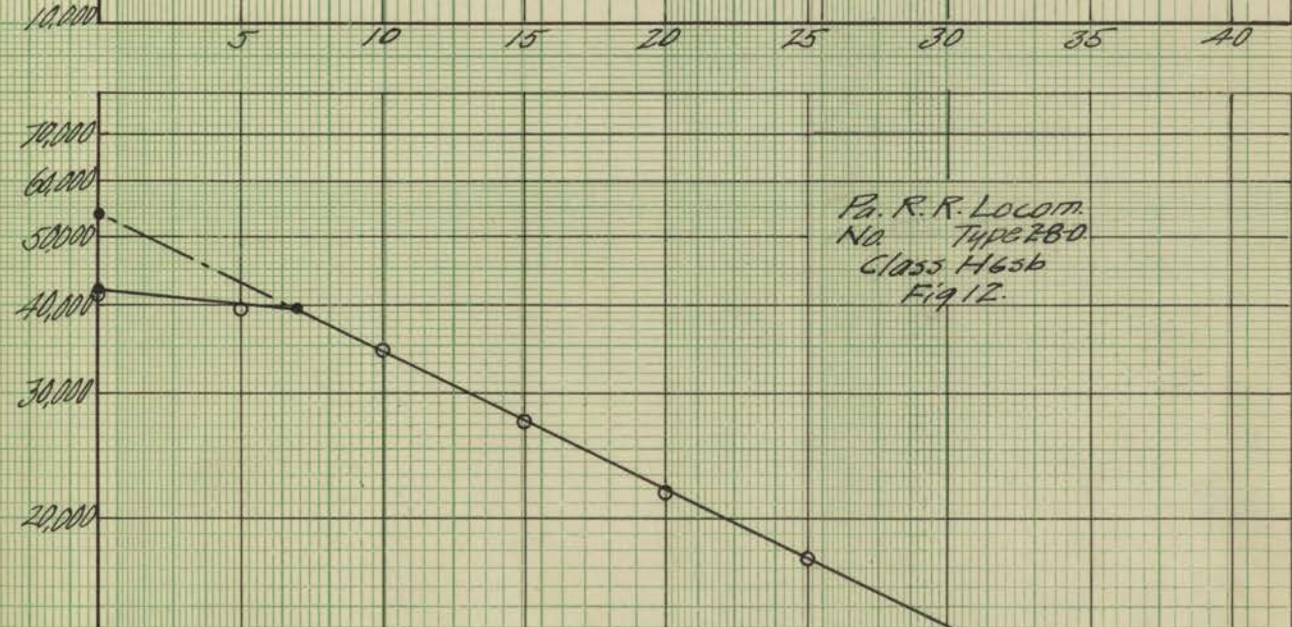
Ordinate: Drawbar Pull behind Tender in lbs.
Abscissa: Speed in miles per hour. (Natural scale).



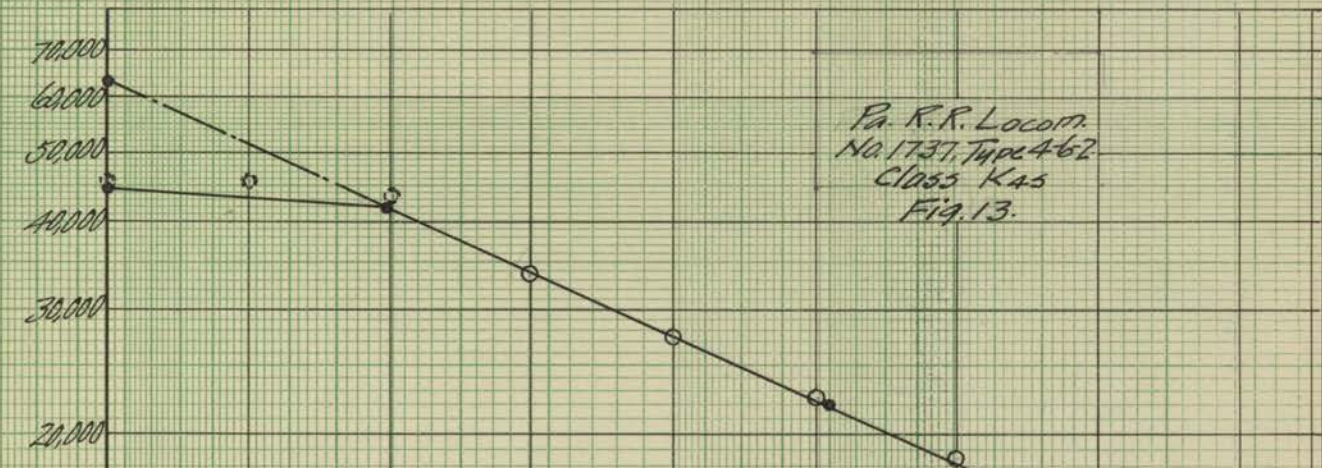


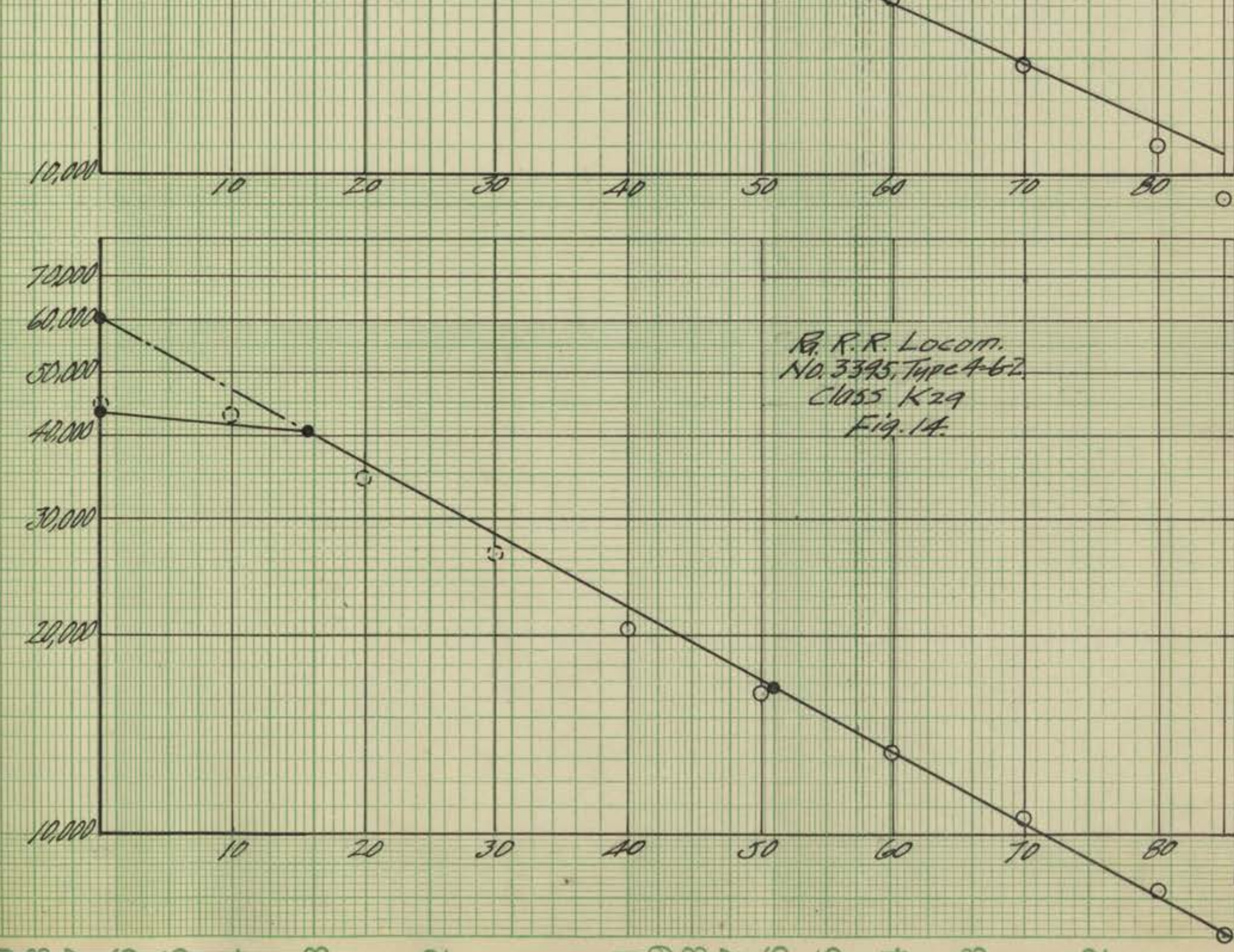


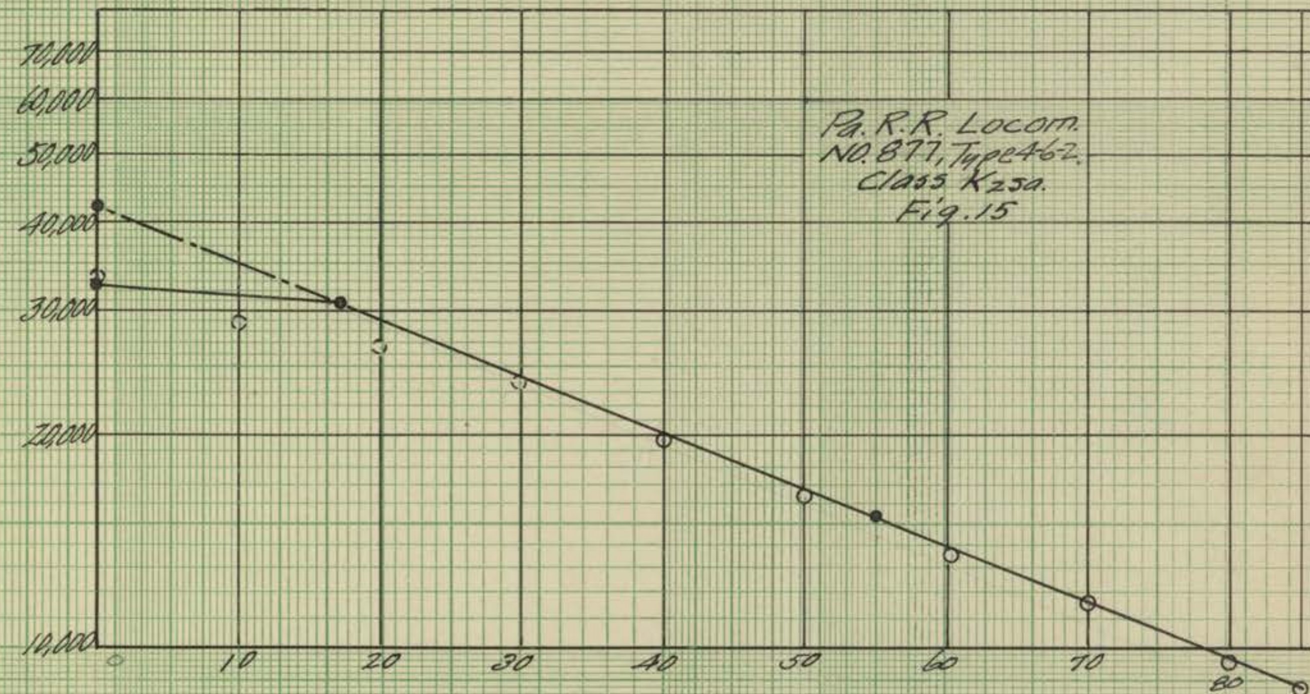
70,000
60,000
50,000
40,000
30,000
20,000
10,000
5
10
15
20
25
30
35
40
Pa. R.R. Loco.
No. Type 2-8-0.
Class H8sb
Fig. 11.

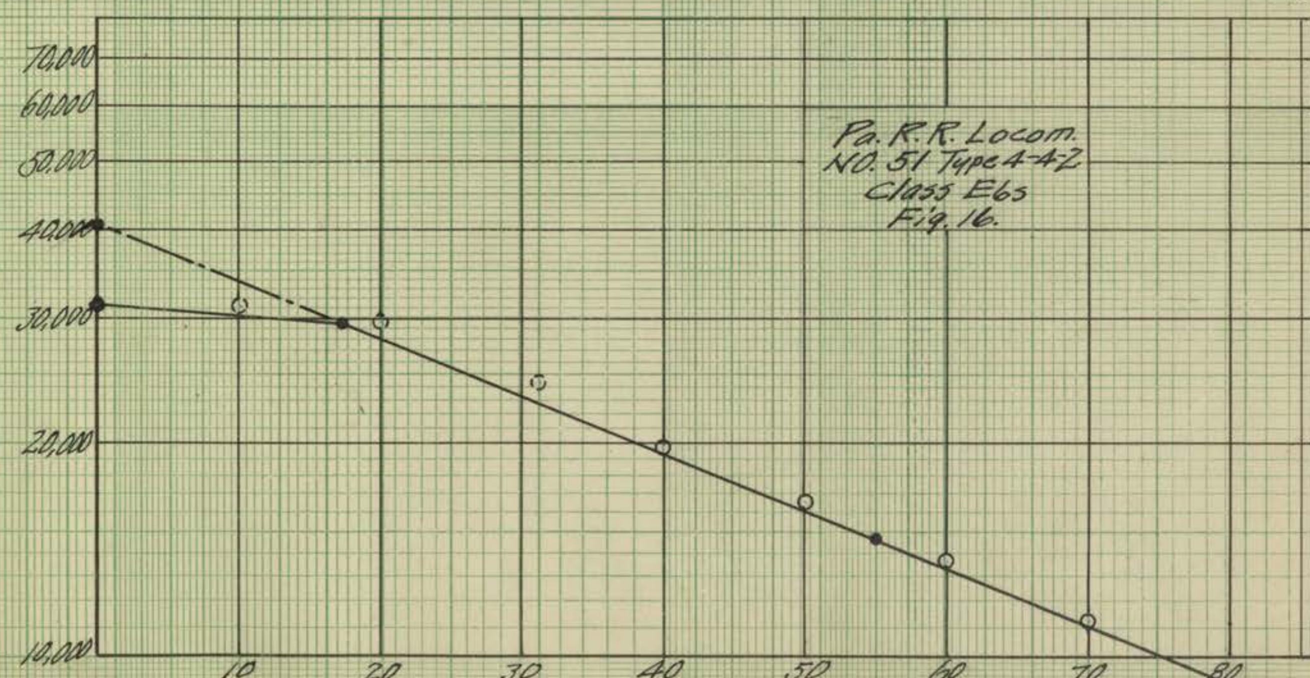


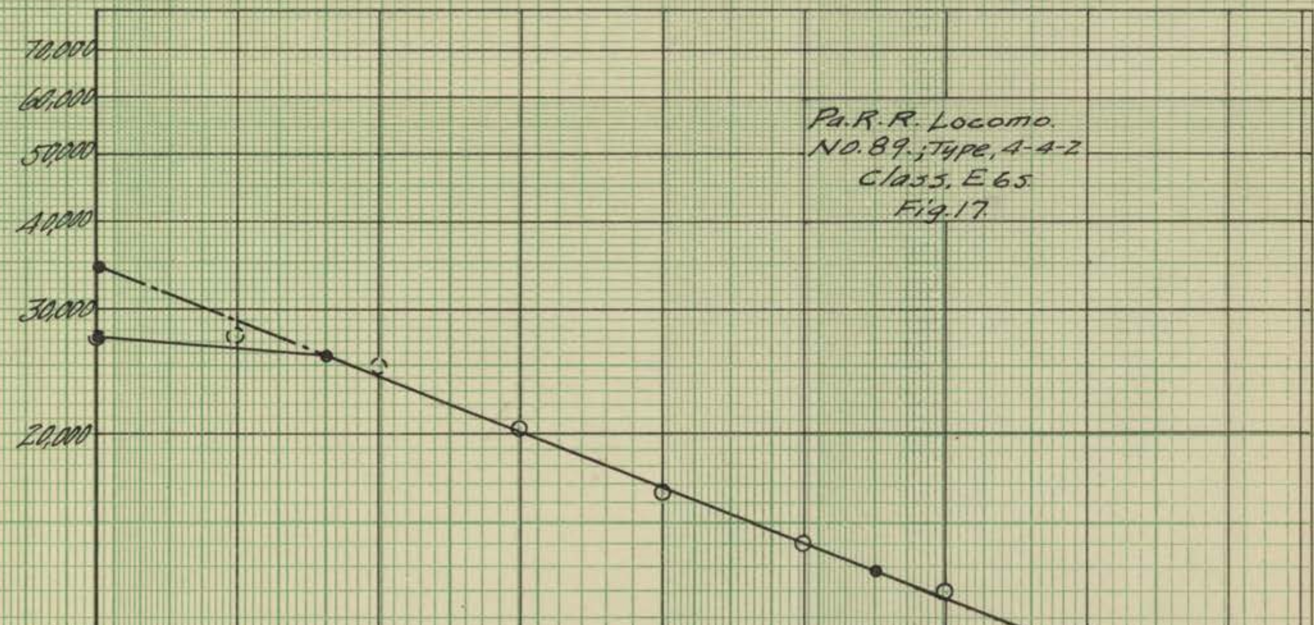
70,000
60,000
50,000
40,000
30,000
20,000
10,000
5
10
15
20
25
30
35
40.
Pa. R.R. Locom.
No. Type 2-8-0.
Class H6sb
Fig. 12.

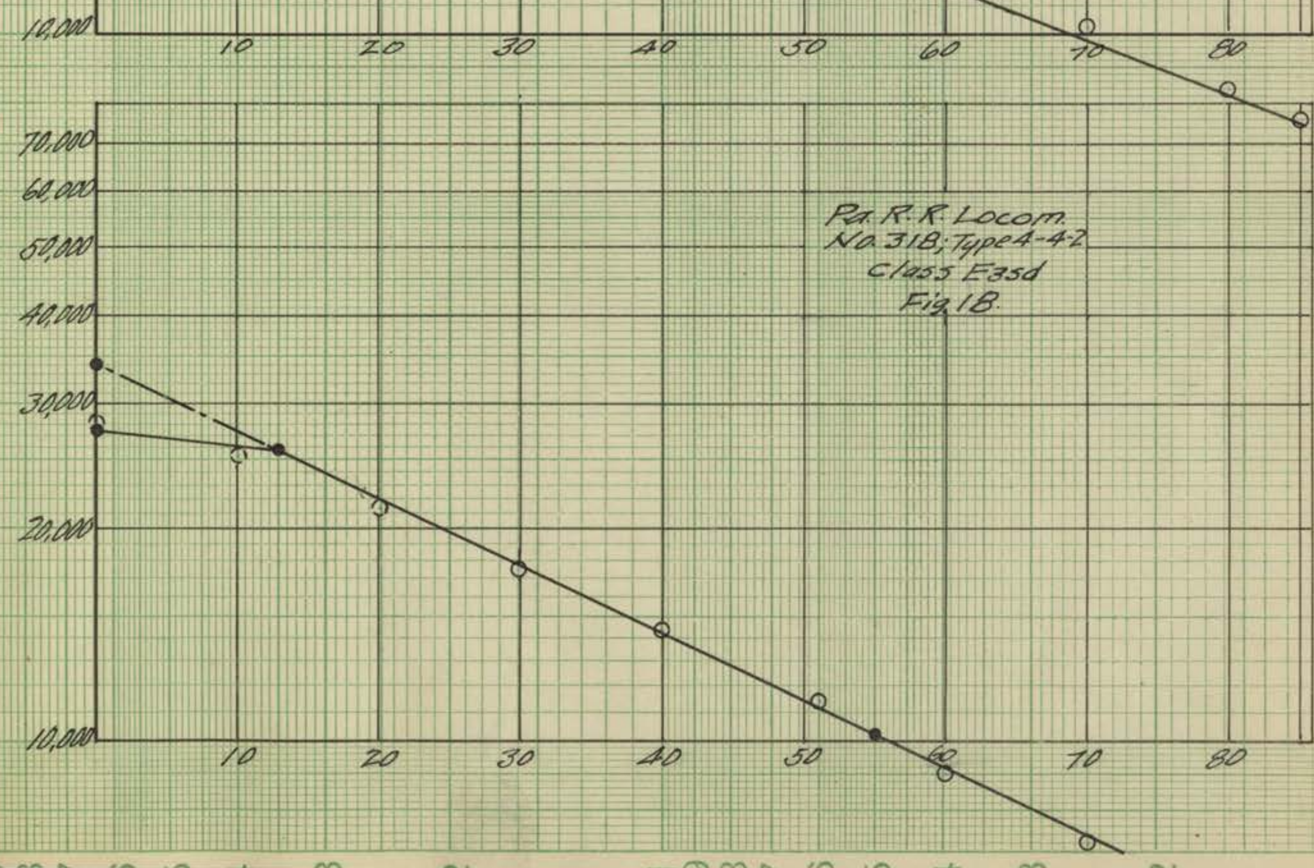

Pa. R.R. Locom.
No. 1737. Type 4-6-2
Class K4s
Fig. 13.
70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80

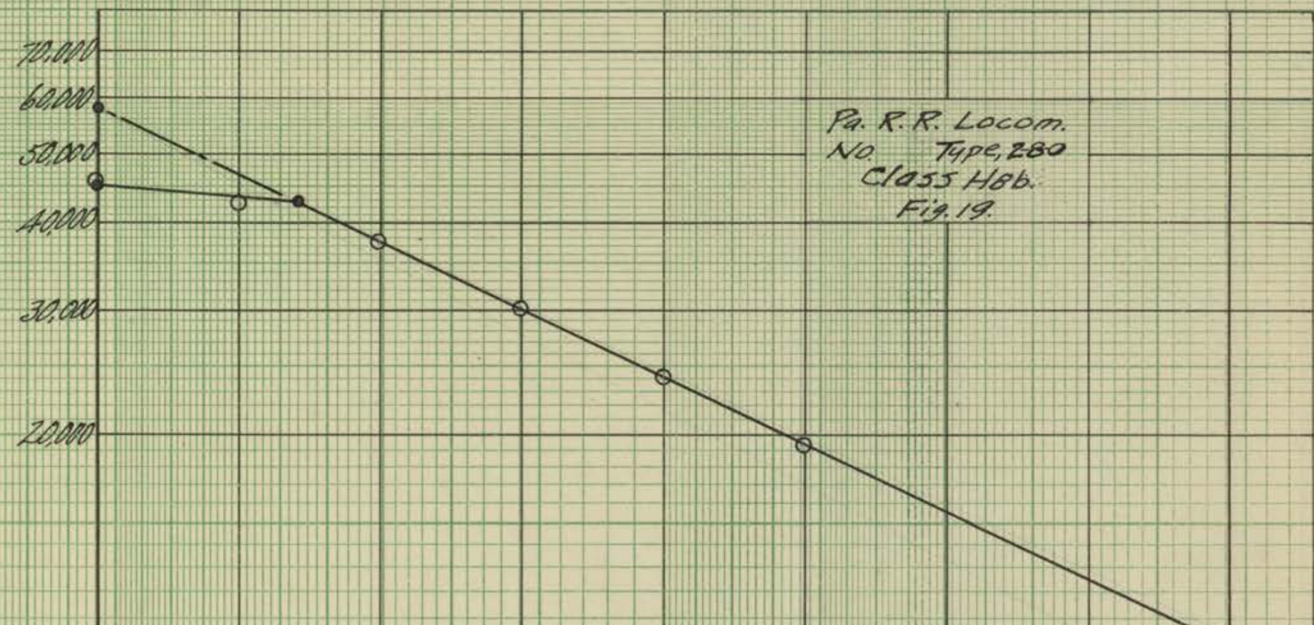

Pa. R.R. Locom.
No. 3395. Type 4-6-2.
Class K29
Fig. 14.
70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80

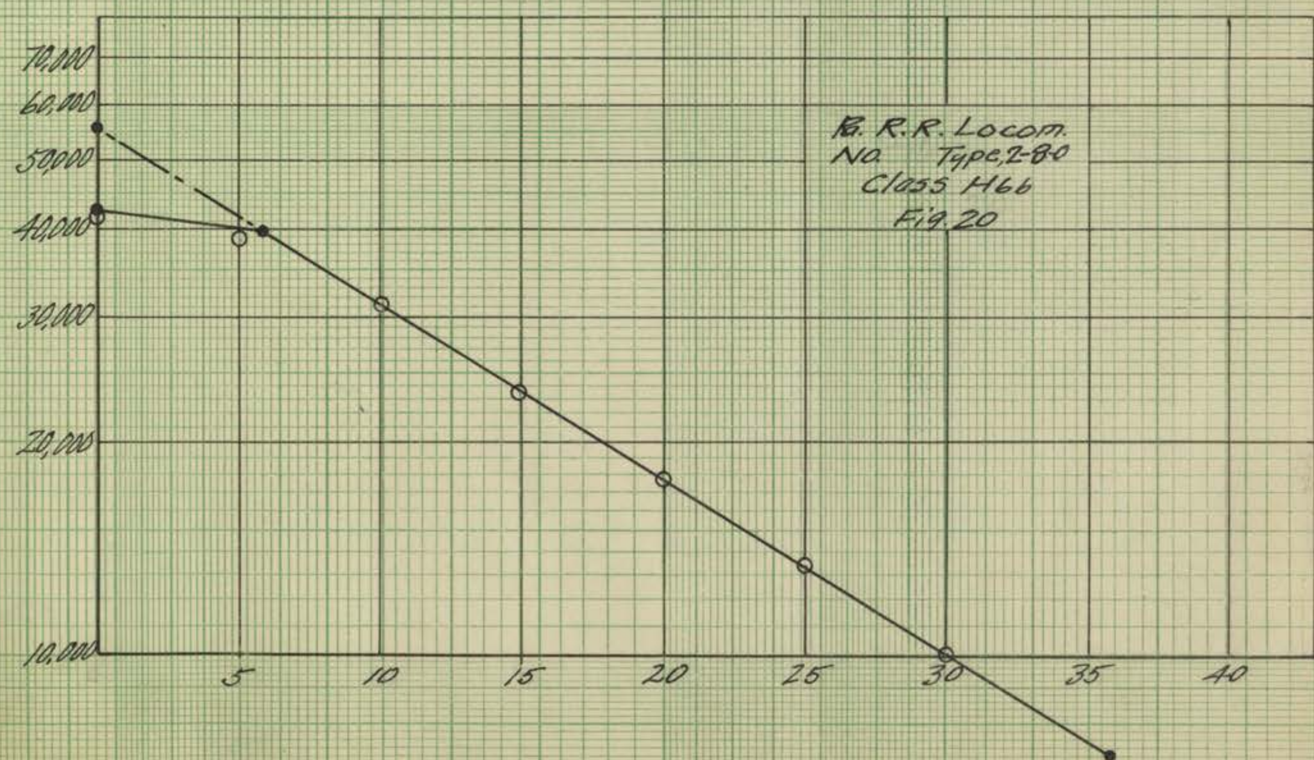

Pa. R.R. Locom.
NO. 877, Type 4-6-2.
Class K2sa.
Fig. 15
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
10
20
30
40
50
60
70
80


Pa. R.R. Locom.
NO. 51 Type 4-4-2
Class E6s
Fig. 16.
70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80

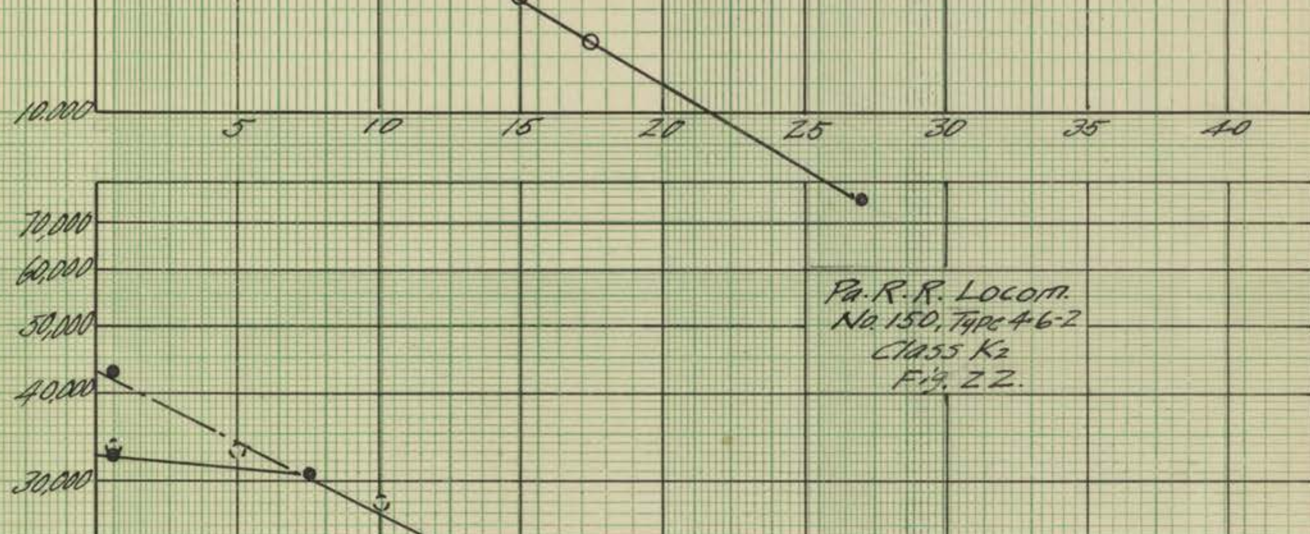

Pa. R. R. Locomo.
No. 89.; Type, 4-4-2
Class, E 6 s.
Fig. 17.
70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80

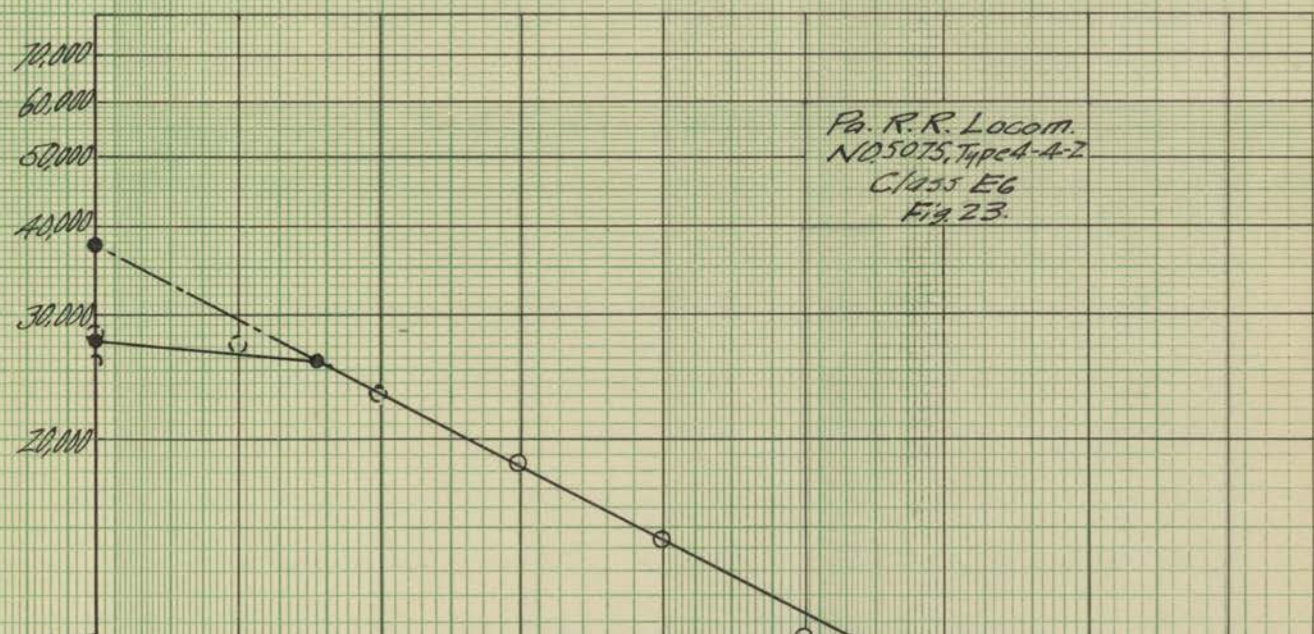

Pa. R. R. Locom.
No. 318; Type 4-4-2
Class E 3 sd
Fig. 18.
70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80


70,000
60,000
50,000
40,000
30,000
20,000
10,000
5
10
15
20
25
30
35
40
Pa. R.R. Locom.
No. Type, 280
Class H8b.
Fig. 19.


70,000
60,000
50,000
40,000
30,000
20,000
10,000
5
10
15
20
25
30
35
40
Pa. R.R. Locom.
No. Type, 2-8-0
Class H6b
Fig. 20


70,000
60,000
50,000
40,000
30,000
20,000
10,000
5
10
15
20
25
30
35
40
Pa. R. R. Locom.
No. Type 2-8-0
Class, H3
Fig. 21.


70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80
Pa. R. R. Locom.
No. 150, Type 4-6-2
Class K2
Fig. 22.


70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80
Pa. R.R. Locom.
No.5075, Type 4-4-2
Class E6
Fig. 23.


70,000
60,000
50,000
40,000
30,000
20,000
10,000
10
20
30
40
50
60
70
80
Pa. R.R. Locom.
No. 3162, Type 4-4-2
Class E2d
Fig. 24.

system when four points of a certain specification are known, and if desired the lines thus determined may be transferred to a cartesian co-ordinate system as shown in Fig. 34, or the relation may also be represented by the two equations of the form:

$$\log(y) = k - m\,x \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

that is, by

$$\log(T) = k' - m'(S), \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

and

$$\log(T) = k'' - m''(S). \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

These equations are the characteristic formulas of the speed-pull relation of steam locomotives, in which T represents the drawbar pull behind tender at a constant rate of evaporation for various piston speeds, S; and k', k'', m' and m'' are constants which are found to vary with the dimensions of cylinders and driving wheels, boiler pressure, kind of steam used, and the hourly equivalent evaporation, which, in turn, varies with the dimensions of the boiler and grate, the quantity of coal fired, etc.

3. Determination of constants. - With locomotives of modern design, the cylinder tractive effort is slightly below the adhesion of the drivers, and the boiler capacity is such that up to a certain limit of speed the boiler generates as much steam as the cylinders demand; beyond that limit, however, the boiler fails to do so but produces practically a constant amount of steam. Thus, the tractive effort of a steam locomotive up to that limiting speed is governed by the performance

of the engine cylinders while beyond that speed it depends upon both boiler and cylinders. As will be seen later, the determination of the constants of the formula for the line representing the drawbar pull below the limiting or transition speed is effected very easily when the line representing the pull above the transition speed is determined, that is, when the constants of the formula, $\log(T) = k'' - m''(S)$, are determined.

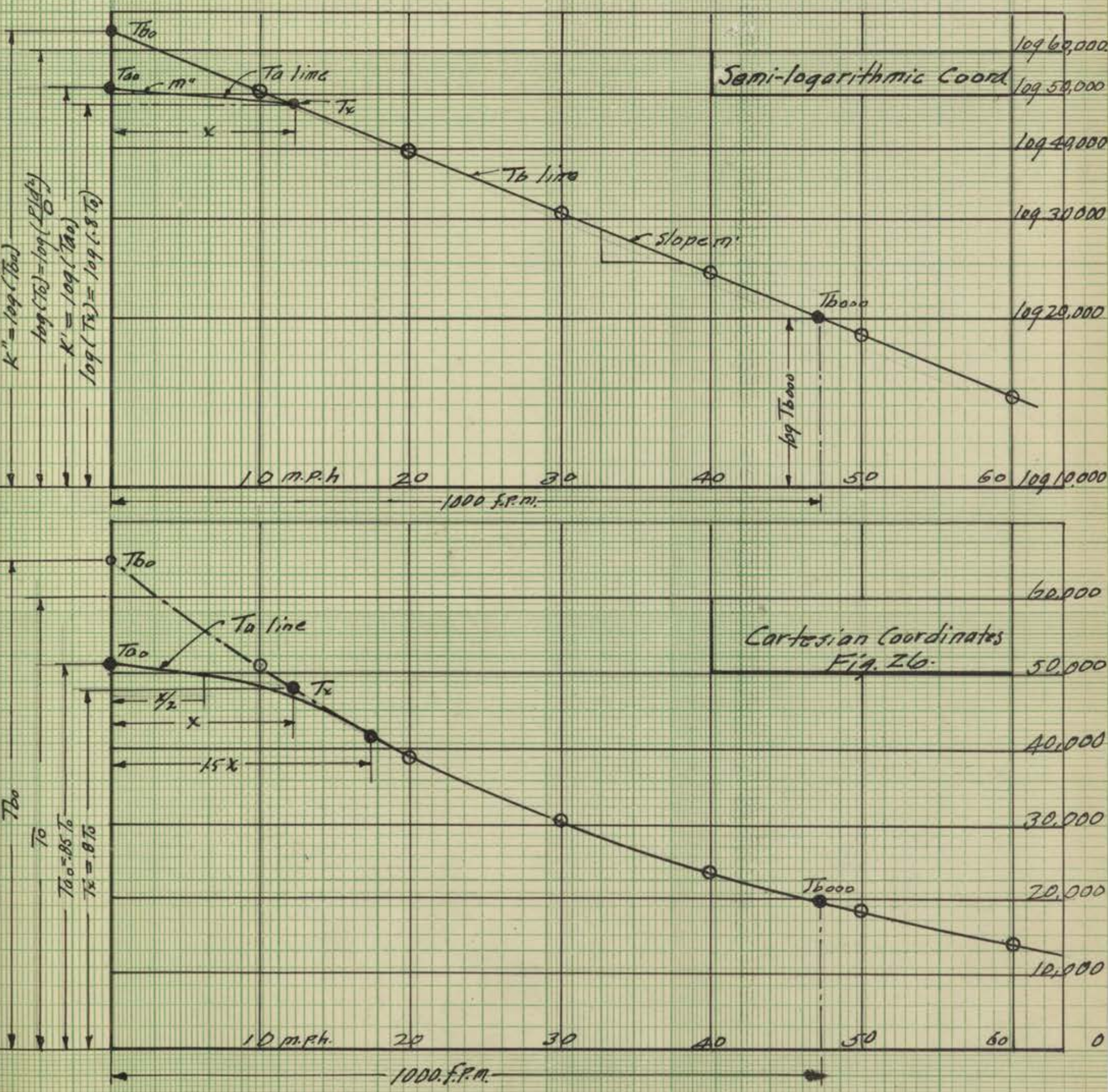
a. Constant k". - After a careful study of the experimental data, it is found that k" (also m") has different values or functions for a superheated steam locomotive and a saturated steam locomotive even when the other conditions are the same for these locomotives.

1). The value of k" for superheated steam locomotives. - Geometrically, the value of k" is the intercept on the T-axis of the straight line which represents the drawbar pull above the transition speed, Tb (see Fig. 25). From the diagrams in Figs. 9 - 24 it was apparent that the various values of k" were not the same and that the Tb lines or their extensions did not meet at a common point on the co-ordinate system.* Then, it was thought that the boiler tractive effort at zero speed per pound of theoretical tractive effort, that is, Tbo/To might be a definite constant for all superheated steam locomotives at least, but it was found not so, as ~~it~~ is shown in Tables

---

* All the diagrams in Fig. 9 - 24 inclusive were first produced on a single semi-logarithmic co-ordinate system of far larger scale.

Speed-Pull Relation on
Semi-logarithmic Coordinates
Fig. 25



IIa and IIb, Appendix. It was next supposed that they must have some definite relation to some dimensions of the engine cylinders and the steam pressure, since the tractive effort beyond the transition speed is influenced by these factors as well as by the rate of evaporation; and the values of Tbo/To were plotted against the values of pld^2, pl^2d^2, pld, etc. without any satisfactory result - P represents the boiler pressure in pounds per sq. in., l the stroke in in., and d the diameter of cylinder in inches. Finally, however, they were plotted against the values of ld and a rather surprising result has been obtained as shown in Fig. 27. This relation is, no doubt, very simple, being represented by a straight line, which agrees very closely with the experimental data. From this diagram it is found that the values of Tbo/To and k" can be expressed as follows:

$$Tbo/To = 0.635 + 0.00075(ld)$$

and

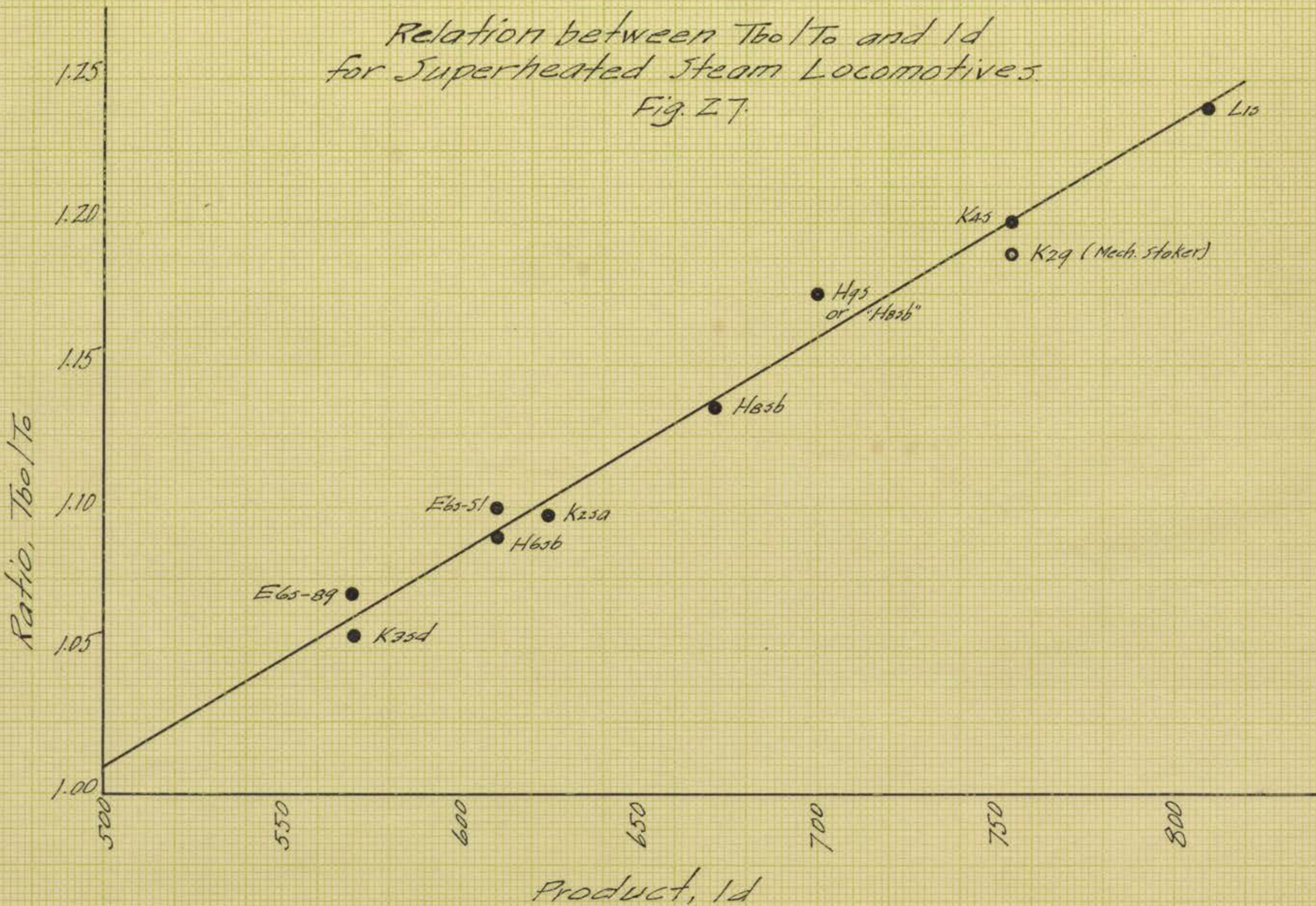
$$Tbo = (pld^2/D)(.635 + .00075\ ld)$$

or

$$k'' = \log(Tbo) = \log((.635 + .00075dl)pld^2/D)$$

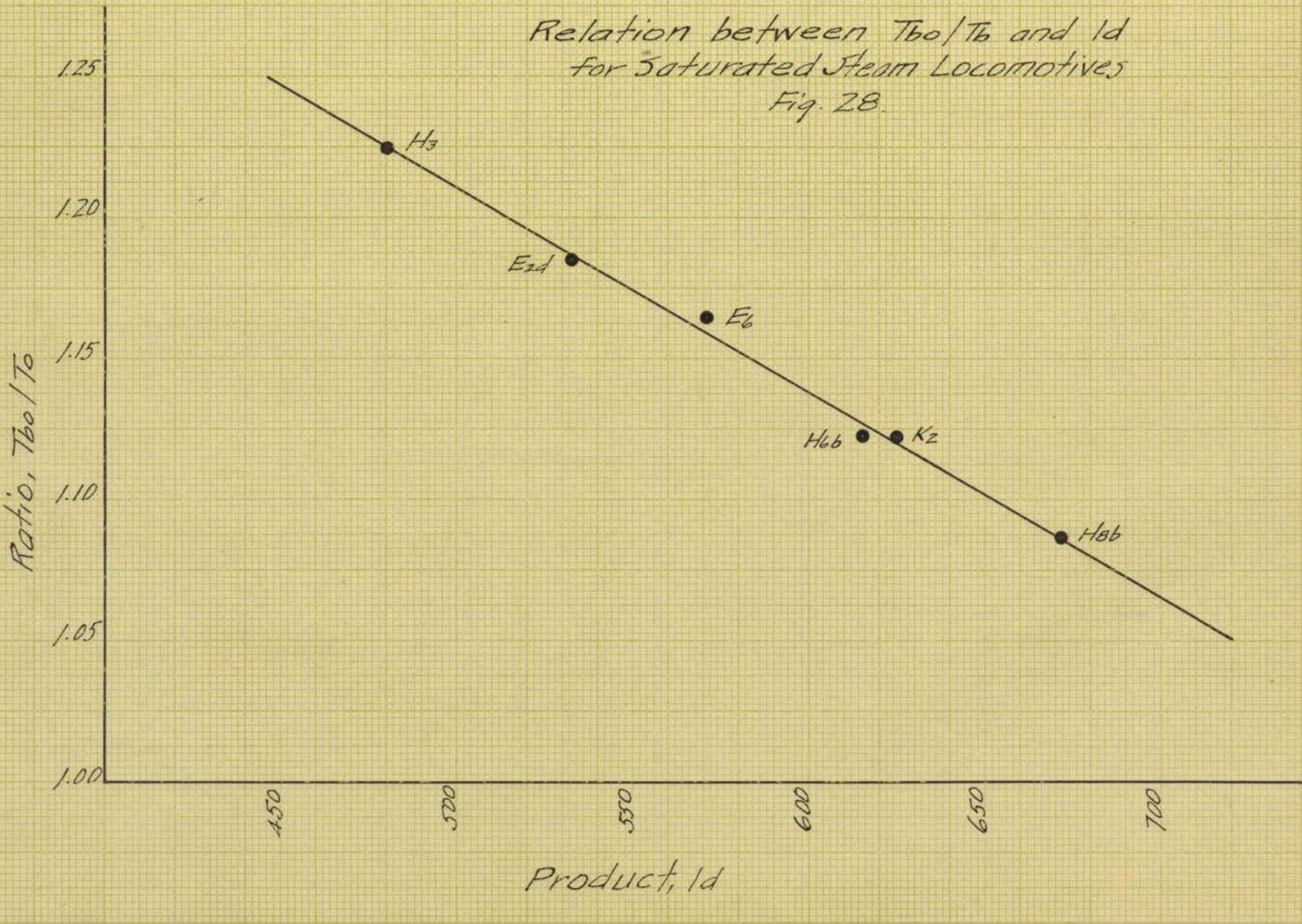
.... for superheated steam locomotives ..... (4)

2). <u>The value of k" for saturated steam locomotives.</u> - In exactly the same way as above, the relation of k" for saturated steam locomotives to the variable ld has been found as shown in Fig. 28. In this case the agreement of the line and the experimental data is much closer than in the previous case and the greatest error is less than 0.5 percent. The values of Tbo/To and k" are found as follows:



Relation between Tbo/To and Id for Superheated Steam Locomotives
Fig. 27.

59


Relation between Tbo/Tb and Id
for Saturated Steam Locomotives
Fig. 28.
Ratio, Tbo/To
1.25
1.20
1.15
1.10
1.05
1.00
450
500
550
600
650
700
Product, Id
H3
E2d
E6
H6b
K2
H8b

$$Tbo/To = 1.575 - 0.00073dl$$

and

$$Tbo = (1.575 - 0.00073\ dl)pld^2/D$$

$$k'' = \log(Tbo) = \log\left[(1.575 - .00073)pld^2/D\right] \quad \text{..... for saturated steam locomotives} \quad \text{.....................} \quad (5)$$

It is of interest to note that the value of Tbo/To for saturated steam locomotives decreases as the product ld increases. No satisfactory explanation has been found but it is a fact found from the most reliable locomotive test results and there must be some reason, which would be of great value for designing cylinders of locomotives using saturated steam.

b. Constant m". -

1). For superheated steam locomotives. - As the diagrams in Figs. 9 to 24 show, the values of m" are not the same for all superheated steam locomotives, and they must be a function of the factors which affect the performance of engines and boiler. In order to find a simple function of m", many trial plots were made and finally the relation shown in Fig. 29 was found. As the diagram shows, the relation found is very simple and definite, the agreement of the line with the experimental data being remarkably close. This relation may be algebraically expressed as follows:

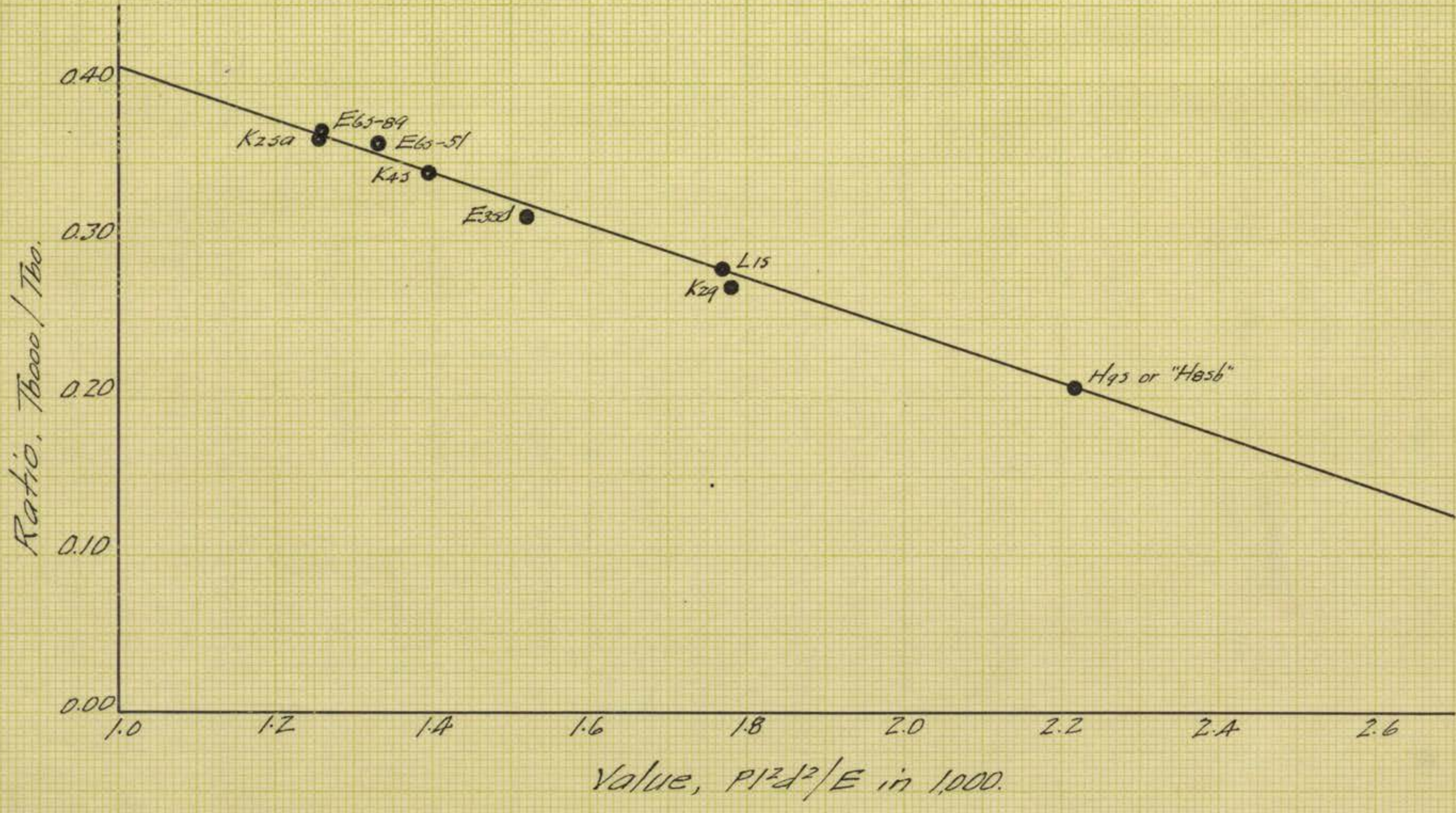
$$Tbooo/Tbo = 0.58 - 0.00017pl^2d^2/E$$

in which Tbooo is the drawbar pull behind tender at the piston speed of 1000 feet per minute, Tbo the same at zero speed*, E

---

* The value of Tbo is not physically realized by any locomotive; it is only a theoretical or imaginary quantity used in this investigation.

Relation between T_{6000}/T_{bo} and Pl^2d^2/E
for Superheated Steam Locomotives
Fig 29.



the equivalent evaporation per hour, and p, l, and d are pressure, stroke and diameter as usual. Then,

$$Tbooo = (.5815 - .00017pl^2d^2/E)Tbo$$

But

$$m'' = \frac{\log(Tbo) - \log(Tbooo)}{1000}$$

Therefore,

$$m'' = -\log(.5815 - .00017pl^2d^2/E)/1000 \text{ ...... for superheated steam locomotives .... (6)}$$

2). <u>The value of m" for saturated steam locomotives</u>.- Expecting some simple relation of m" to the value pl^2d^2/E, the plot was made as shown in Fig. 30, but the agreement of the straight line with the data was not so satisfactory as in the case shown in Fig. 31, in which the ratio Tbooo/Tbo has been plotted against the value of pl^2/E instead of pl^2d^2/E. Besides these two plots, several other plots with abscissa of different combinations of p, l, d and E were tried without any satisfactory result, and the plot in Fig. 31 has been finally adopted. This relation may also be expressed as follows:

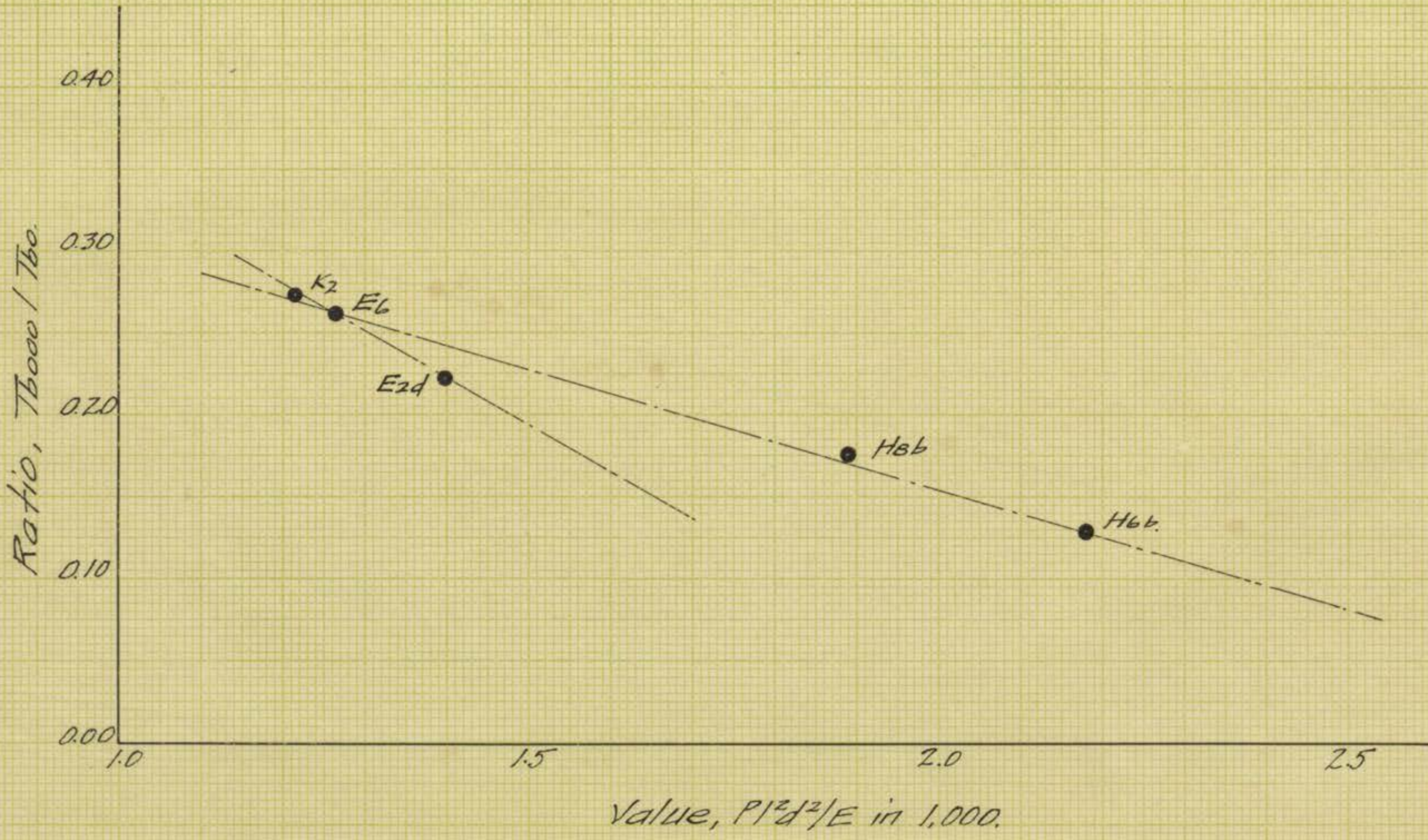
$$Tbooo/Tbo = 0.539 - 0.148pd^2/E$$

and hence

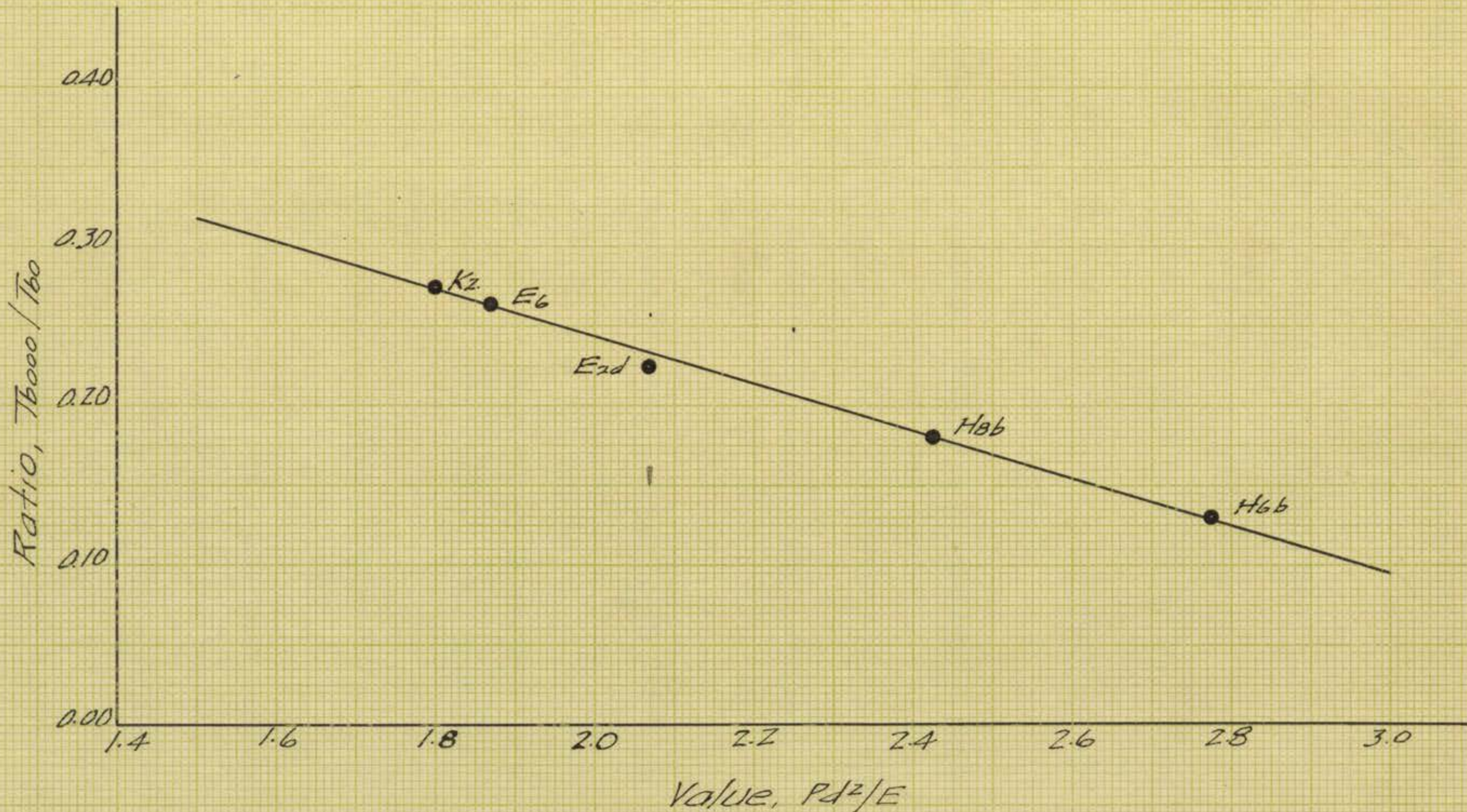
$$m'' = -\log(.539 - .148pd^2/E)/1000 \text{ ..... for saturated steam locomotives .... (7)}$$

4. <u>Formulas for speed-pull relations above the transition speed</u>. a. For superheated steam locomotives. - Substituting the values of k" and m" of the equations (4) and (6) in the characteristic formula logT = k" - m"S, we have

Relation Between T_{6000}/T_{60} and Pl^2d^2/E
For Saturated Steam Locomotives
Fig. 30.



Relation between T_{6000}/T_{60} and Pd^2/E
for Saturated Steam Locomotives.
Fig. 31.



$$\log(Tb) = \log\left[(0.635+.000075dl)\frac{pld^2}{D}\right] + \left[\log(0.5815 - .00017\frac{pd^2l^2}{E})\right]\frac{S}{1000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

$$= \log\left[To(0.635 + .000075dl)\right] + \log\left[(0.5815 - .00017\frac{lD}{E}To)^{\frac{S}{1000}}\right]$$

$$Tb = To(0.635 + .000075dl)(0.5815 - .00017\frac{ld}{E}To)^{\frac{S}{1000}} \ldots \text{(I')}$$

b. For saturated steam locomotives. - Similarly, for locomotives using saturated steam, we have

$$\log(Tb) = \log\left[(1.575 - .00073dl)\frac{pld^2}{D}\right] + \left[\log(0.539 - .0148\frac{Pd^2}{E})\right]\frac{S}{1000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

$$= \log\left[To(1.575 - .00073dl)\right] + \log\left[(0.539 - 0.148\frac{D}{El}To)^{\frac{S}{1000}}\right]$$

$$Tb = To(1.575 - .00073dl)(0.539 - 0.148\frac{D}{El}To)^{\frac{S}{1000}} . \text{(II')}$$

The formulas I, I', II and II' appear to be somewhat complicated, but when the values of p, l, d, D and E of a locomotive are known, they become very simple. For instance, the formulas I and I' become

$$\log(Tb) = \log A + (S/1000)\log B$$

and

$$Tb = AB^s$$

respectively, where s = S/1000, and A and B are some numerical constants. The base of the above equation is, of course, e(= 2.718+), but I and II are in such a form that a logarithmic table of any base can be readily used without any modification of these formulas. The formulas I' and II' are in very convenient form when a slide rule with a "loglog" scale on it is available.

5. Formula for speed-pull relation below the transition speed. - The experimental data of tractive effort at lower speed are not so complete as those for higher speed. The parts of the speed-pull diagrams in Fig. 9 - 24, which are represented by broken circles correspond to the parts represented by dotted lines in the original diagrams in the Pennsylvania bulletins and cannot be much relied upon. There are, however, the data of several road tests with a dynamometer car (see Figs. 9, 10, 11, 12, 17, 18 and 19) to which greater weight was given in the determination of the constants in $Ta = k' - m'S$, the characteristic formula for the pull below the transition speed.

Constant k'. - As shown in Fig. 25, k' is equivalent to logTao, that is, the logarithm of the actual drawbar pull at zero speed. The long accepted formula for starting tractive effort,

$$T = \frac{.85pld^2}{D}$$

appears (see Figs. 9 - 24 inclusive) to hold good for modern locomotives using superheated steam as well as for those using saturated steam, if we consider it as the drawbar pull behind tender instead of the "cylinder" tractive effort. Then, we have readily

$$k' = \log(0.85pld^2/D) \quad \text{for locomotives using superheated or saturated steam} \quad \ldots\ldots\ldots\ldots \quad (8)$$

Constant m'. - The Ta lines of the diagrams in Figs. 9 - 24 inclusive, have been drawn through the points, k', on the T-axis, with slopes which may be represented by the equation

$$m' = \frac{\log(Tao) - \log(Tx)}{x}$$

in which Tx is the drawbar pull numerically equal to 0.8To or $.8pld^2/D$, but it is located on the Tb line at x f.p.m. piston speed. The value of x can be easily found when the Tb line is drawn on either a semi-logarithmic or a cartesian co-ordinate system; but if it is necessary to find x analytically it can be deduced from Formula I or II by simply substituting $.8pld^2/D$ for Tb. We have, then

$$m' = \frac{\log(.85pld^2/D) - \log(.8pld^2/D)}{x}$$

$$= (1/x)\log(.85/.80)$$

or

$$m' = (1/x)\log(1.06)$$ for locomotives using superheated or saturated steam (9)

The formula. - Substituting, then, the values of k' and m' in the characteristic formula, Ta = k' - m'S, we get for a locomotive using superheated steam or saturated steam,

$$\log(Ta) = \log(.85pld^2/D) - \log(1.06)\frac{S}{x} \quad \ldots\ldots\ldots \quad \text{(III)}$$

or

$$Ta = (0.85pld^2/D)(.951)^{\frac{S}{x}} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III')}$$

These formulas define a curve which is practically a straight line on the cartesian co-ordinate system. For practical purposes it is more convenient and advisable to replace this curve by a straight line drawn through Tao and Tx on cartesian co-ordinates. Furthermore, it will be seen by inspection of the diagrams on Figs. 9 - 24, that although the Ta and Tb lines represent very closely the experimental data there plotted; the corner at Tx is somewhat too sharp, and a smooth curve which is tangent at .5x and 1.5x to the Ta and Tb curves agrees better with the experimental data.

6. <u>Examples and test of the formulas.</u> - As examples of the application of the formulas and also as proof of their validity, we will first take an Illinois Central Railroad consolidation locomotive and then a Chicago, Rock Island & Pacific Railroad mountain type locomotive, whose speed-pull relations are determined by actual tests and can be used for checking the values computed from the formulas.

I.C.R.R. Locomotive No. 958* - saturated steam.

Cylinders .... 22" x 30"
Drivers .... 63" dia.

Test No.	Speed m.p.h.	Piston Speed f.p.m.	Boiler Pres. lbs/sq.in.	Equivalent Evap'n. lbs./hr.	Drawbar Pull lbs.
2094	20.1	536	196.3	54,336	25,225
2098	30.4	810	191.5	57,954	17,660
2089	41.9	1118	194.0	54,989	11,831
			avg. 194.2	avg. 55,700	

<u>Calculation:-</u>

$$Tb = To(1.575 - .00073dl)(.539 - .148ToD/El)^{s}$$

$$= 44,750 \times 1.093(.288)^{s}$$

$$= 48,900(.288)^{s} \qquad \text{where } s = S/1000$$

Then, $Tb = 48,900 \times .513 = 25,100$ lbs. at 536 f.p.m. piston speed.

$= 48,900 \times .365 = 17,840$ lbs. at 810 f.p.m. piston speed

$= 48,900 \times .248 = 12,120$ lbs. at 1118 f.p.m. piston speed

*Bulletin No. 82, Eng. Exp. Station, University of Illinois.

Comparison:-

Speed m.p.h.	Drawbar Pull, lbs. by Test	Drawbar Pull, lbs. by Formula	Difference lbs.	Percent Error.
20.1	25,225	25,100	-125	-0.5
30.4	17,660	17,840	180	+1.0
41.9	11,831	12,120	289	+2.4

The comparisons show that at 41.9 m.p.h. the estimated value is 2.4 percent too large. This is partly due to the fact that the average values of equivalent evaporation and boiler pressure used in the calculation were greater than the actual values, as will be seen in the table above. A similar explanation may be made for the errors at the speeds 20.1 and 30.4 m.p.h. The agreement of the estimated values within a maximum error of 2.4 percent, with the actual test results of a locomotive whose dimensions are radically different from those of the locomotives used in the development of the formulas, is encouraging evidence of their validity.

C.R.I. & P.R.R. Locomotive No. 999.

Mountain type, superheated steam.
Cylinders .... 28" x 28"
Boiler pressure ..... 185 lbs. sq. in.
Equiv. evap. per hour* 60500 lbs.

Test Data.

Speed, m.p.h.	Drawbar Pull, lbs.
11.6	45700
15.5	42500
22.1	39000
27.0	32400
30.0	29300
35.6	25000
40.1	21900
45.0	19200
49.9	17900
54.9	15600

---

* From the tests the average actual evaporation was 46,400 lb. (see E. G. Young's thesis, 1916, p. 107).

Calculation:-

$$Tb = (pld^2/D)(.635 + .000075dl)(.5815 - .00017pl^2d^2/E)^s.$$

$$= 58800(1.223)(.275)^s$$

$$= 72000(.275)^s$$

$$= 19800 \quad \text{when } s = 1 \text{ or } 44.0 \text{ m.p.h.}$$

$$= 72000 \quad \text{when } s =) \text{ or zero m.p.h.}$$

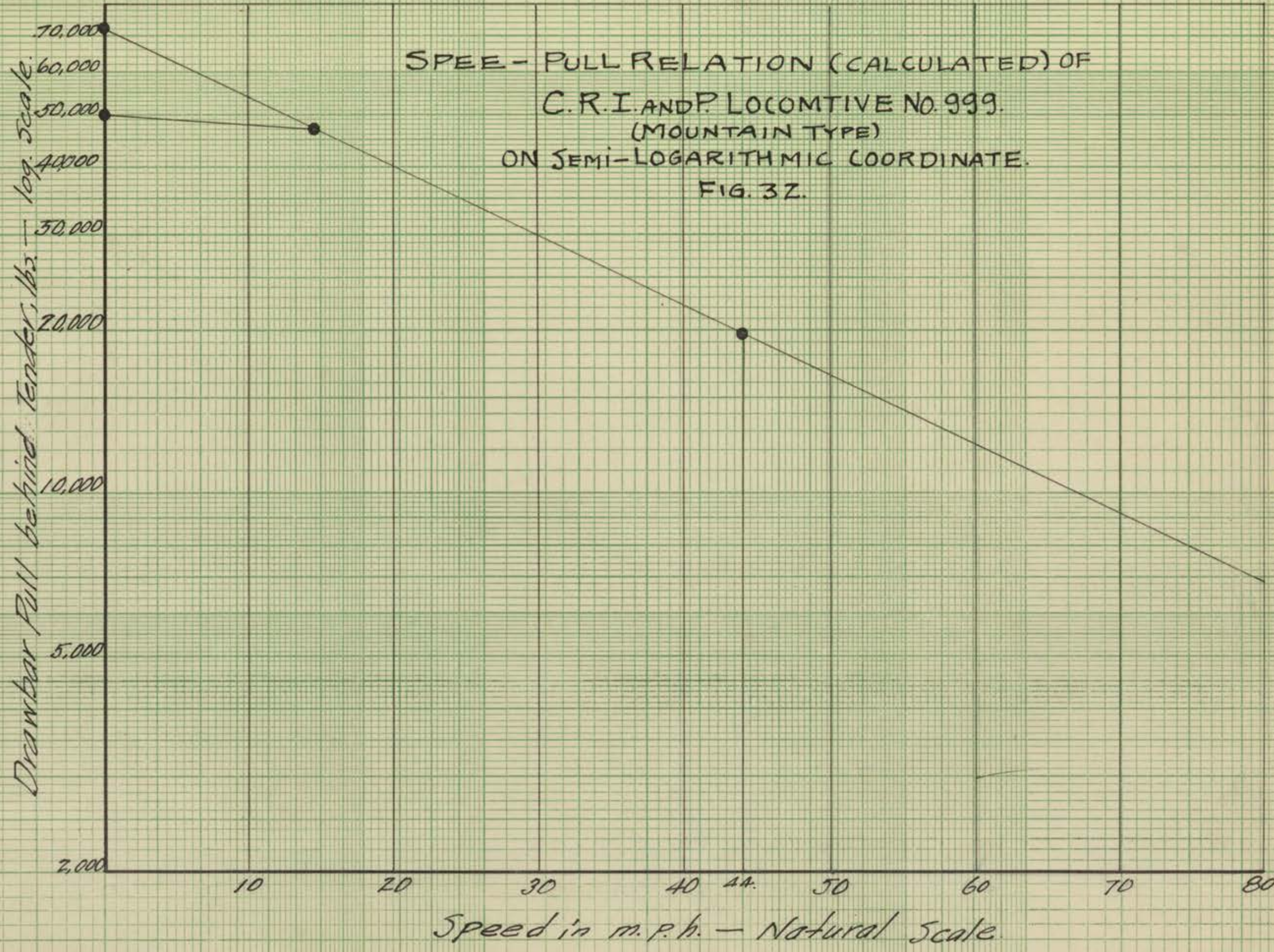
Then, locating these two points on a semi-logarithmic co-ordinate system and drawing through the points a straight line as shown in Fig. 32, we can find on this diagram values of Tb at various speeds without further computation.

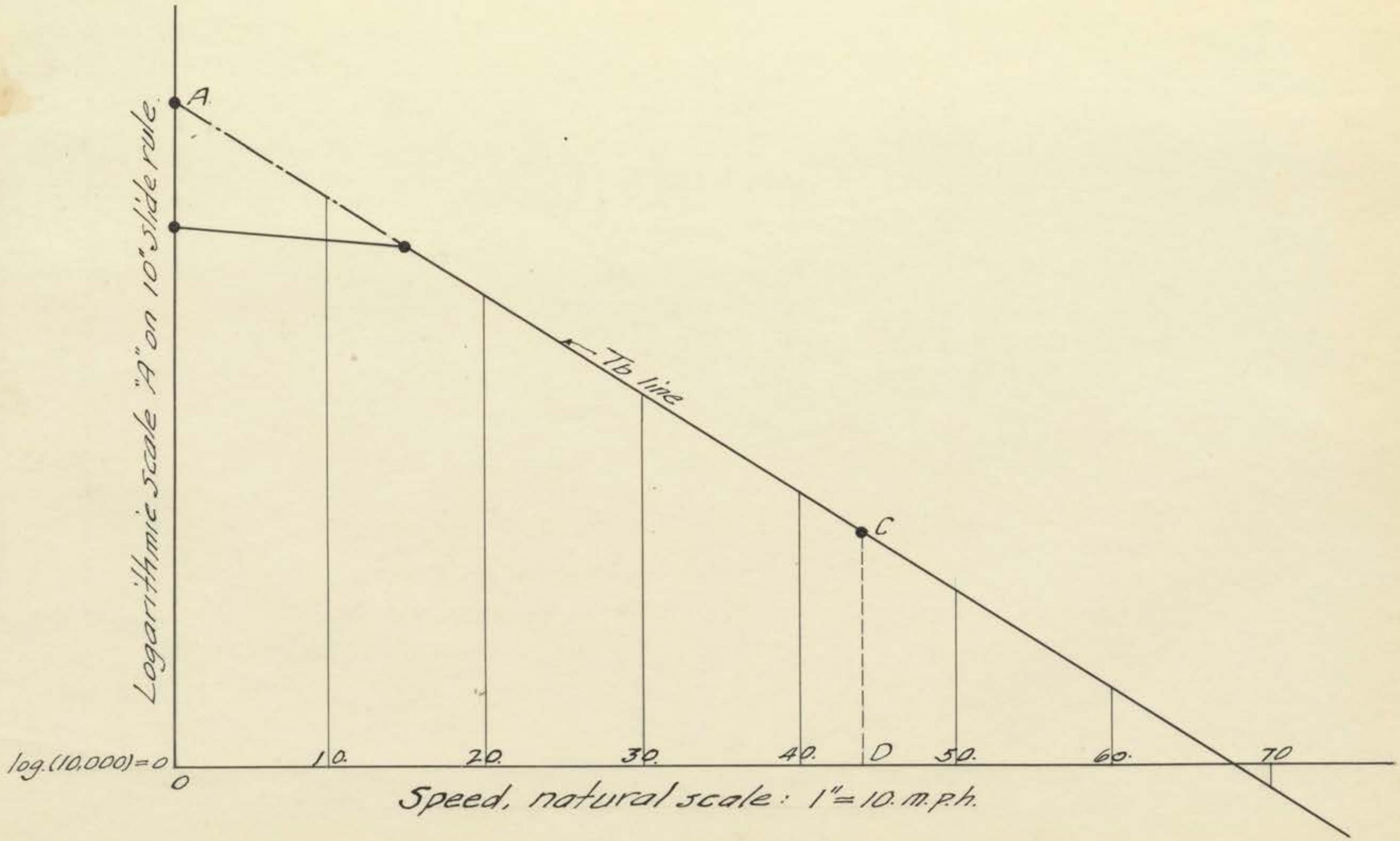
If logarithmic paper or semi-logarithmic paper is not available, the values of Tb can be computed as shown in the first example. Another convenient method is to locate the two points with the scale (logarithmic) on a slide rule, and drawing a straight line through the points read the ordinates with that scale. For instance, in Fig. 33, OA is equal to 72000 on "A" scale of an ordinary 10-inch slide rule, and similarly CD is equal to 19800. The value of Tb or the ordinate of the line AC at any speed can be found by means of a pair of dividers and the "A" scale.

By one of these methods the points plotted in the diagram in Fig. 34 have been obtained. Then, locating Tx = 47000 lbs. on Tb line and Tao = 50000 on T-axis, the Ta line has been drawn. The speed-pull diagram is completed by insertion of the smooth curve tangent to the Ta and Tb lines at 0.5x and 1.5x.

B. The Evaporation of Steam in Locomotive Boilers.

1. Adjusted heating surface. - The amount of


SPEE - PULL RELATION (CALCULATED) OF
C.R.I. AND P. LOCOMTIVE NO. 999.
(MOUNTAIN TYPE)
ON SEMI-LOGARITHMIC COORDINATE.
FIG. 32.
Drawbar Pull behind Tender, lbs. — log. Scale.
70,000
60,000
50,000
40,000
30,000
20,000
10,000
5,000
2,000
10
20
30
40
44.
50
60
70
80
Speed in m.p.h. — Natural Scale



Logarithmic scale "A" on 10" slide rule.
A.
C
← Tb line
log.(10,000) = 0
0
10.
20.
30.
40.
D
50.
60.
70
Speed, natural scale: 1" = 10. m.p.h.

Fig 33.

Speed-Pull Relation of C.R.I and P.R.R.
Mountain Type Locomotive No. 999.
Fig. 34.

steam evaporated in a locomotive boiler depends, first, upon the ability of the heating surface to transmit heat under different conditions of the heat supply and, second, upon the amount of heat generated in the furnace. Generally speaking, the ability of the heating surface to transmit heat is proportional to the number of square feet of its area, but this area cannot be the mere geometrical area of the heating surfaces or the flat sum of firebox, flue and superheating surfaces in square feet; because, as recent experiments show, on account of the high temperature a unit area of firebox heating surface is several times as effective as that of tube heating surface, and a unit area of tube heating surface transmits also several times as much heat as a unit area of superheating surface on account of the better thermal conductivity of water as compared with steam. Therefore, it is clear that to deal with the evaporating capacity of locomotives whose ratios of firebox heating surface to tube heating surface and to superheating surface are all different, a certain unit of heating surfaces which is the measure of the effectiveness but not the mere geometrical area must be adopted. For this purpose, after careful study of the experiments and investigations* on this subject, a measure called "adjusted heating

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* Test of a Jacobs-Shupert Boiler" by W.F.M. Goss; "Étude Expérimentale de la Vaporisation dans les Chaudières de Locomotives," by M. A. Henry; "Heat Transmission" by Dudly; "On the Transmission of Heat into Steam Boilers", Hedrick & Fessenden; "The Transmission of Heat into Steam Boilers" by Kreisinger and Walter; and several other books and articles on this subject.

surface" was adopted. In this measure of heating surface, the effectiveness of firebox heating surface is taken as the standard and one square foot of its heating surface is one square foot of adjusted heating surface also. The ratio of effectiveness of firebox to tube heating surface per unit area varies considerably with the heat supply or the amount of coal fired per hour; but five (5) is taken to be a fair average value for ordinary ranges of firing rate.* The results of numerous experiments show that heat transmitted through a unit area of superheating surface amounts to about 25 to 30 percent of that through a unit area of water heating surfaces ** (tubes and firebox) and is estimated to be 50 percent of that through tube heating surface or 10 percent of firebox heating surface. Then, the adjusted heating surface becomes,

$$H = Hf + Ht/5 + Hs/10 \quad .... \quad (10)$$

where Hf represents the firebox heating surface, Ht the tube h.s., and Hs the superheating surface of the locomotive. In establishing an ideal measure of the effectiveness of heating surfaces like the adjusted heating surface proposed, it might be proper to take in account also the different effectiveness of tube heating surfaces according to their

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* Cole: "Locomotive Ratios", American Locomotive Co. Bulletin No. 1017, p. 7; J. J. Anthony: "Firebox Efficiency", Ry. Review, Oct. 28, 1916, vol. 59, p. 581
Richmond Railroad Club, Oct. 16, 1916
Railway Age Gazette, vol. 61, No. 16, p. 695, Oct. 20, 1916
New England RR. Club, Oct. 10, 1916
Ry. Mech. Eng., vol. 90, No. 9 - p. 443; Sept. 1916.

** Pa. R.R. Test Dept. Bulletins Nos. 10, 11, 18, 19, 21, 24, 27, 28 and 29.

length, diameter, thickness, spacing, location, etc.; however, practically no experimental data on these points are available, except the temperature measurement along the tubes. Fortunately, however, current methods of locomotive design do not present radical differences in these respects and they may be ignored without introducing errors greater in magnitude than those which inhere on other steps in our process.

2. The coal factor. - The amount of heat generated in the furnace to supply the heating surfaces depends upon the amount of coal fired and the rate of combustion,but it is also greatly influenced by the skill of the fireman and by the quality of the coal. The degree of skill varies with different firemen, and even with the same fireman it varies at different times, and this personal equation can hardly be expressed in a mathematical formula. The rate of combustion involves the degree of completeness in combustion - the amounts of carbon monoxide, oxygen, sparks or cinders, and combustible in the ashes. The relation of the rate of combustion to the heat generated or to the evaporation will be taken up later, while here only the influence of the quality of coal or its thermal value on the evaporation will be studied. Due to the absence of data of evaporation experiments made under exactly similar conditions but with coal of different thermal values, the determination of a definite relation of the thermal value of coal to the amount of heat generated or to the evaporation is very difficult. Generally it is believed by many engineers that either the evaporating power of a unit weight of coal is

directly proportional to its B.t.u. content, or that it is practically independent of the thermal value, that is, any grade of coal evaporates practically the same amount of water. Mr. E. G. Young*, however, after careful study of the experimental data of stationary boilers produced a formula

$$f = (x - 3000)/11500.$$

in which f is the factor or coefficient which multiplied by the evaporation of the standard coal - 14500 B.t.u. per pound coal - gives the evaporation per pound of coal, the B.t.u. content of which is x. This formula is, no doubt, more logical than the assumptions above cited, that is, f is equal to a constant times x or f is always unity. A study of this problem with the experimental data of about twenty-five different locomotives in addition to all the data Mr. Young employed in his investigation, leads us to believe that his formula is correct in form; but that a slight change in the numerical constants, that is,

$f = (x - 4500)/10000$ for coals whose B.t.u. is less than 14500; and f is unity for coal whose B.t.u. content is above 14500. .... (11)

will give better results in the estimation of evaporation in locomotive boilers if not also for stationary boilers. The data for this conclusion will be found in Tables IIIa, IIIb, IIIc and diagrams in Figs. 40 and 41, in the Appendix.

3. <u>Equivalent evaporation in locomotive boilers at the firing rate of 5000 lbs. dry coal per hour</u>. - Although the total heat generated per hour in a locomotive firebox per

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*Thesis: "Factors affecting tractive effort of Steam locomotives", U. of I., 1916, for M.S. in Railway Engineering.

pound of dry coal is influenced by the rate of combustion per square foot of grate area, it may be considered as the same at a definite firing rate*, and the evaporation depends only upon the ability of the heating surfaces to absorb the heat. With this belief, the equivalent evaporationsper pound of coal corrected to the standard 14500 B.t.u. by formula (11) for different locomotives, were plotted against the adjusted heating surface as shown in Fig. 36, and a straight line was drawn to represent the points as closely as possible. The agreement of the line with the plotted data is remarkably good for an investigation of this sort. The points for E6s-51 and "H8sb", however, are far away from the line although the average of these two points falls on the line. Explanation for the positions of these points was sought in the grate area, ratio of heating surface to grate area, heating surface, length of tubes, etc. of the boilers, but no satisfactory justification was found. It would not be due, however, to the skill of firemen since only approved firemen are employed in the tests. As to the size of the coals no definite information is available, but the coal used on "H8sb" was "a run of mine coal used in freight service on this road" while that used on E6s-51 was coal "mined by the Penn Gas Coal Company

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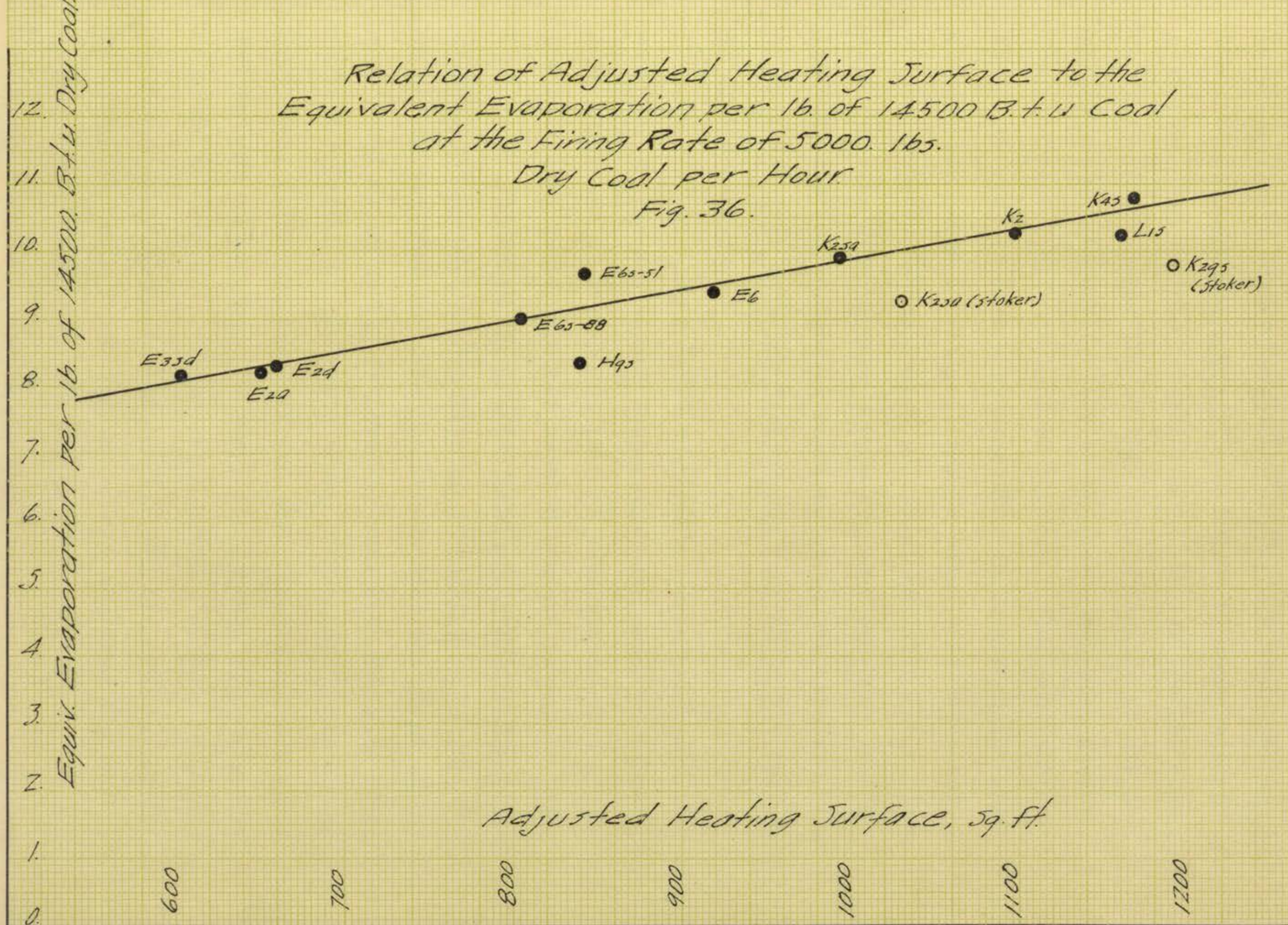
* In this paper, the term "rate of firing" or "firing rate" is employed to express the amount of coal fired per hour per locomotive in pounds, while the term "rate of combustion" of "combustion rate" is used to express the amount of coal burned per square foot of grate per hour in pounds.



Relation of Adjusted Heating Surface to the Equivalent Evaporation per lb. of 14500 B.t.u. Coal at the Firing Rate of 5000. lbs. Dry Coal per Hour.

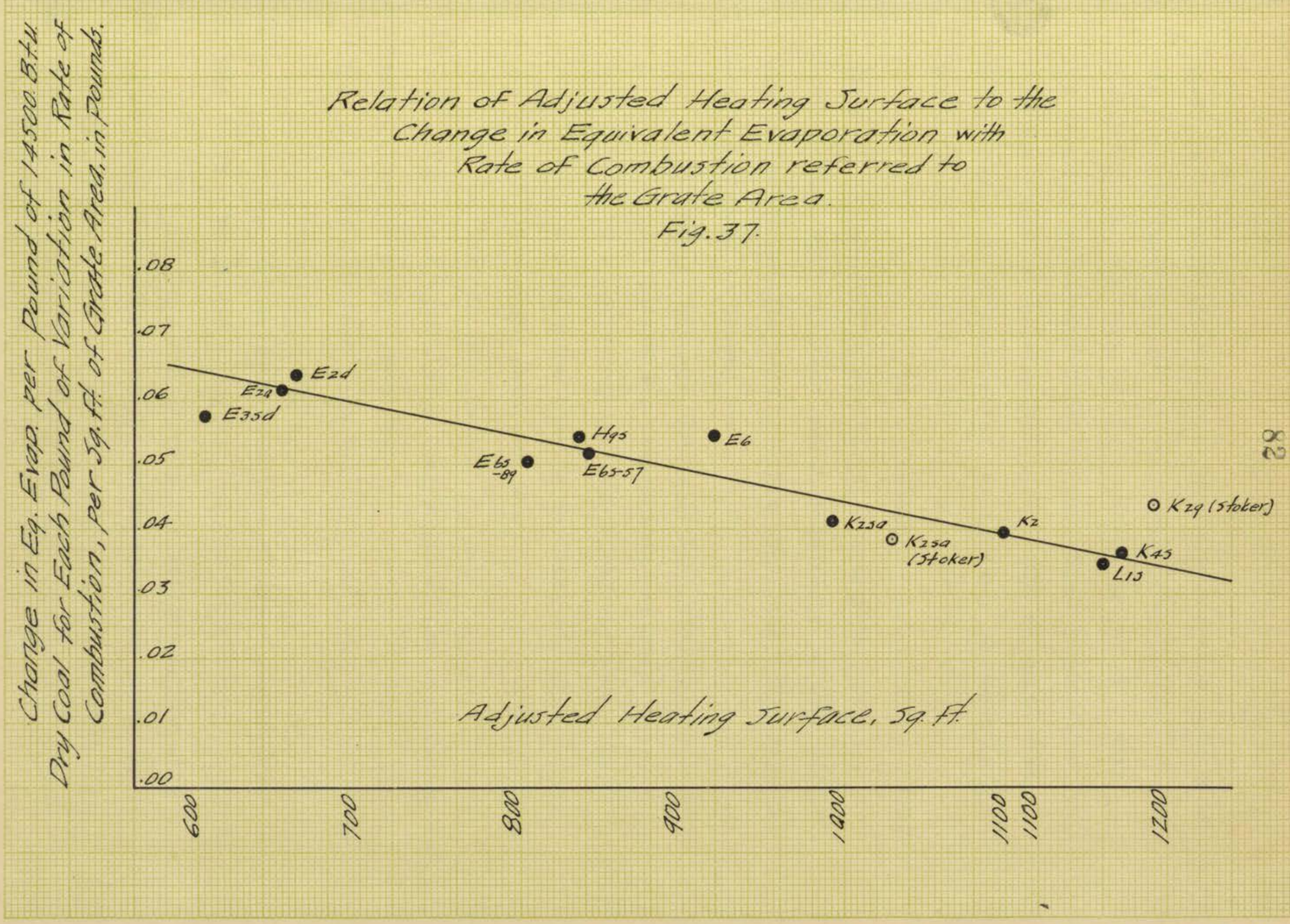
Fig. 36.

.... it is fairly representative of the coal used on the road in <u>passenger service</u>." It may be of some interest to note here that in this diagram the points for freight locomotives all lie below the average line while the points for all passenger locomotives lie on or above the line, except the locomotives with mechanical stokers. Thus some unknown factor or factors have a considerable influence on the evaporation per pound of coal. It may be taken that the relation of the equivalent evaporation per pound of 14500 B.t.u. coal at the firing rate of 5000 pounds dry coal per hour to the adjusted heating surface is represented by the straight line in Fig. 36, (for data see Table V and Fig. 42 in the Appendix) or by the equation

$$e_1 = 5.335 + .00453H \quad .... (12)$$

where e_1 is the equivalent evaporation per pound of 14500 B.t.u. coal in pounds at the firing rate of 5000 pounds of dry coal per hour, and H the adjusted heating surface in square feet.

4. <u>Variation of the equivalent evaporation per pound of dry coal fired, with the rate of combustion</u>. - As alluded to before, the heat generated in the furnace per pound of dry coal fired is generally believed to vary with the rate of combustion referred to the grate area. Consequently the equivalent evaporation per pound of dry coal fired must vary with the rate of combustion. Further, this change in evaporation is found to vary with the adjusted heating surface by a definite function as demonstrated in Fig. 37, the data for which are given in Table V and Fig. 43 of the Appendix. In this diagram the agreement of the straight line and the plotted



Relation of Adjusted Heating Surface to the Change in Equivalent Evaporation with Rate of Combustion referred to the Grate Area.
Fig. 37.
Change in Eq. Evap. per Pound of 14500 B.t.u. Dry Coal for Each Pound of Variation in Rate of Combustion, per Sq. Ft. of Grate Area, in Pounds.
.08
.07
.06
.05
.04
.03
.02
.01
.00
E2d
E2a
E35d
H95
E6
E65-89
E65-57
K25a
K25a (Stoker)
K2
K29 (Stoker)
K45
L13
Adjusted Heating Surface, Sq. Ft.
600
700
800
900
1000
1100
1100
1200

data is not very close. This, however, does not much affect the final estimated value of equivalent evaporation. For instance, take the case of the greatest deviation - E3sd. The error is about .006 pound of equivalent evaporation for each one pound of increase per square foot of grate area. The mathematical expression of the relation represented by the straight line in Fig. 37, is found to be

$$e_1' = .096 - .000052H \quad \ldots\ldots\ldots\ldots\ldots (13)$$

where e_1' is decrease in equivalent evaporation per pound of 14500 B.t.u. coal for each pound of increase in the rate of combustion per square foot of grate area, and H the adjusted heating surface in square feet.

5. <u>The formula for the equivalent evaporation</u>. - Let:

E = equivalent evaporation per hour, lbs.

W = weight of dry coal fired per hour, lbs.

H = adjusted heating surface, sq. ft.

G = grate area, sq. ft.

f = the coal factor

e = equiv. evap. per pound of dry coal fired, lbs.

and $r = (W - 5000)/G$.

Then from the formulas, (12) and (13), we have

$$e = f(e_1 - re_1')$$
$$= f(5.335 + .00453H - r(.096 - .000052H))$$

and $$E = Wf((5.335 + .00453H) - r(.096 - .000052H)) \quad \ldots\ldots (IV)$$

This is the final formula for estimating the equivalent evaporation for superheated or saturated steam locomotives. It contains, as variables, the weight of dry coal fired, W; the rate of combustion, W/G; the quality of the coal, f; and also the adjusted heating surface in which the relative effectiveness of firebox, tubes and superheating surfaces are considered.

6. Example of the formula IV. - To illustrate the use of the formula developed in the preceding article, let us take Illinois Central Railroad locomotive No. 958, whose evaporation data are known and can be used for a check.

I.C.R.R. Locomotive No. 958*.

Firebox heating surface .... 168 sq. ft.
Tube heating surface .... 3511 sq. ft.
Grate area .... 49.55 sq. ft.

Test No.	Wt. of Dry Coal per Hr., W.	B.t.u. per lb. of dry coal.	Equivalent Evapora'n. lb/hr.,E.	Eq. Evap. by Formula IV	Error Per-Cent
2078	4707	12517	34431	36200	5.2
2032	5352	12242	36967	37000	0.0
2092	5640	12620	41770	39700	5.2
2074	6015	12575	48700	49200	3.6

Calculation:-

$$H = 168 + 3541/5 = 876.2 \text{ sq. ft.}$$

Then, $$E = Wf(5.335 + 3.97 - r(.096 -.0455))$$

$$= Wf(9.305 -.0505r)$$

Test No. 2078, W = 4707, B.t.u. = 12517

$$f = (12517 - 4500)/10000 = .8017$$

$$r = (4707 - 5000)/ 49.55 = -5.9$$

and $$E = 4707 \times .8017(9.305 - .0505\times(-5.9))$$

$$= 3770 \times 9.603$$

$$= 36200. \text{ lbs. per hr.}$$

Similar calculations were made for Test Nos. 2032, 2092 and 2074, and the estimated values are shown in the above table,

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* Engineering Experiment Station, U. of I. Bulletin No. 82.

column 5. The maximum error is about 5 percent within the range of the firing rate. This percentage is not large, and the result of the estimation is satisfactory when we consider the greatly varying performance of locomotive boilers even under apparently the same conditions. Further, the comparison of the boiler performance of the "958" with that of other locomotives shows that the former is somewhat unusual, having comparatively low efficiency at the low rate of combustion and comparatively high efficiency at higher rates of combustion. (see line ab in Fig. 42.) This is the reason why the estimated value of E is too large at the lower rate of combustion and too small at the higher rate. However, the small error in the application of the formula to the unusual case proves its value.

C. The Complete Formulas for the Speed-pull Relation of Steam Locomotives.

1. Formula for superheated steam locomotives. -

$$Ta = 0.85To(1 - .05ToS/x) \text{ when } S < x, \text{ and}$$

$$Tb = To(.635 + .000075dl)(.5615 - .00017TolD/E)^{s} \text{ when } S > x.$$

Where Ta and Tb = Drawbar pull behind tender, lbs.

S = Piston speed in feet per minute

s = S/1000

p = Boiler pressure, lbs. per sq. in.

l = stroke of piston, in.

d = diameter of cylinders, in.

D = Diameter of drivers, in.

$To = pld^2/D$

x = Piston speed at which $T = 0.8To$, and

E = Equivalent evaporation from and at 212 deg. F per hour, lbs. which may be estimated by the formula,

$$E = Wf((5.335 + .00453H) - (W/G - 5000/G)(.096 - .000053H))$$

where W = Weight of dry coal fired per hour, lbs.

G = Grate area, sq. ft.

f = The coal-factor, which is equal to $(B-4500)/10000$, when the thermal value per pound of dry coal fired, B, is less than 14500 B.t.u., and is unity when it is greater than 14500,

H = "Adjusted heating surface", which is $H = Hf + Ht/5 + Hs/10$, where Hf is the firebox heating surface including arch tube h.s., Ht the tube (water side) h.s., and Hs the superheating surface all in sq. ft.

The sharp corner of the speed-pull diagram constructed with these formulas should be replaced by a smooth curve tangent at 0.5x and 1.5x speeds.

2. Formulas for saturated steam locomotives. -

$T = 0.85To(1 - .05ToS/x)$ when $S < x$, and

$T = To(1.575 - .00073dl)(0.539 - .148ToD/lE)^S$ when $S > x$,

where the symbols are the same as before. The formulas for E, f and H are also the same, except that Hs in the formula for adjusted heating surface becomes zero. The sharp corner of the diagram should be replaced by a smooth curve tangent at 0.5x and 1.5x as in the case of superheated steam locomotives.

3. <u>An example of the application of the formulas</u>. - As a further example of the application of the formulas, Illinois Central Railroad Mikado locomotive, the drawbar pull of which (as shown by the points in Fig. 38) has been determined by a dynamometer car test made for the purpose of determining the ultimate capacity of the locomotive on ruling grades and the effect of rail washer on train resistance, will be considered. The purpose of the test being such and the operating conditions ever changing, the evaporation, firing rate, etc. were not determined. Later, however, it became desirable to check the result with some formula. A formula which had been adapted by a prominent railway engineering society was tried first, but they did not check each other very satisfactorily. The test data cannot be suspected, although thepoints plotted diverge considerably at higher speeds, as the experiment was made with an accurate dynamometer car under expert supervision, and the computed results have been checked very carefully. The following calculation of drawbar pull will be, therefore, neither to check the test result nor to test the formula - the accuracy of the formulas has been demonstrated in the previous examples - but simply to show a method for estimating the tractive effort at any speed and at any rate of firing.

Data of the locomotive No. 1748:-

Cylinders	27" x 30"
Drivers	63" dia.
Heating surface, firebox ...	235 sq. ft.
" " , tubes	3835 " "
" " , superheater	1093 " "
Grate area	70 " "
Boiler pressure	175 lb. per sq. in.

The thermal value of the fuel is not definitely known and is assumed in the following calculation to be 13500 B.t.u. per pound of dry coal. The coal was mined near Central City, Ky. (Tests made by the U.S. Geological Survey of coal mined at this point show a B.t.u. of 13500.) The firing rate was not determined, but it seemed to be very high - the locomotive was often fired by two men, that is, one fireman and a helper as the writer witnessed - and in the calculation five different firing rates 4000, 5000, 6000, 7000 and 8000 pounds dry coal per hour are assumed.

Calculation:-

$H = 235 + 3835/5 + 1093/10 = 1111.3$ sq. ft.

$f = (13500 - 4500)/10000 = 0.9$

Then, $E = 0.9W((5.335 + 5.05) - r(.096 - .0578))$

where $r = (W - 5000)/G$

$= 0.9W(10.375 - .0382r)$

$= 3600(10.375 +.546) = \underline{39400}$. lbs. when $W = 4000$. lb.

$= 4500(10.375 \pm 0) = \underline{46700}$. " " $W = 5000$ "

$= 5400(10.375 -.546) = \underline{53100}$. " " $W = 6000$ "

$= 6300(10.375-.1.092)= \underline{58500}$. " " $W = 7000$ "

$= 7200(10.375-.1.638) = \underline{62800}$. " " $W = 8000$ "

Then when $W = 4000$ lbs. per hour

$Tb = 60600(.635 + .607)(.5815 - .495)^s$

$= 75350(.0865)^s$

$= 75350(.0864)^0 = 75350$ lb. at speed $s=0$ or 0 m.p.h.

$= 75350(.0864)^1 = 6500$ lb. at $S = 1000$ f.p.m.

or 37.5 m.p.h.

when $W = 5000$ lbs. per hour,

$$Tb = 75350(0.5815 - .4175)^s$$

$$= 75350(.164)^1 = 12350 \text{ lbs. at } 37.5 \text{ m.p.h.}$$

when $W = 6000$ lbs. per hour

$$Tb = 75350(0.5815 -.367)^s$$

$$= 16050 \text{ lbs. at } 37.5 \text{ m.p.h.}$$

when $W = 7000$ lbs. per hour

$$Tb = 73350(.5815 - .3345)^s$$

$$= 75350(.247) = 18650 \text{ lbs. at } 37.5 \text{ m.p.h.}$$

when $W = 8000$ lbs. per hour

$$Tb = 75350(.5815 - .3145)^s$$

$$= 75350(.267) = 20150 \text{ lbs. at } 37.5 \text{ m.p.h.}$$

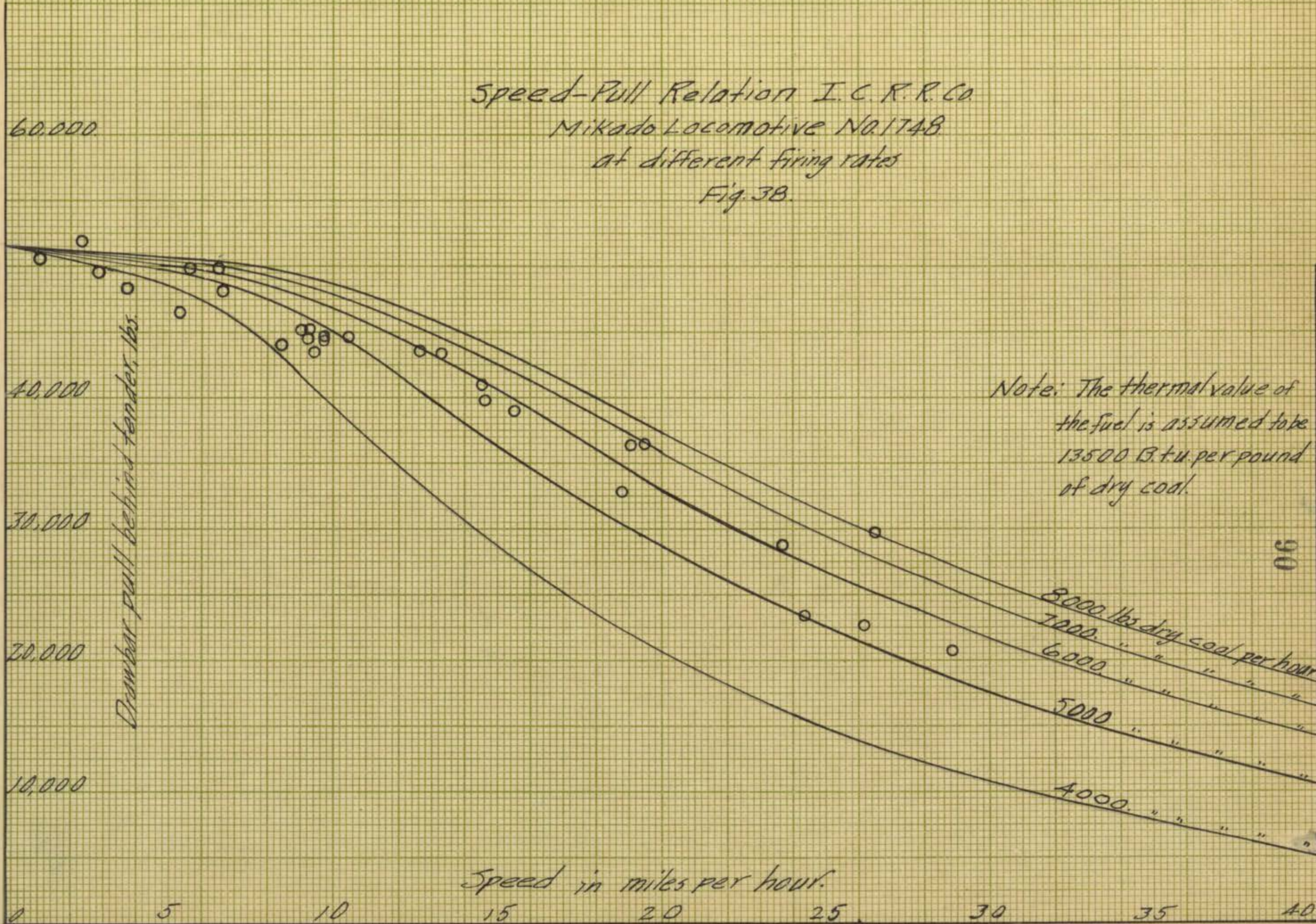
Further, for Ta,

$$0.85pld^2/D = 51500 \text{ lbs.}$$

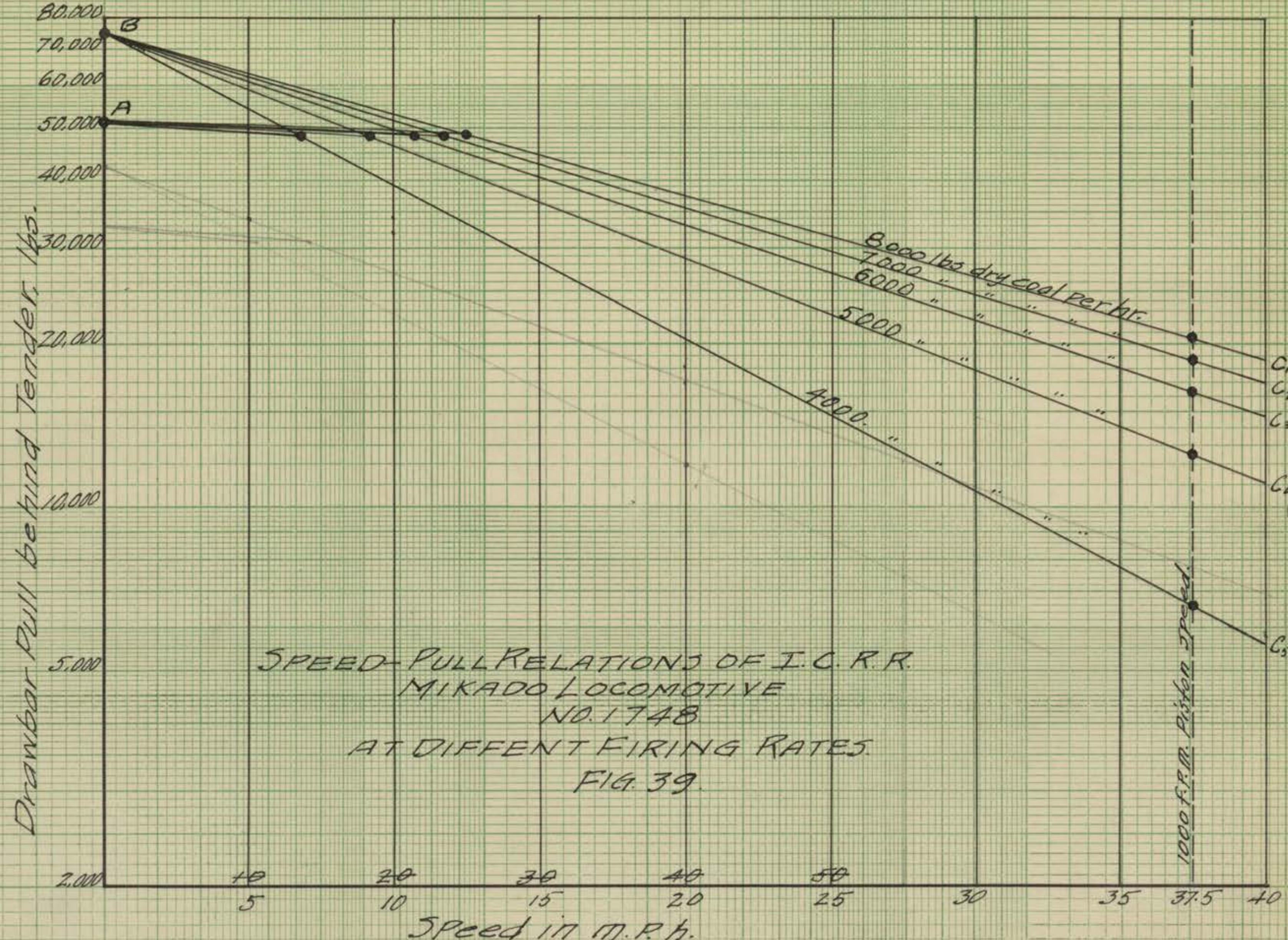
$$0.80pld^2/D = 48500 \text{ lbs.}$$

With these data the straight lines $B'C_1$, etc. of the semi-logarithmic co-ordinate system in Fig. 39 were drawn, and the curves in Fig. 38 are produced from these straight lines and the corners are replaced by the smooth curves tangent at 0.5x and 1.5x speeds.

It may be noticed in Fig. 38 that none of the five curves perfectly represents the experimental data plotted at the entire rate of speed but the points fall within the zones of the different firing rates or evaporation rates. This is due to the fact that the test was made at random firing rates and the curves, as they stand, represent the relations at several constant firing rates. The series of curves will, however, serve in estimating the tractive effort at any speed



Speed-Pull Relation I.C.R.R.Co.
Mikado Locomotive No.1748
at different firing rates
Fig. 38.
60,000
40,000
30,000
20,000
10,000
Drawbar pull behind tender, lbs.
Speed in miles per hour.
0
5
10
15
20
25
30
35
40
Note: The thermal value of the fuel is assumed to be 13500 B.t.u. per pound of dry coal.
8000 lbs dry coal per hour
7000 " " " "
6000 " " " "
5000 " " " "
4000 " " " "



SPEED-PULL RELATIONS OF I.C.R.R.
MIKADO LOCOMOTIVE
NO. 1748
AT DIFFENT FIRING RATES.
FIG. 39.
Drawbar Pull behind Tender, lbs.
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
5,000
2,000
Speed in m.p.h.
5
10
15
20
25
30
35
37.5
40
B
A
8,000 lbs dry coal per hr.
7,000 " " " "
6,000 " " " "
5,000 " " " "
4000 " " " "
1000 f.p.m. Piston Speed.
C1
C2
C3
C4
C5

and also at any firing rate, which will be very valuable data in the solution of problems such as fuel consumption, selection of locomotives, tonnage rating, etc.

VI. RESISTANCES TO THE MOTION OF RAILWAY TRAINS.

A. Train Resistance and its Importance.

It is said that, although there is no reason to doubt the validity of Newton's first law of motion it could not be demonstrated by purely physical experiments because a body in motion is always attended by some force or forces which act against the motion of the body. This criticism serves also as the statement of another physical law, that is, the law of existence of resisting force to the motion of a body. A railway train in motion cannot be an exception to this law, even when it is running on a smooth, level and straight track in perfectly still air. In fact, the motion of even a light local train is resisted by many thousand pounds of this resisting force and in the case of a very heavy freight train it is not unusual for the resistance to exceed a hundred thousand pounds. In order to keep a train in motion at speed this great resistance must be kept neutralized at the expense of an equal amount of locomotive tractive force. Thus one pound of reduction in train resistance is precisely equivalent to the one pound of increase in the tractive effort of the locomotive in their economic value. The Lake Shore and Michigan Southern, in 1873 estimated that $785,000. a year could be saved by reducing the train resistance by 25 percent.* The importance of a knowledge of train resistance arises in two ways. First, from the standpoint of the engineering departments, it is important because by correct analysis of train resistance they can recognize the elements of equipment and track which cause train resistances and can compare their relative importance. With this knowledge they can properly estimate the justifiable amount of investment for improvement of the parts in order to reduce resistances. Improvement of this sort is, no doubt, equally as important as that of motive power for the purpose of increasing their tractive effort. Secondly, the knowledge of the value or amount of train resistance is very important for the transportation department. Only with this data established can they effect a proper and economical tonnage rating of locomotives. In fact, this is the most important field of application and all the train resistance experiments were primarily made for this purpose.

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* Proc. A.R.E. and M.W.A., vol. 8, p. 188. (1907).

B. Classification of Train Resistances.

In its most general sense by "train resistances" we mean any force or forces, the resultant of which, acting directly opposite to the effective resultant of the motive forces, retards or tends to retard the motion of a railway train. There are several different ways of classifying train resistances, but the following is regarded as the most concise and convenient classification especially for their analytical investigation.

I. Inherent Resistances, or "Train-Resistance"

(a). Journal resistance.

(b). Rolling resistance.

(c). Atmospheric resistance.

(d). Miscellaneous resistance.

II. Incidental Resistance.

(a). Grade resistance.

(b). Acceleration resistance.

(c). Curve resistance.

(d). Wind resistance.

(e). Miscellaneous resistance.

III. Resistances peculiar to Locomotives.

(a). Machine friction, etc. of steam locomotives.

(b). Gear loss, etc. of electric locomotives.

In the paragraphs to follow a brief analysis of these resistances will be made, and in the next chapter a detailed study of the train-resistance or inherent resistances will be attempted.

C. Inherent Resistances.

1. The journal resistance is that due to the friction existing between the lubricated surfaces of the axle journal and the bearing and is sometimes called axle resistance. The nature of this frictional force is considered to be exactly the same as the journal resistance of a steam engine shaft, a turbine shaft, a dynamo shaft, etc., although the magnitude differs considerably on account of different methods of lubrication and the use of different lubricants. The journal resistance of railway trains is a very important element of train resistance because of its magnitude. As will be seen later, the resistance of trains - passenger or freight - at low speed, say below 10 or 15 m.p.h., is almost entirely due to this journal resistance, and even at 15 - 40 m.p.h. the journal friction is still the major part of the train resistance. The economic value of reducing this resistance is such that some advocate the use of ball or roller bearings in the journal boxes of railway cars.*

2. The rolling resistance includes (1) the rolling friction which is offered when wheels roll on a perfectly straight, level and rigid rail or track, and (2) the track resistance, which is due to (a) the imperfect alignment and joints of rails, (b) due to the work done by the wheels on the rails or by the train depressing the elastic rails or track continuously while it runs on the track. The rolling resistance varies greatly with the conditions of the track, it is,

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* Proc. Inst. C.E., v. 153, p. 227; Thurston's Friction and Lost Work, p. 376.

however, thought to be a very small part of the train resistance for well constructed and well maintained track, while it is of such magnitude for poor track that railway managers do not spare money for track improvement mainly to reduce this class of resistance.

3. The atmospheric resistance is the resistance of still air to a train in motion. This resistance may be further differentiated as (1) the head air resistance or the reactional force of the impact of the front end of a train in motion against still air; (2) the rear suction, due to the partial vacuum created by the train at its rear end; and (3) the skin friction or the surface friction on the sides, top and bottom of the train. As will appear, the atmospheric resistance is considerably influenced by the area and shape of the head and rear ends and also the condition of the sides, top, and bottom surfaces of the locomotive and cars; and improvement in this direction is slowly but ceaselessly progressing especially for high speed trains since the atmospheric resistance is the major part of train resistance at higher speeds.

4. The miscellaneous resistance includes all the inherent resistances which are not included in the first three classes of resistance mentioned above. Owing to the difficulty of separately investigating these resistances, we at present know little about their exact nature and magnitude. They, however, seem to be due to the complicated internal frictions of the train in motion, such as those between the couplers, side bearings, and energy consumed by the vibration, and also to

external friction or the increase in rolling resistance, all due to the oscillation and concussion of the train*.

D. Train-Resistance Formulas and Diagrams.

The proper method of determining the aggregate amount of these inherent resistances or train-resistance is the road test with a dynamometer car, although it is not impossible to determine it by other methods, such as a road test without a dynamometer car, a "run off" test, and a "gravity" test, the last two of which are the methods usually employed in continental Europe. There are about seventy different formulas, among which the following are the most reliable and the most extensively used in this country.

(1). Freight train resistance. - The result of most elaborate freight train resistance tests made on a well constructed and well maintained track under the service conditions usual to ordinary train operations**, is shown in Figs. 45 and and 46. The formula, (1) represents very closely the entire

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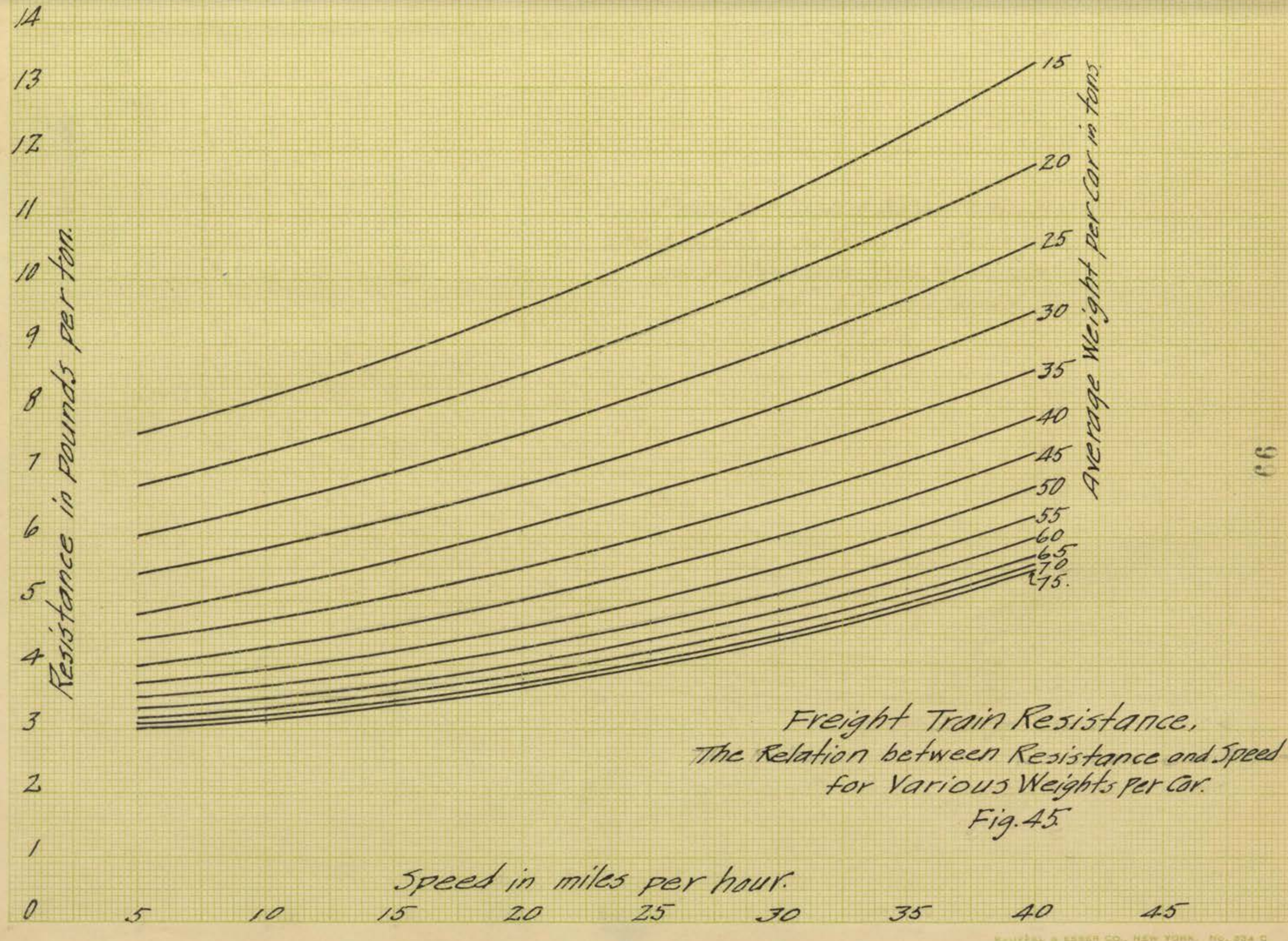
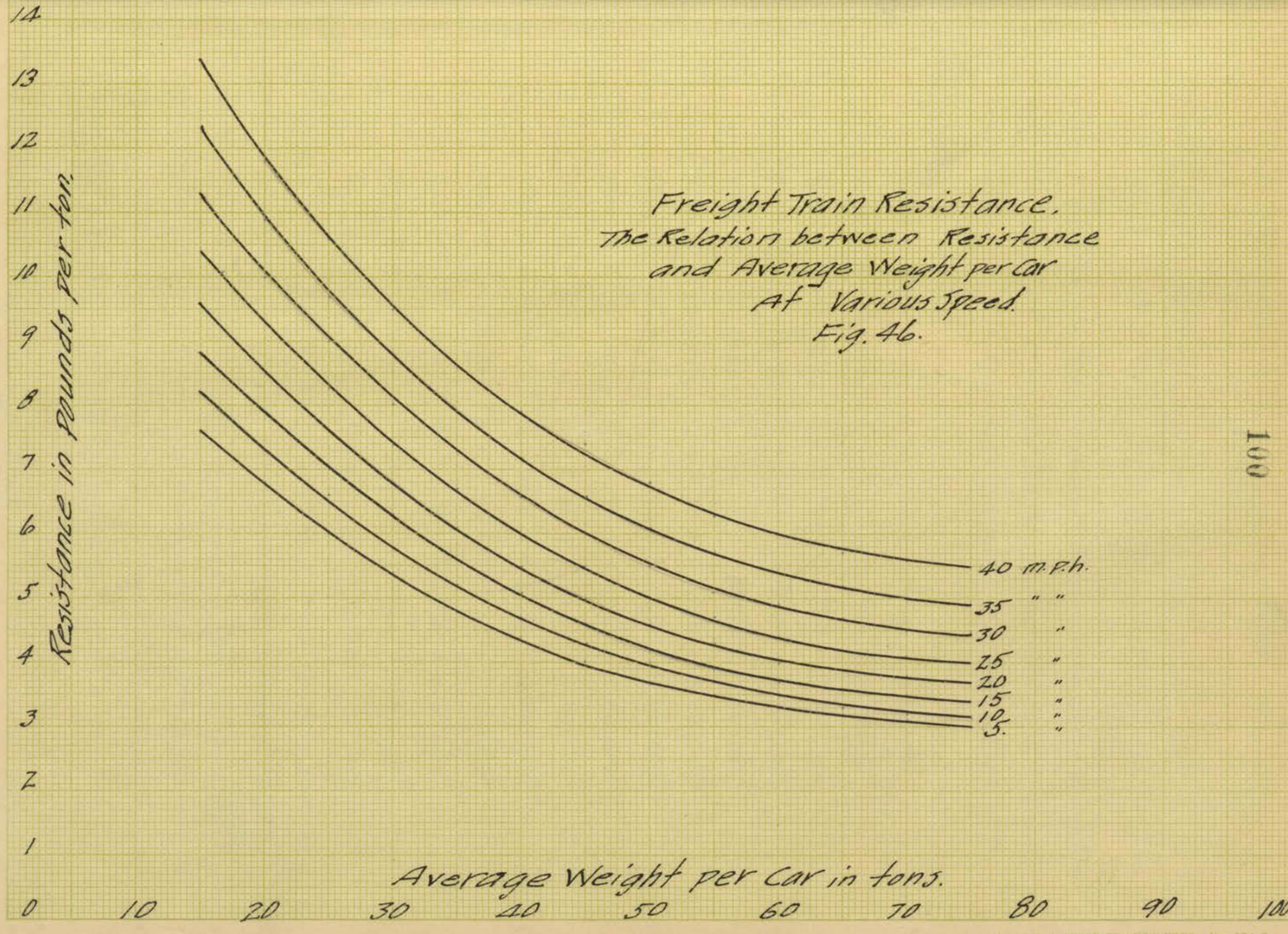
* Strahl: "Is the endway oscillation of locomotive a disturbing movement?", Bul.Int.Ry.Cong.,Jan. 1908; George Marie: "Les oscillations du materiel dues au materiel liu-meme et les grandes vitesses des chemins der fer", Rev.Gen.des Chemins des Fer, May 1907; Mehlis: "Theoretische Betrachtungen ueber die Schwingungen von Schnellfahrenden D-Zugwagen und deren practische Messung", Classer's Ander Gew. und Bau., May 1, 1908; Waddigen: "Untersuchungen ueber das unruhige Laufen der Drehgestellwagen", Glasser's An.der Gew. und Bau., Mar. 1,1909; "Das Wanken der Lokomotiven ~~un~~ unter Berucksichtigung des Federspiels", Zeit.d.Ver.deut.Ing., April 3, 1909.

** E. C. Schmidt: "Freight Train Resistance", Eng. Exp. Station, University of Illinois, Bulletin No. 43.



Freight Train Resistance,
The Relation between Resistance and Speed
for Various Weights per Car.
Fig. 45.


Freight Train Resistance.
The Relation between Resistance
and Average Weight per Car
At Various Speed.
Fig. 46.
Resistance in Pounds per ton.
Average Weight per Car in tons.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
10
20
30
40
50
60
70
80
90
100
40 m.p.h.
35 " "
30 "
25 "
20 "
15 "
10 "
5. "

result of the test.

$$R = \frac{S + 36.6 - 0.031W}{4.08 + 0.152W} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where R denotes the train resistance in pounds per ton, S the speed in miles per hour, and W the average car weight of the train in tons. The following formulas give the values of R within 1/2 percent.

When $W = 15$ tons, $R = 7.15 + 0.085S + .00175S^2$

When $W = 20$ tons, $R = 6.30 + 0.087S + 0.00126S^2$

When $W = 25$ tons, $R = 5.60 + 0.077S + 0.00116S^2$

When $W = 30$ tons, $R = 5.02 + 0.066S + 0.00116S^2$

When $W = 35$ tons, $R = 4.49 + 0.060S + 0.00108S^2$

When $W = 40$ tons, $R = 4.15 + 0.041S + 0.00134S^2$

When $W = 45$ tons, $R = 3.82 + 0.031S + 0.00140S^2$

When $W = 50$ tons, $R = 3.56 + 0.024S + 0.00140S^2$

When $W = 55$ tons, $R = 3.38 + 0.016S + 0.00142S^2$

When $W = 60$ tons, $R = 3.19 + 0.016S + 0.00132S^2$

When $W = 65$ tons, $R = 3.06 + 0.014S + 0.00130S^2$

When $W = 70$ tons, $R = 2.92 + 0.021S + 0.00111S^2$

When $W = 75$ tons, $R = 2.87 + 0.019S + 0.00113S^2$

2. Passenger train resistance. - in 1915, a series of passenger train resistance tests* was undertaken with regular service passenger trains on the well ballasted tracks of the New York Division and on the W.J.& S.R.R., Pa.R.R. Co. The followingformula represents the results very closely and can be

---

* Pennsylvania Railroad Test Department, Bulletin No. 26, page 25.

used for the estimation of train resistance on well constructed and well maintained tracks:

$$R = \frac{100}{W} + 1.5 + \frac{V(V + 16)}{100\sqrt{W}} \quad \dots\dots\dots\dots (2)$$

in which V is the speed in miles per hour, W the average weight per car in tons, and R the "average maximum" resistance in lbs. per ton.

The results of other passenger train resistance tests*, which are regarded as applicable to trunk line passenger service are exhibited in Figs. 47 and 48.

3. Train resistance formula used in electric traction**-

$$R = \frac{50}{\sqrt{W}} + 0.03S + \frac{0.00aS^2}{W}\left(1 + \frac{n - 1}{10}\right) \quad \dots\dots\dots\dots (3)$$

where R represents the train resistance, lbs. per ton; W the car weight in tons; a the cross section of car, sq. ft.; and n the number of cars. The diagram in Fig. 49 is based on this formula.

E. Incidental Resistances.

1. Grade resistance. - When a train is running on an up-grade it encounters a certain resistance additional to that on level track. This extra resistance is called grade resistance. As mentioned before, this resistance is due to the force expended to increase the potential energy of the train or to elevate its position, and it has been demonstrated that the

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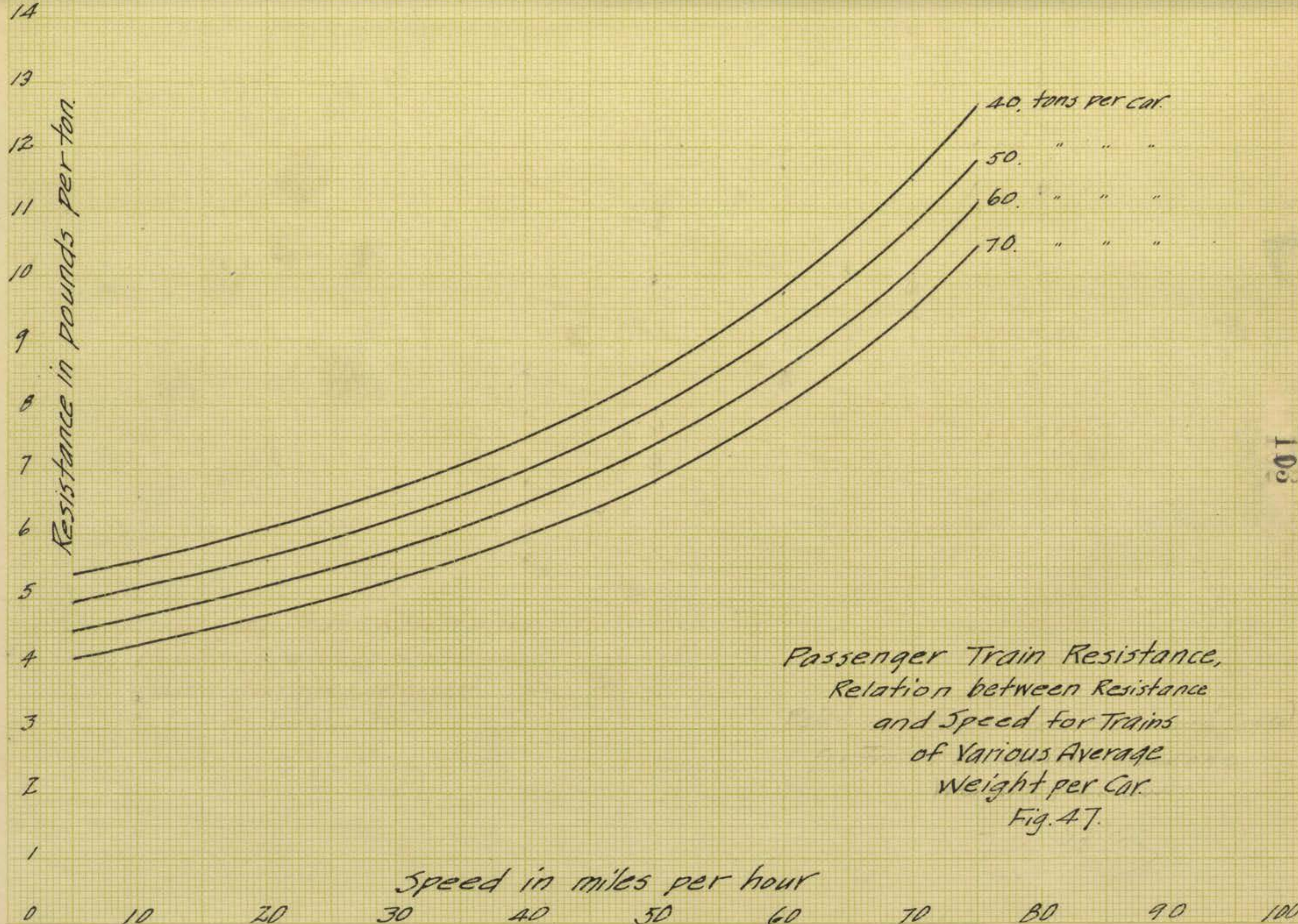
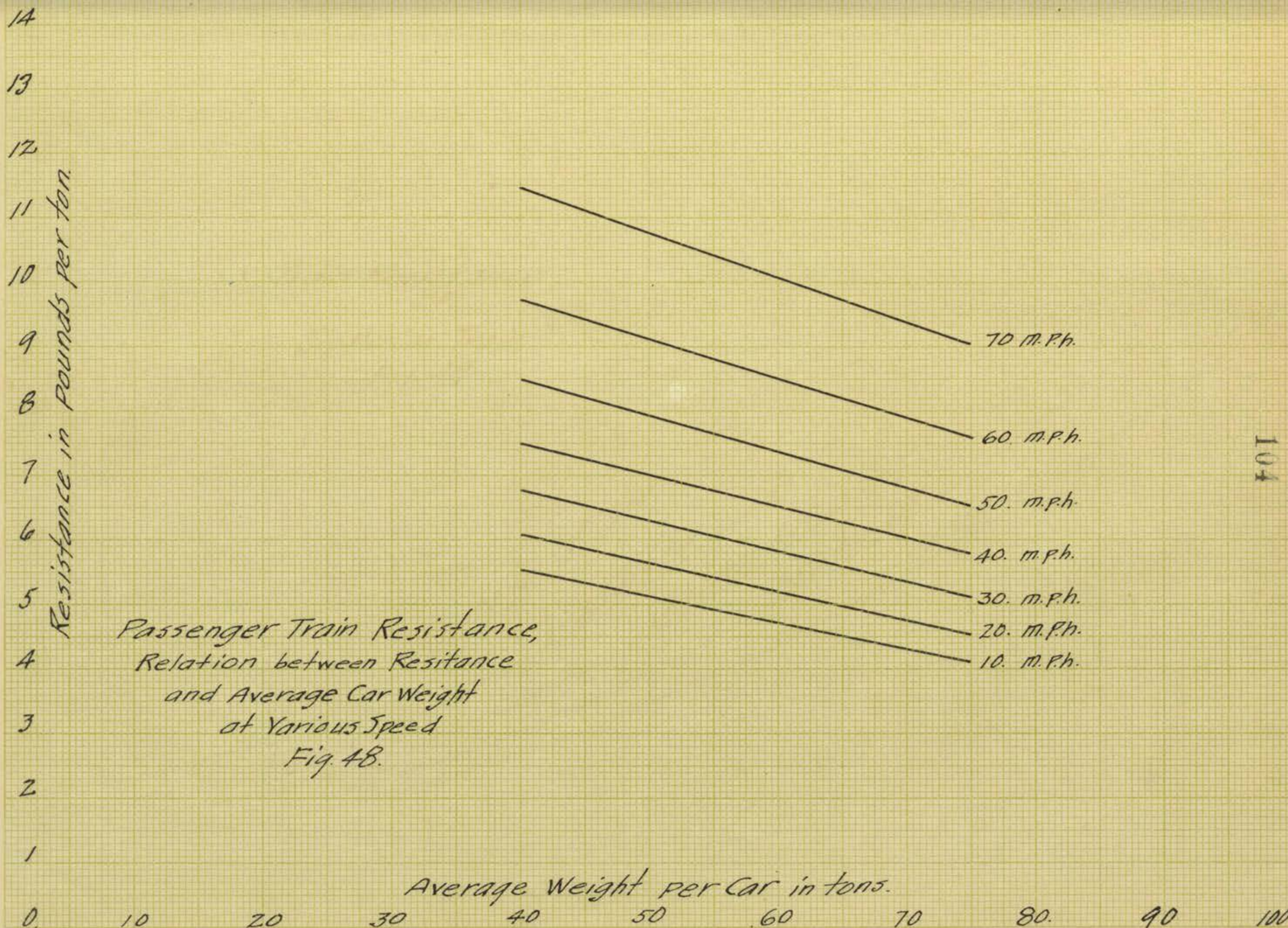
* E. C. Schmidt and H. H. Dunn: "~~The~~ Passenger Train Resistance" Bul. A.R.E.A., vol. 18, p. 689. (Feb. 1917).

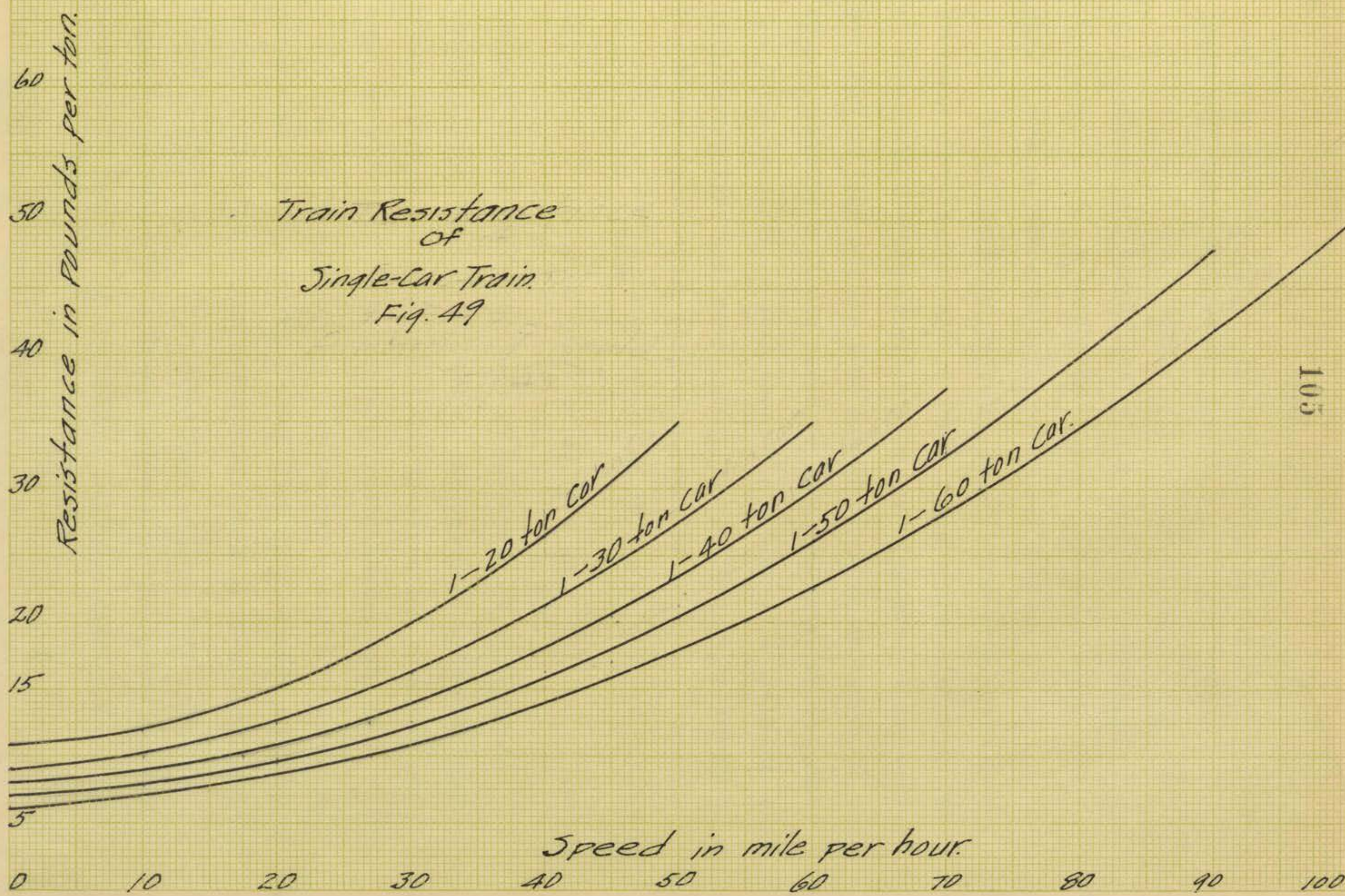
** A. H. Armstrong: "Electric Traction", Standard Handbook for Electrical Engineers, Section 13.



Passenger Train Resistance, Relation between Resistance and Speed for Trains of Various Average Weight per Car.
Fig. 47.



Passenger Train Resistance,
Relation between Resitance
and Average Car Weight
at Various Speed
Fig. 48.



Train Resistance
of
Single-Car Train
Fig. 49

grade resistance in lbs. per ton is

$$R_g = 20.G. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

if G represents the gradient of the track in percent, or

$$R_g = 0.379f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4')$$

when f denotes the gradient in feet per mile.

2. Acceleration resistance. - It has been shown in a previous chapter that when a train changes from higher to lower speed its inertia force acts in the direction of its motion, and that it requires the same amount of force in order to change from a lower to a higher speed or to accelerate the motion. The force required to accelerate the motion of a train is called acceleration resistance, and its magnitude is expressed by the formula previously developed, that is,

$$R_a = (91.05 + 145.5N/W)A \quad \ldots\ldots\ldots\ldots \quad (5)$$

or

$$R_a = (66.7 + 106.7\frac{N}{W})(\frac{V_2^2 - V_1^2}{S}) \quad \ldots\ldots\ldots \quad (5')$$

where N is the number of cars in the train, W the average weight of car in tons, V_1 and V_2 the speed in miles per hour, S the distance in feet, and A the acceleration in miles per hour per second. For approximate formulas see page 27.

3. Curve resistance. - Curve resistance is the extra resistance experienced by a train when it runs over a curved track in addition to the train resistance on level and tangent track in still air. This resistance is due, first, to the slipping of the outside wheels forward or the inside wheels backward on account of the unequal distance displaced by the

wheels of equal diameter* fixed rigidly on a common axle; and second, to the rubbing or grinding action of wheel flanges when the elevation of the outer rails is not sufficient and when excessive**. Another factor which properly is to be attributed to curve resistance is the resistance due to the friction of center pins and side bearings in order to adjust the position of the truck bodies to the curvature on entering the curve and to the tangent on leaving it.# When compared with grade resistance curve resistance is generally very small; nevertheless it is worthy of attention for there are cases in which a series of curves limits tonnage as a ruling grade does.## The exact nature and magnitude of curve resistance are not known. There are, however, several formulas proposed:

$$R_c = 0.5D \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$R_c = 0.8D \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

---

* The tread of wheels is usually coned so that the slipping on curves may be avoided, but it is difficult to have a proper taper of the cone which avoids the slipping on curves of any degree and for any speed of a train, further even if the wheel could be coned ideally it would become out of shape by wearing; thus the slipping is unavoidable.

** The elevation of outer rail on a curve can be done for only one definite speed, but trains of different speeds have to run on the same track, so the grinding action is also unavoidable in practice.

Pennsylvania Railroad Test Dept. Bulletin No. 26, p. 16; and George Marié: "Les oscillation du materiel des chemins de fer à l'entrée en courbe et à la sortie", Mem. Soc. Civ. de France, Nov. 1905.

A. M. Wellington: "Economic Theory of Railway Location, p. 327; and E. Spirgatis: "Berechnung der Fahrzeiten ...", p.5.

in which R_c is the curve resistance in pounds per ton and D the degree of the curve. The formula (6) is one which had been popularly used for many years before the formula (7) was recommended by the committee on Economics of Railway Location, and it is still used by electric railway engineers and some steam railway engineers. The formula seems to give very satisfactory results when applied to sharp curves such as those used in street railways, but too low for the curves generally found on main lines of steam roads, for which (7) is more generally applicable.* The following formula

$$R_c = 0.4 + (0.21 + 0.035A)D \quad \ldots\ldots\ldots\ldots \quad (8)$$

which takes into consideration the wheel base of truck, A was developed by Dean W. G. Raymond** of the State University of Iowa and is claimed to give very satisfactory results. He states, "It is probable that the results of the formula", referring to the formula (8), "should be somewhat increased for very low velocities, and possibly diminished for very high velocities, since the coefficient of sliding friction varies with the velocity of sliding" and he gives the following formula for high speed passenger trains:

$$R_c = (.4 + (.21 + .035A)D)(1 - \frac{s - 20}{200}) \quad \ldots\ldots\ldots \quad (8')$$

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* Pennsylvania Railroad Test Dept. Bulletin No. 26, p. 16.

** W. G. Raymond: Element of Railroad Engineering, p. 177. For another curve resistance formula which takes in consideration the wheel base and also the gauge of tracks, see S. Whinery, Transactions of the American Society of Mechanical Engineers, April, 1878, or Thurston's "Friction and Lost Work", p. 213.

where s is the speed of train in m.p.h., and R_c, A, and D are as defined before. It is doubtful, however, as both he and Wellington* state , whether curve resistance decreases with speed. As a direct contrast, we have the following formula** which indicates an increase in curve resistance with speed:

$$R_c = 0.058SC \quad .... (9)$$

In formula 9 Rc represents the curve resistance, pounds per ton; S the speed, miles per hour; and C the degree of curvature.

The coefficient of friction between non-lubricated surfaces like those of wheels and rails and of brake shoes and wheels, decreases with speed as will be seen in Fig. 50. The speed of slipping of wheels is about 2 m.p.h. when a train is running on a 10 degree-curve at 60 m.p.h. and the variation of the coefficient with speed is very great. The speed of the wheel flange is, however, the same as that of the train, and the coefficient decreases slowly with the speed. So far these facts confirm the statements of Raymond and others. But it is not true when we take in consideration the effect of centrifugal force. Suppose the superelevation of a track with 10 degree curvature, for instance, is designed for 50 miles per hour train speed. Then any train running at the speed 50 m.p.h. has no lateral pressure on the rails, and the curve resistance is entirely due to the slipping of either or both of the outer and inner wheels. A train running at 80 m.p.h.,

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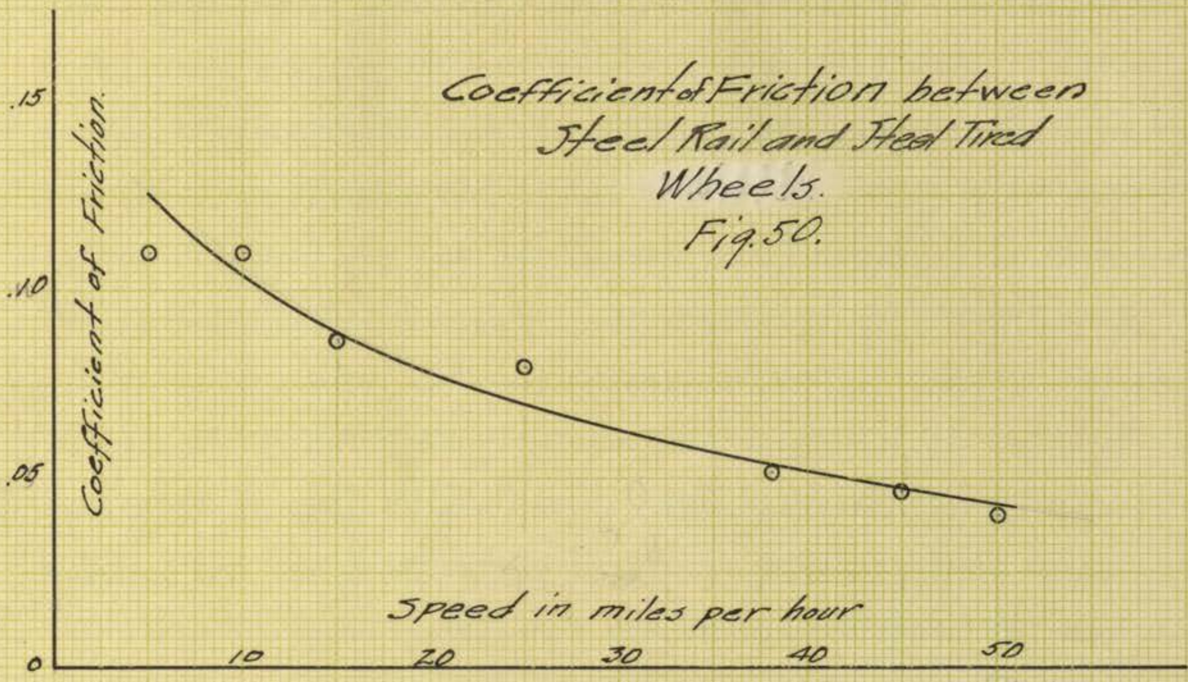
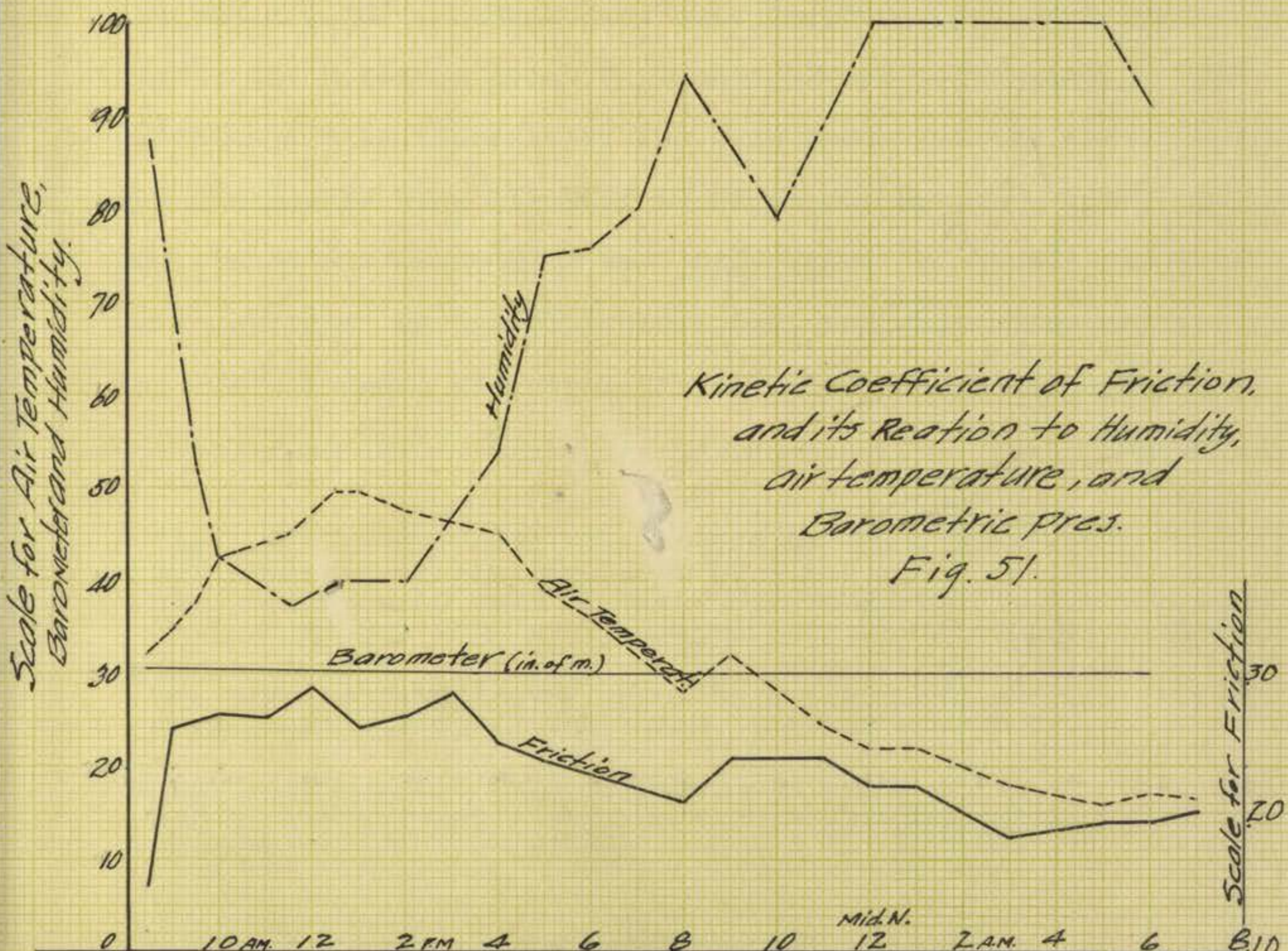
* Wellington's Economic Theory of Railway Location, p. 911.
** Schmidt and Dunn: The Tractive Resistance on Curves of a 28-ton Electric Car., Eng. Exp. Sta. U.of I. Bulletin No. 92.


Coefficient of Friction between
Steel Rail and Steel Tired
Wheels.
Fig. 50.
Coefficient of Friction.
Speed in miles per hour
.15
.10
.05
0
10
20
30
40
50


Kinetic Coefficient of Friction,
and its Reation to Humidity,
air temperature, and
Barometric pres.
Fig. 51.
Scale for Air Temperature,
Barometer and Humidity.
Scale for Friction
Humidity
Air Temperat.
Barometer (in. of m.)
Friction
100
90
80
70
60
50
40
30
20
10
0
10 AM.
12
2 PM
4
6
8
10
Mid.N.
12
2 AM.
4
6
8
30
20
10

however, has lateral pressure on the outer rails, which equals $(80 - 60)^2$ times a certain constant divided by the radius of curvature or about 210 pounds per ton in this case. If we assume the coefficient of friction at 80 miles per hour as 0.10, the curve resistance due to the flange action is 21 pounds per ton or 2.1 lbs. per ton per degree at 80 m.p.h. In this case the slipping will, probably, occur only on the inner rails since the pressure on the inner rails is reduced by about 10 percent more or less, depending upon the position of the center of gravity of the car. The slipping speed is about 2.5 m.p.h. and the coefficient is about 0.20 at this speed. The reduction in curve resistance due to the decrease in the weight on inner rails is about 1.3 pounds* per ton or .13 lbs. per ton per degree. Hence the net increase in curve resistance due to the speed is about 2 pounds per ton per degree. The resistance will increase by the same amount if the train runs on the same curve at a speed of 40 miles per hour. Thus, curve resistance increases with the difference of actual speed and rated speed** of the superelevation, and quite independently of the actual speed of a train. This does not, however, mean

---

* 210 x 0.20 x (2.5/80) = 1.3 since actual displacement is only 2.5 while the train displaces 80 miles in an hour. Due to the clearance of 3.8" to 3/4" between wheel flanges and rails, wheels may not slip continuously but slip certain distances and then roll, then slip and roll again. etc. Thus the actual time of slipping is equal to 2.5/80, and average resistance throughout the time is 1.3 lbs. instead of 4.2 lbs.

** By "rated speed" is meant here the speed at which the resultand of centrifugal force and the weight passes through the center line of the track.

that the curve resistance is least at the rated speed of a curved track. The outer leading wheel of each truck runs against the rails even when the actual speed is considerably lower than the rated speed. At a certain sufficiently low speed - perhaps about 10 or 20 m.p.h. below the rated speed - the leading wheels too will not continuously grind either outer or inner rails. This is the speed at which the curve resistance is lowest, and below this speed the resistance will increase as the speed decreases, since the gravity force attracts the inner wheels to rails and increases the resistance.

4. Wind resistance. - Sometimes railway cars left on tracks in an open yard are started by the pressure of storm winds overcoming the starting resistance of the car. It was said that George Westinghouse got his first idea of the air brake by observing a railroad train stalled by strong wind. Thus we see what wind resistance means to the motion of railroad trains. If the wind blows exactly in the direction of the motion of a train it assists the motion, but if the relative direction is shifted the normal component will augment the flange friction to such an extent that the effect is decidedly worse* than the direct head wind, although we have not data sufficient to determine the exact relation of its direction and speed to the wind resistance. If the wind is blowing exactly opposite to the motion of the train, the wind

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* For further information see "The effect of cold weather upon train resistance and tonnage rating", by E.C. Schmidt and F. W. Marquis, Eng. Exp. Station, U. of I. Bul. No. 59. and Proc. A.R.M.M. vol. 47, 1914.

resistance may roughly be estimated by the formula,

$$R_w = 0.0025AV^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

in which R_w is the resistance in lbs., A the exposed area in sq. ft., and V the speed in m.p.h. The occurrance of such strong wind is uncertain and rare, and only in practical tonnage rating in yards a certain allowance is made for this resistance.

5. Miscellaneous incidental resistances.* - Abnormally low temperature, say below 32 deg. F., reduces the viscocity of lubricants, and thus increases materially the journal resistance. The density of the air at zero deg. F. is about 14 percent greater than that of at 70 deg. F. and its effect on the resistance of high speed trains must be considerable. Further, the low temperature lowers the efficiency and capacity of the locomotive, which is equivalent to an increase in train resistance. Moreover, the adhesion of drivers, on which the tractive effort at low speeds depends, is greatly reduced by moisture or snow on the rails. All these elements must be borne in mind for actual tonnage ratings.

F. Resistances Peculiar to Locomotives.

1. A steam locomotive has a complicated power transmission mechanism between the pistons and the drawbar. The loss of energy or the resistance in this mechanism is called machine friction. Investigation** shows that this resistance

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* For further information, see "The effect of cold weather upon train resistance and tonnage rating", by E.C.Schmidt and F.W.Marquis, Eng. Exp. Sta. bulletin No. 59, Univ. of Ill.

** American Locomotive Company, Bulletin No. 1001, p. 4.

amounts to about 22.2 pounds per ton of weight on the driving wheels. Later experiments*, however, indicate that this estimate is too low and 25 pounds per ton, or the following formula#, which includes the rolling resistance of the drivers, represent more closely the result of actual tests.

$$R_m = 18.7W + 80.D \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(11)}$$

in which R_m is the machine friction, including the rolling resistance of the drivers in pounds per ton of the total weight on drivers W; and D the number of driving wheels. The train resistance of the trucks of the locomotive proper and of the tender can be estimated by considering them as freight cars of appropriate weight by means of the formula, (1) on page 101. Besides these resistances the locomotive coupled to the head of a train encounters always the head air resistance, which may be expressed by the formula,

$$R_a = 0.0025AV^2.$$

2. An electric locomotive. - Although there is no experimental data available on the machine friction of electric locomotives, it is generally thought to be, due to the absence of heavy reciprocating mass and the uniformity of the torque, less than that of a steam locomotive of the same power. Electric locomotives, however, have special heavy rotating masses - motors and gears or main rods-which require extra energy in accelerating the train. Other resistances of electric locomotives

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* Laboratory Tests of a Consolidation Locomotive, by Schmidt and others, Engineering Experiment Station Bulletin No. 82, U.of I.

Proceedings American Railway Engineering Association, vol. 11, part 2, p. 644.

may be estimated the same way as for steam locomotives.

All the incidental resistances, such as grade, curve, and acceleration resistances must also be considered for all locomotives - steam or electric.

VII. DERIVATION OF A GENERAL FORMULA FOR TRAIN RESISTANCE.

A. Importance of a Rational Train Resistance Formula.

The subject of train resistance is one that has attract- the attention of railway engineers ever since the inception of railway transportation.* The need of more accurate knowledge of

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* Nicholas Wood: "Practical treatise on railroads, and internal communication in general", p. 169-201, 1st ed. London (1825)
Thomas Tredgold: "A practical treatise on railroads and carriage", p. 39-62, 1st ed., London (1825).

De Pambour: "Traite theorique et practique des machines locomotives", p. 134, 1st ed. Paris (1835).

this subject was, however more fully realized when the busy but prosperous period of trunk line expansion closed and the attention of railway managers turned to scientific and economical railway train operation. Many series of painstaking and careful experiments on train resistance have been made by prominent engineers and able investigators at different times and places, and no less than seventy different train resistance formulas have been proposed each claiming accuracy and reliability. In spite of this fact, however, much uncertainty still exists, the results and formulas differing widely and yet leaving unexplained the causes and reasons for the variations. Not only do their numerical results diverge widely, but the formulas are of forms very different in their fundamental functions. In the practical application of train resistance formulas accuracy is, no doubt, the most important requirement and their rationality need not be so much insisted upon. But a formula which gives accurate numerical results and which is consistent in form with the underlying theories is certainly more valuable than an ordinary empirical formula, which has no physical meaning.

In this chapter an attempt is made, first, to determine a characteristic equation of inherent train resistance as a function of speed, weight of cars, and several other factors which have an important influence on train resistance, by means of an analytical study of elementary resistances such as journal, rolling, and atmospheric resistances; and then to determine the constants in the equation by means of the results of the most reliable train resistance experiments, so that the formula thus derived may be consistent with the theories and that it may at the same time furnish accurate numerical values of train resistance under given conditions.

B. Journal Resistance.

1. Bearing friction and journal resistance. - In discussions of journal resistance, the terms, "bearing" or "journal friction" and "journal resistance" are often taken as synonyms. It is, however, convenient to have the meaning of these terms distinguished. Bearing or journal friction is the frictional force whose point of application is on the contact surface of the journal and bearing, while journal resistance is that force measured at the tread of the wheel; and they have the following relation:

$$R_j = \frac{a}{d} F_j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where R_j represents journal resistance, F_j the journal friction, and a and d the diameters of journal and wheel respectively.

The Equation (1) may be rewritten as follows:

$$R_j = 2000\frac{a}{d}m \text{ pounds per ton} \quad \ldots\ldots\ldots \quad (2)$$

in which m denotes the coefficient of journal friction. The values of a and d can be determined without trouble, hence we can determine by means of (2) the journal resistance as soon as we get the value of m. To determine the precise value of m under various conditions, however, is not an easy problem. Many valuable results of experiments undertaken by several authorities on the subject of friction have been published* and its magnitude under different conditions, as well as some of its auxiliary laws, are known; but the general law which governs the variation of m is not yet known.

2. Coefficient of friction as a function of speed, pressure and temperature. - A careful study of the published results of experimental investigations leads to the conclusions: first, that when other conditions remain constant the relation between the coefficient of friction, m, and the velocity of the journal, v, can be expressed by an equation of the form,

$$m = a\sqrt{v}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)^{**}$$

where a is a certain constant; and second, that when other things are constant, the coefficient of friction is an inverse function of the intensity of bearing pressure, p, or

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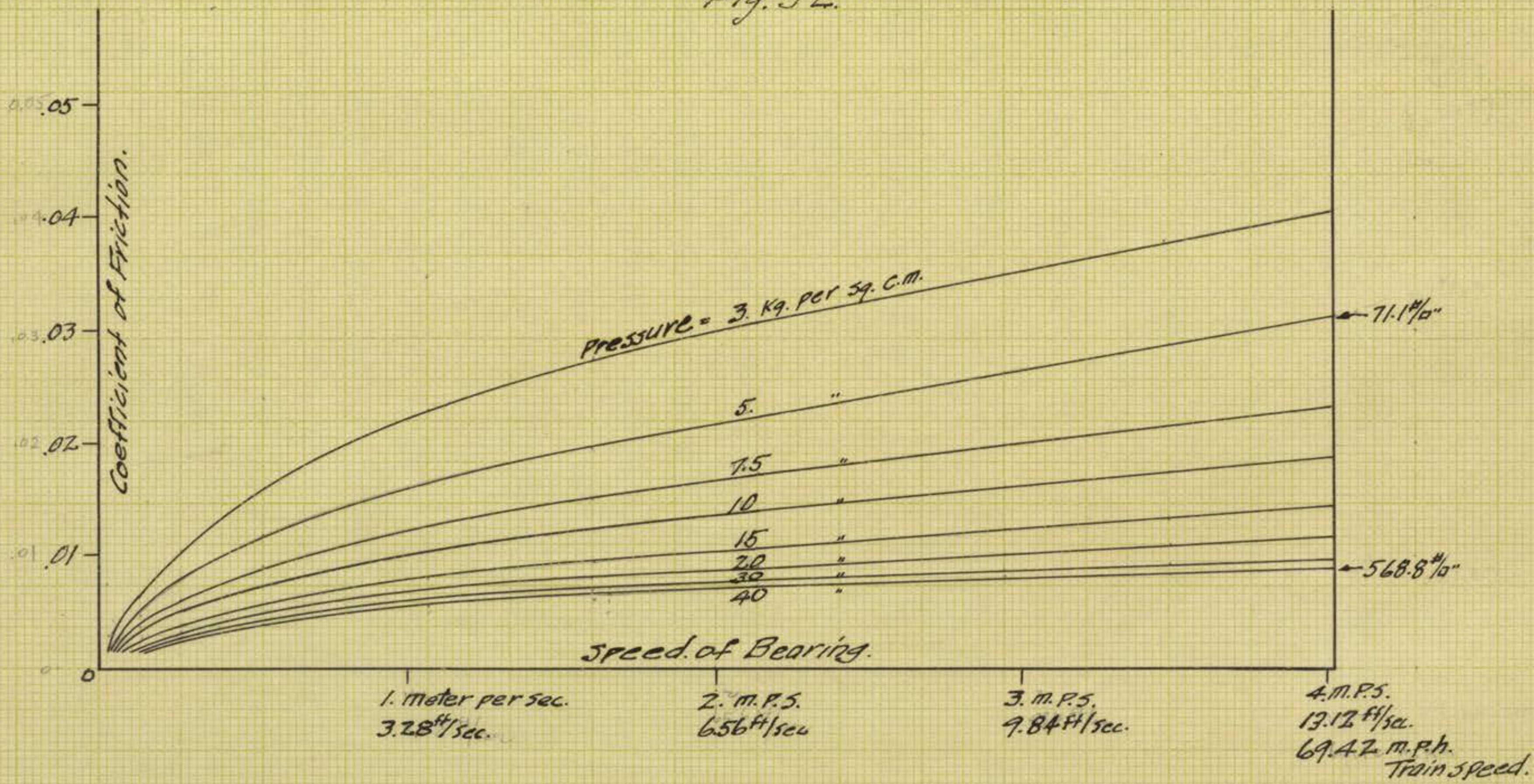
* R. H. Thurston's "A Treatise on Friction and Lost Work", Wiley & Sons, publishers.
R. Stribeck: "Die wesentlichen Eigenschaften der Gliet - und Rollenlager", Zeit.d.Ver. Deut. Ing., vol. 46 (1902) pp. 1341, 1432, and 1463.
O. Lasche: "On Bearings for High Speeds", Traction and Transmission, vol. 2, No. 22, (1903).
J. E. Denton: "Special Experiments with Lubricants", Tras. A.S.M.E. vol. 12, p. 405, (1891).
G. Goodman: "Recent Reserches in Friction", Proc.I.C.E. V. 85, p. 376 (1886).
C.J.H.Woodbury: "Measurements of Friction of Lubricating Oils" Engineering, vol.   p. 532, (1884).
B. Tower: "Theory of Lubrication", Phil. Trans. 1886; Inst. M.E. Sept. 28, 1883 or Engineering, vol. 36, p.451, (1883).

** The form is correct for speeds up to about 50 or 60 m.p.h. except for very low speeds, i.e., below 4 m.p.h. when cars are heavily loaded and 1 m.p.h. when they are lightly loaded. It is also fairly correct for speeds between 60 and 100 m.p.h. (see Stribeck's, Lasche's and Thurston's works. The diagrams in Fig. 52 is reproduced from Stribeck's figure 10 on page 1437.

Variation of Coefficient of Friction
with Speed and Pressure at
Constant Journal Tempe.
R. Stribeck.
Fig. 52.



$$\underline{\underline{m}} = \frac{\text{constant}}{p + b}, \qquad \text{.....................} \; (4)^*$$

where b is a certain constant. Then combining (3) and (4), we get

$$\underline{\underline{m}} = \frac{a' \sqrt{v}}{p + b}, \qquad \text{.......................} \; (4')$$

Further, the results of the experiments indicate that when other things remain constant, the coefficient varies inversely as the temperature of the journal, t, that is,

$$\underline{\underline{m}} = \frac{\text{constant}}{t} \qquad \text{.....................} \; (5)^{**}$$

Combining, then (4') and (5), we have

$$\underline{\underline{m}} = \frac{a'' \sqrt{v}}{(p + b)t} \qquad \text{......................} \; (5')$$

In equation (5') p, v and t are mathematically all independent variables, but physically t depends upon both p and v. When we consider a very wide range of pressure, the journal temperature is certainly a function of the intensity of journal pressure but under ordinary service conditions of car loading, t may be considered as independent of p. The increase in journal temperature due to increased speed of journal, however, should not be overlooked. The experimental data on journal temperatures indicate, as shown in Fig. 54, that the relation may be quite well expressed by an equation of the form

$$t^2 = kv \quad \text{or} \quad t = k\sqrt{v}$$

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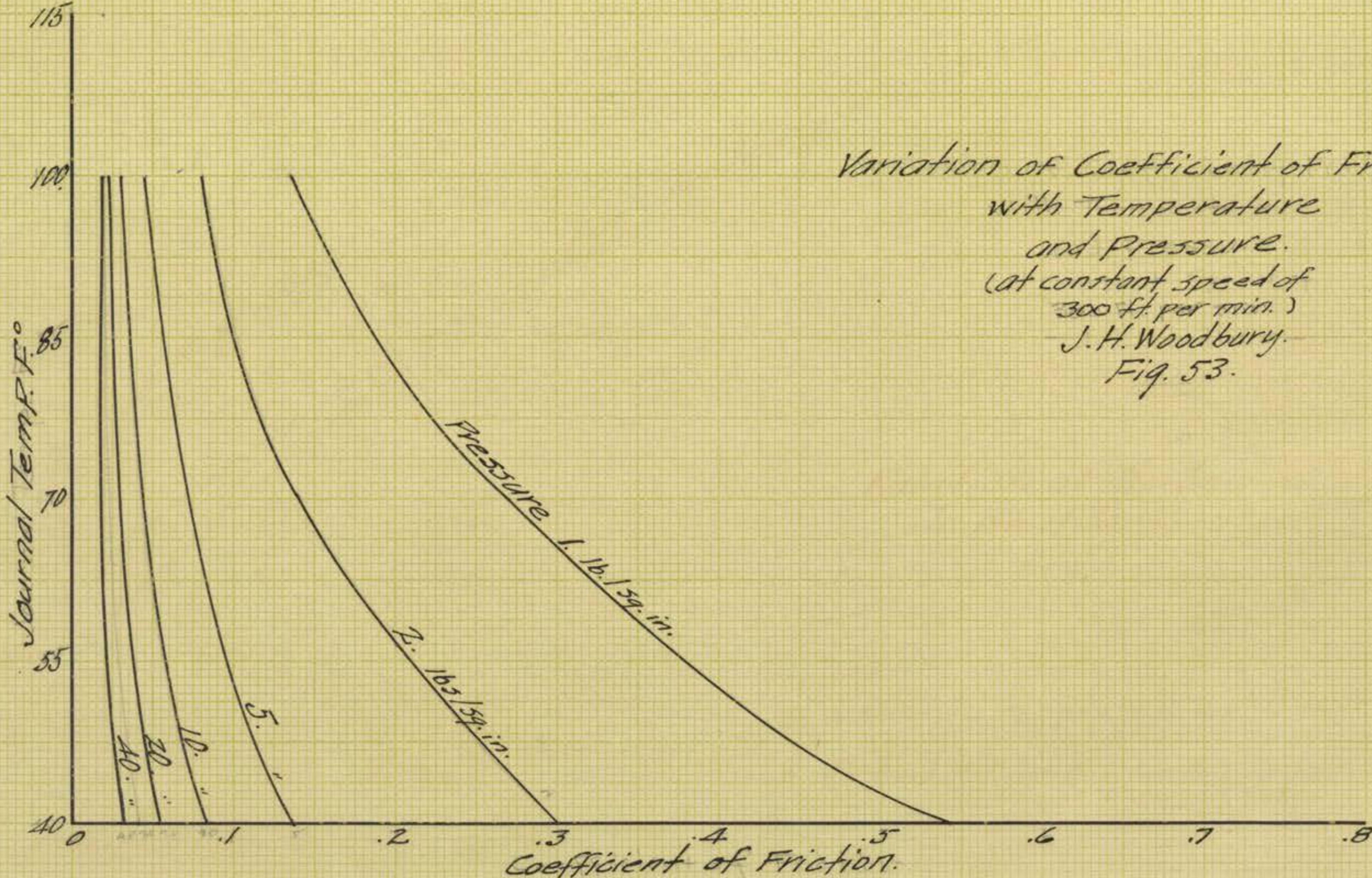
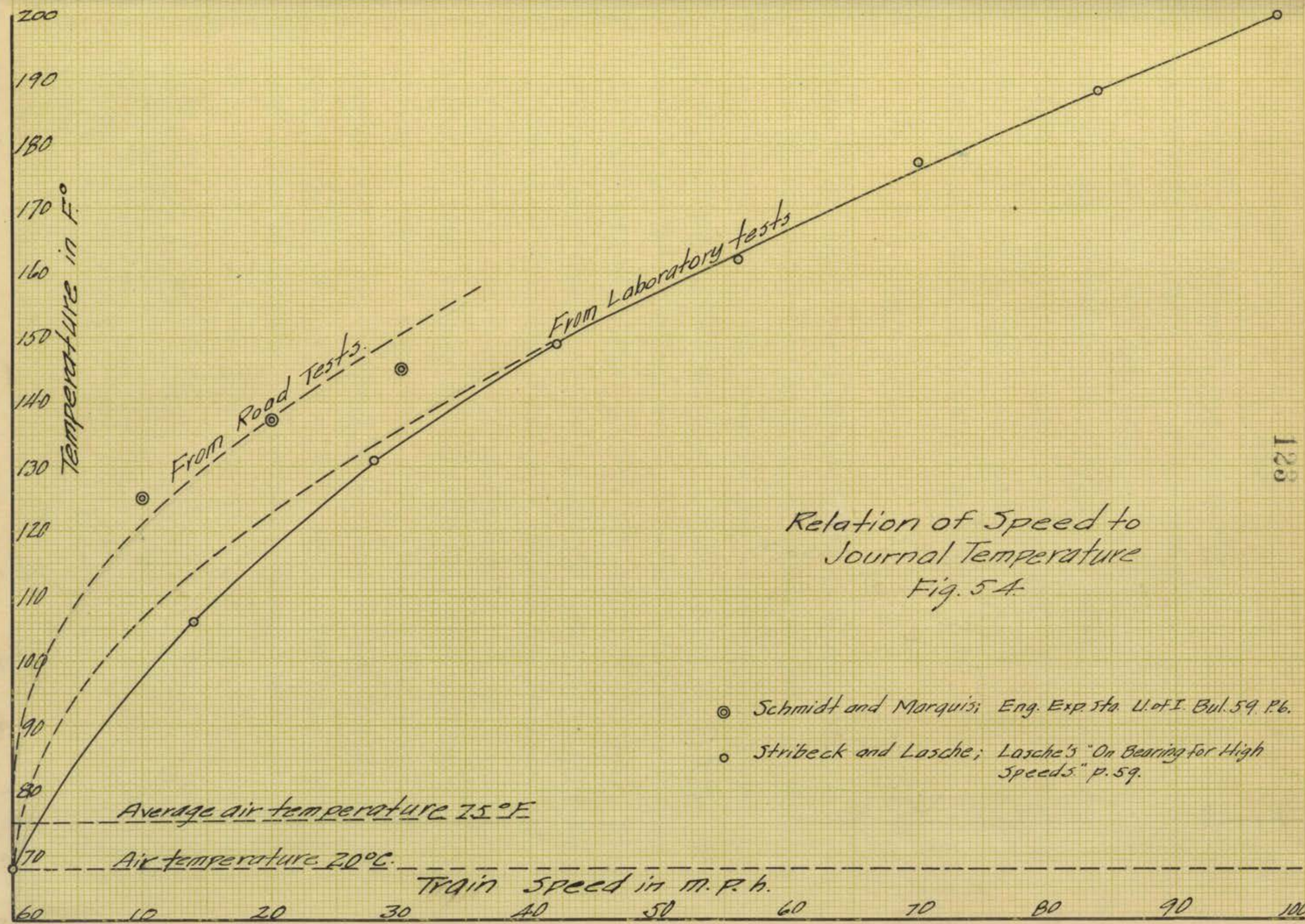
* Thurston gives a formula $\underline{\underline{m}}$ = constant/$p^{\frac{2}{3}}$, but (4) is practically (geometrically) equal to this formula and it is easier to apply.

** See Fig. 53.



Variation of Coefficient of Friction
with Temperature
and Pressure.
(at constant speed of
300 ft. per min.)
J. H. Woodbury.
Fig. 53.



Relation of Speed to
Journal Temperature
Fig. 54.

in which* v is the speed, k a constant, and t the rise in journal temperature above that of the atmosphere. We consider in this connection only difference in temperature between the journal and the atmosphere and postpone for consideration under "effects of weather" the changes in journal resistance due to changes in temperature of the surrounding air. Then, substituting this for t in (5'), we get simply

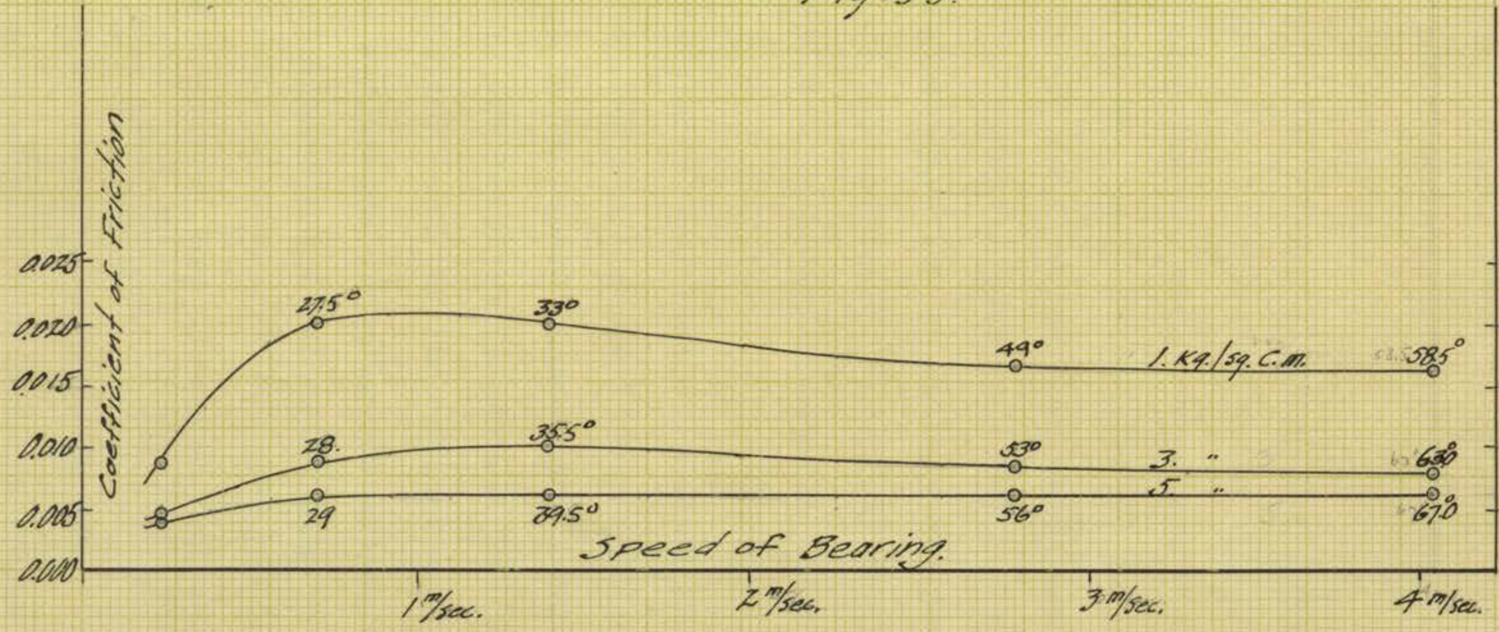
$$\underline{\underline{m}} = \frac{k'}{p + b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (I)$$

where k' is some other constant. This is a characteristic equation of the coefficient of friction between lubricated surfaces as a function of speed, pressure and temperature, although t and v do not appear in the equation. The diagrams in Fig. 55, which came to the notice of the writer after he had derived equation (I) from other materials, support the equation very well. The diagrams show that if we let the journal temperature rise freely without artificially warming or cooling the journal*, the coefficient of friction will increase with the speed until the journal attains a speed of about one meter per second or until the train attains a speed of about 12 m.p.h., provided the pressure is very small, say below five kilograms per square centimeter, corresponding to a car weight of nine tons. Above that speed the coefficient is constant. Although the diagram does not show the

---

* In case of a railway car journal, when the train is running, the air acts like a cooling device on a stationary journal. The experimental data shown in Fig. 54, indicate that the temperature rises in practically the same way as in the case of a stationary journal. This may be due to the fact that the relative speed of the air near a train is not so great as Nipher's experiment shows.

Diagram showing Behavior of
Coefficient of Friction
when a journal is not artificially cooled.
R. Stribeck.
Fig. 55.



curves for higher pressures such as are common in railroad practice - that is about 10 to 45 kg. per cm. - it can be easily reasoned that at such high pressures the coefficient is independent of speed and temperature (combined) but that it is an inverse function of pressure.

3. Review of opinions on journal resistance. - The characteristic equation (I) has been based mainly upon the results of general experiments, which were made without any particular reference to the journal friction of railway cars. There may be, however, many respects in which car journals differ from others. In order to determine whether the equation is applicable without modification to railway car journals, the following review of the opinions* of railway engineers who have had valuable experience in train resistance investigation is made.

Crawford** states that "the frictional resistance varies considerably. The friction expressed in pounds per ton of drawbar pull increases as the speed increases". Armstrong[ø] says: "The laws governing the friction of journal bearing are fairly well understood and the fact that such bearing friction constitutes part of the resistance opposing the motion of a train, need introduce no undetermined factors. Such friction losses

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* Only a few are cited here, but the writer considers that he has made a very thorough and careful study of the opinions expressed in the available publications.

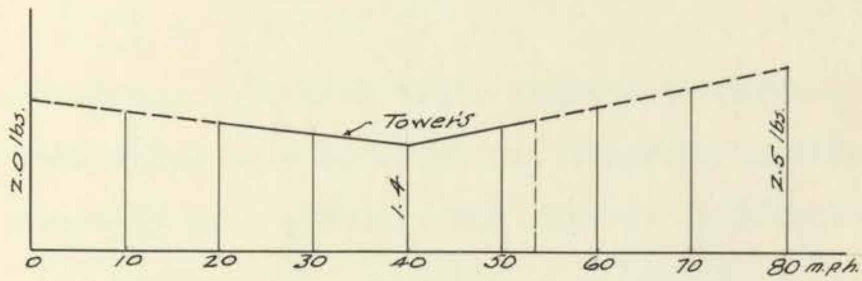
** Proceedings A.R.E.A., vol. 8, p. 224. (1907).

ø "Electric Traction", Standard Handbook for Electrical Engineers, p. 951.

decrease with the pressure on the bearing and are a function of speed. Hence the expression f = A + BS, ... $f = \frac{a}{\sqrt{W}} + BS$".

Thus Crawford and Armstrong believe that journal resistance increases with speed. They may be right if they are referring to resistance when the trains are undergoing rapid acceleration.

Aspinall*, after his study of the experimental results of Tower concluded that the journal resistance of English railway coaches could be expressed as the following diagram shows:



Tower's experiment covers a very small speed range and is very incomplete compared with some of the more recent tests. In his train resistance formulas, Aspinall assumed that the journal resistance is independent of speed and also of bearing pressure.

Referring to his train resistance formula $R = 3.2 + 0.077V + 0.0025V^2$, Barbier** states "The constant 3.2 represents

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* J.F. Aspinall: "Train Resistance", Minutes of Proceedings of Inst. of Civil Eng., vol. 147, p. 155. (1901).

** Cited in Proc. A.R.E.A., v. 8, p.219, (1907). See also "Resistance a la Traction", Le Genie Civil, 1897-98, v. 32, p.377.

the different fixed resistances of the cars, such as friction of axles, the rolling resistance of the tires and the shock, whether periodical or accidental, between the wheels and the rail. .... The friction of the journal is the most important element of the fixed resistance, if we consider that the train and the track are in good condition".

Referring to B. Tower's and W. Stroudley's experiments, Carus-Wilson states*, "From these tests it appears that under the conditions usual in ordinary railway practice, journal friction is independent of both speed and load, and may be taken to be a constant quantity depending only upon the wheel and journal diameters ....".

Blood in his "Rational Train Resistance Formula"ø discusses journal resistance only briefly; but seems to consider that it varies inversely with pressure and that it is independent of speed.

An Engineering News editorial** says, "The general law of friction is also well determined that at very high speeds the lubricants are so well carried around between the metallic surfaces that the friction is greatly reduced, and may almost become evanescent. Mr. J. W. Cloud and others have the opinion that at high speeds the journal friction proper may be less even than 2 lbs. per ton".

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* C.A. Carus-Wilson: "The Predetermination of Train-Resistance" Min. of Proc. of I. of C.E., vol. 171, p. 227 (1908).

ø Transactions A.S.M.E., vol. 24, p.

** The Engineering News, 1890, p. 506, and also Bul. A.R.E.A. and M.W.A., vol. 8, p. 201.

Wellington* advocates the use of a formula in the form of $R_j = a/v + b$ for journal resistance when the train is started from rest after a long stop; and he considers that R_j is constant when a train has been continuously moving, i.e., when the journal boxes are warm. Referring to pad or siphon lubrication, he says, "The latter being more like the ordinary lubrication in railroad service, we may say, without sensible error, that the coefficient of journal friction is approximately constant for velocities of 15 to 50 miles per hour. This has been the assumption which all investigators of railroad friction, to date, have been compelled to make, and it is, in some respects, fortunate that it proves not far from truth."

Authorities thus apparently differ in their ideas of journal resistance, but a careful consideration of these opinions makes it clear that most of them are not inconsistent with our characteristic equation (I), while others define merely particular cases of the equation, that is, some consider it is independent of pressure, others it is independent of speed, and so on. Most of these differences of opinion can be reconciled and but few of them are inconsistent with equation (I) when we take into consideration all the limitations or assumptions surrounding them. In view of this fact and in view of the irreproachable character of the experimental data which has been cited in its support, the writer believes that formula (I) may be accepted as a general characteristic equation for the friction of railroad journals.

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* Wellington's "Economic Theory of Railway Location", p. 923.

C. Rolling Resistances.

1. Rolling resistance due to rolling friction. - The resistance due to the rolling friction of wheels on rigid rails may be computed by the formula,

$$R_r = f \frac{W}{d},$$

or

$$= f \frac{2000}{d} \text{ lbs. per ton} \quad \ldots\ldots (6)$$

where R_r denotes the resistance, f the coefficient of rolling friction, d the radius of wheels, and W weight on the wheel. The law of variation of the coefficient under different conditions is not yet well established. It is easy, however, to show that the coefficient, which is an abstract number, corresponds exactly to a physical length, l, which is shown in Fig. 56. Certain investigators have assumed that f is equal to l', one-half of the total length of the contact surface of wheel and rail, instead of l. This assumption, however, leads to absurd results, i.e., that the resistance due to rolling friction is from 15 to 65 lbs. per ton. This indicates that the

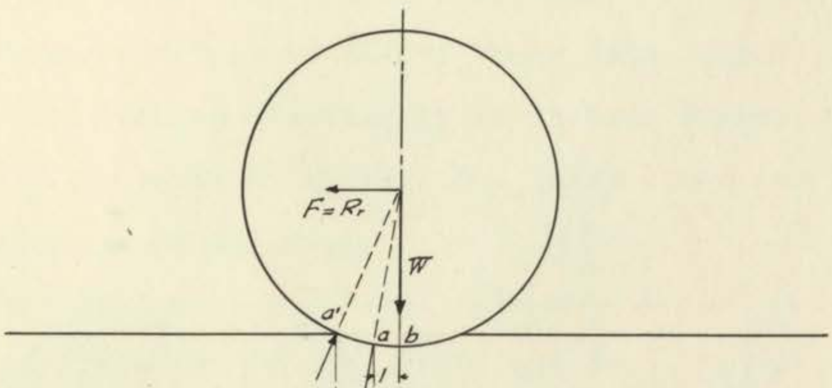

Fig. 56.

center of moments should not be chosen at a' since the resultant of the rolling resistance does not act at a' but somewhere between a' and b, say at a. It is clear that if we assume that the pressure per unit area of the contact surface is uniform from a' to b, and that the width of the surface is also uniform, then l is one-half of l'. In fact the specific pressure near b must be much greater than that at a', because the rail is compressed to much greater extent at b than at a'. (see Fig. 49.) Further the width near b of the contact surface is much greater than that at or near a'. Therefore, the point of application of the resultant is very close to b; and not at a' as some persons have assumed.

The law of variation of the coefficient of rolling friction with pressure has not been conclusively determined. In fact, some consider that there is no such variation, but this would not be true when a wide range of pressure is considered. In Fig. 61, is shown a straight line AB which represents very closely the variation of the longitudinal length of the contact surface of wheel and rail under different wheel loads.* If we assume, as is most probable, that l is proportional to l', and that the diameter of the wheel is constant, these data indicate that the coefficient of rolling friction is an inverse function of pressure as shown by curve CD in Fig. 61. This curve can be expressed by an equation of the form,

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* Railway Age Gazette, May, 7, 1909; and Proc. Pittsburgh R.R. Club, Nov. 1907, an article by George B. Fowler.

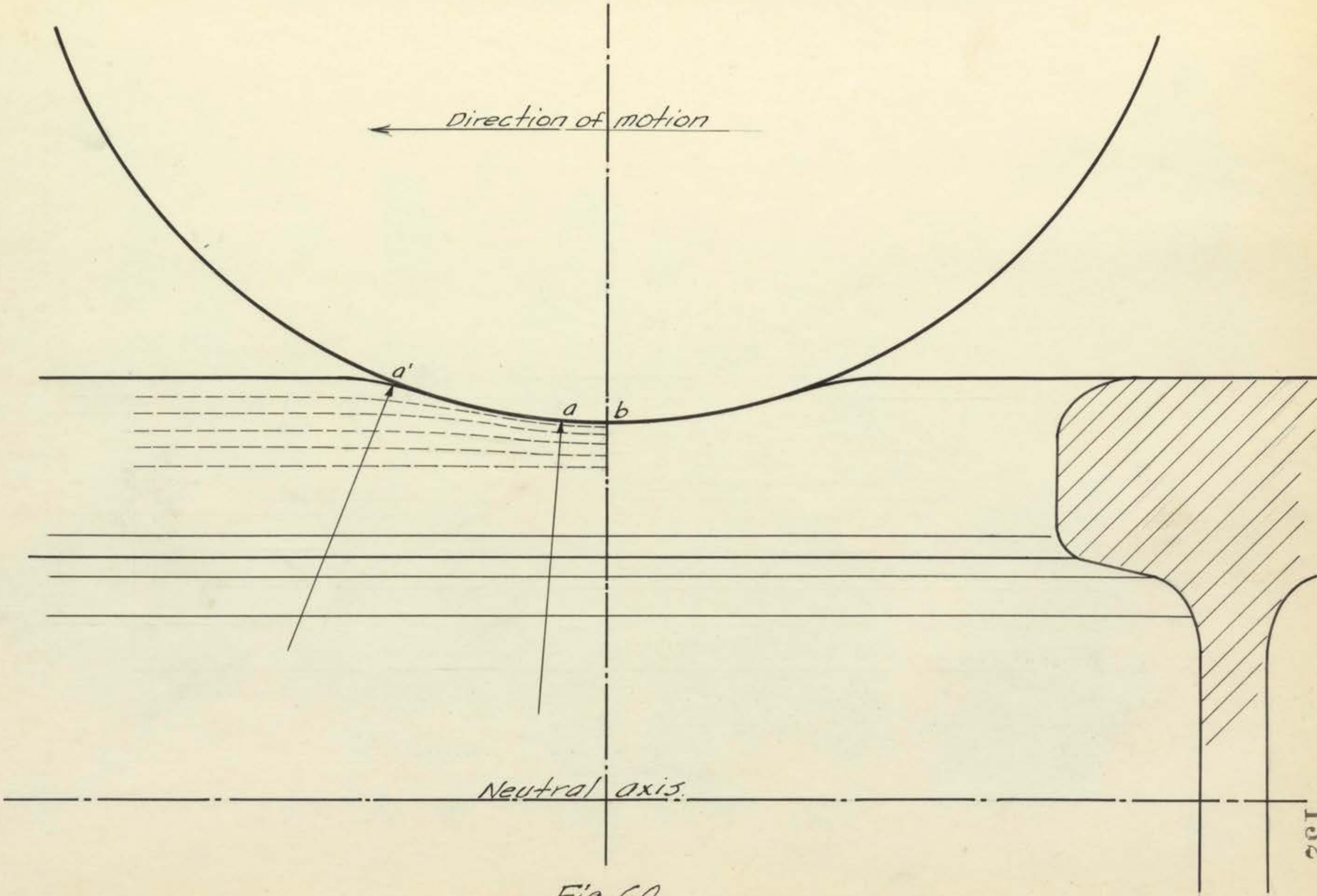

Fig. 60.

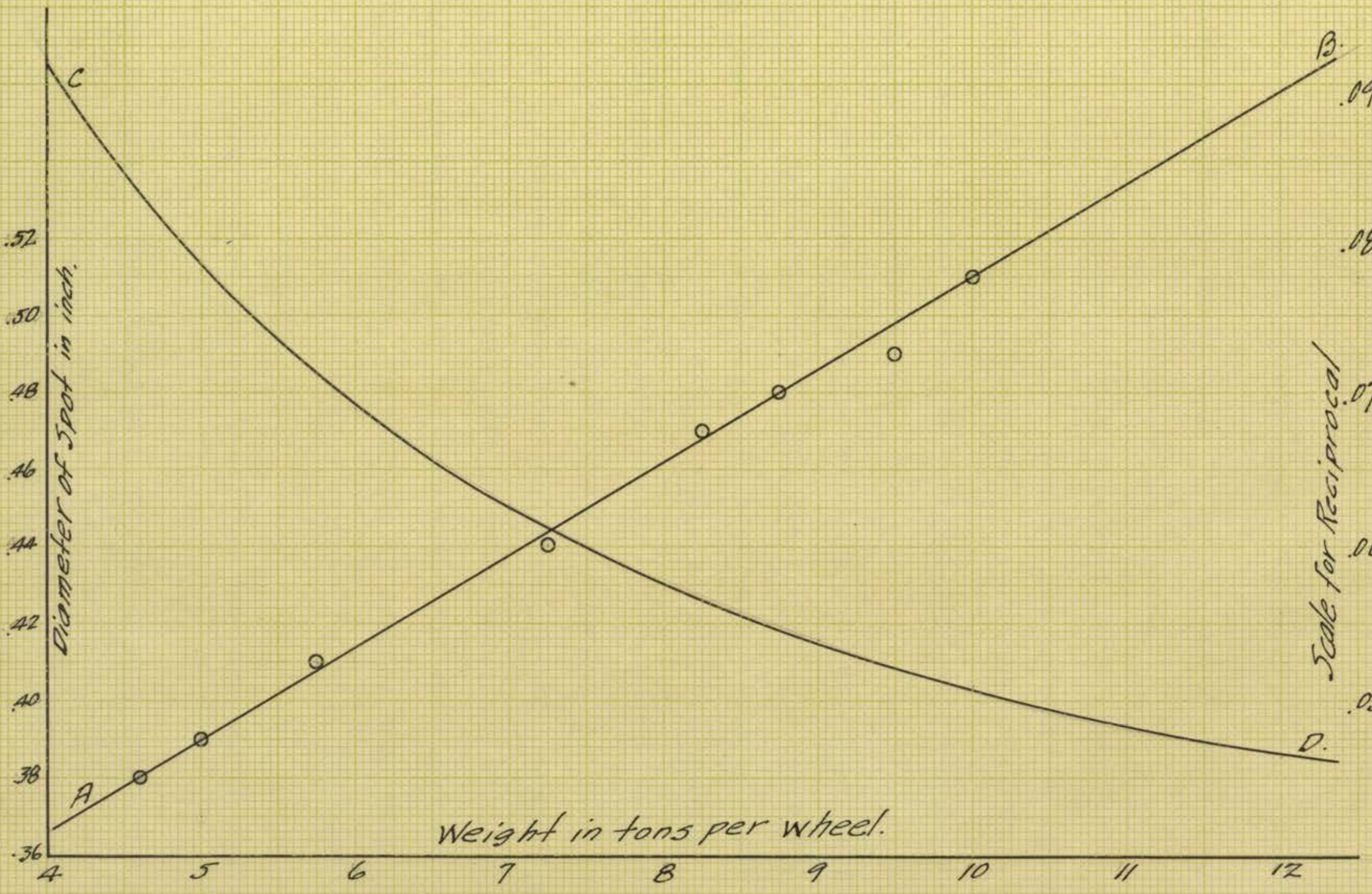
C
B.
A
D.
.52
.50
.48
.46
.44
.42
.40
.38
.36
Diameter of Spot in inch.
.09
.08
.07
.06
.05
Scale for Reciprocal
4
5
6
7
8
9
10
11
12
Weight in tons per wheel.

Fig. 61.

$$f = \frac{k}{P + c}$$

or

$$R_r = \frac{A''}{P + c''} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where P represents wheel load, k, c, A", and c" certain constants, and f and R_r are as defined before.

The influence of wheel diameter on rolling friction has been clearly demonstrated by Vial*. The diameter of railway car wheels, however, varies very little - from 33 to 38 or sometimes 40 inches - and may be considered as constant for the purposes of this discussion.

The speed of a train will have some effect on rolling friction. Referring to Fig. 60, there must be a force similar to that acting at a on the right of b which tends to push the wheel forward while the former tends to push it backward. This is true when the wheel stands still. If, however, the wheel is rolling with a certain speed, the force behind the wheel must act or the depressed part ba" must spring up with a speed greater than that of the front part in order to assist in the rolling of the wheel. We do not know with what speed the depressed part ba" can so recover, but it may be assumed that this force is insignificant at any speed when compared with the opposing force of the rolling friction.

Here a considerable space has been devoted to analyse

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* F. K. Vial: "Standardization of chilled iron crane wheels", paper presented at the annual meeting, Dec. 1914, of A.S.M.E. p. 17. The numerical results obtained by him are wrong because he assumed that the resultant acts at the remote end of the contact surfaces.

rolling friction. This is due to the fact some persons consider that rolling friction is as great as 15 to 65 lbs. per ton, while others believe it amounts only to about one-tenth pound per ton, a negligible value in train resistance. Although the determination of the value l is very difficult, it is found to be from 0.007 to 0.02 inch.* With these values, from formula (7), we get values of rolling friction which vary from 0.85 to 2.42 lbs. per ton.

2. Track resistance.** The resistance due to the work done by the wheels in depressing the rails may be analyzed as follows:

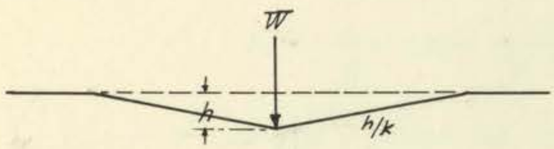

Fig. 62.

Let W be the weight in tons on the axle, h the depression under the axle in feet, h/k the inclination of the rail produced by the depression. Then, the maximum intensity where the depression is greatest, is

$$= \frac{W}{k} \text{ tons per lineal foot.}$$

and the work done by the depression is

$$= \frac{1}{2}h\frac{W}{k}.$$

The rate of doing this work as the depression advances at v feet per second is

* Maurer's Technical Mechanics, p. 297.

** C.A.Carus-Wilson: "Predetermination of Train Resistance", Proc. Inst. C.E., vol. 153.

$$= \frac{1}{2} w \frac{h}{k} v.$$

If T be the tractive force and v the speed, Tv is the rate of doing work, and

$$Tv = \frac{1}{2} w \frac{h}{k} v,$$

$$T = \frac{1}{2} \frac{wh}{k}$$

or the corresponding resistance,

$R = \frac{1}{2} 2000 \frac{h}{k}$ per ton when the rails recover or spring back to their original position before the following wheel arrives. If, however, the rails depressed by the first pair of wheels remain depressed until all the cars pass,

$$R = \frac{1}{2} 2000 \frac{h}{k\ N\ a} \text{ per ton} \quad \ldots\ldots\ldots\ldots \quad (8)$$

in which N is the number of cars and a the number of axles in each car.

The resistance due to the additional depression at weak rail joints may be also estimated by a similar formula,

$$R = \frac{1}{2} 2000 \frac{h'}{k'}$$

in which h'/k' is the depression at the joint.

Whether the rails depressed by the first pair of wheels of a train remain depressed until all other wheels pass, is a topic discussed by many engineers. The general opinion, however, is that the rails recover their original position before the following wheels arrive if the speed is low; but that they remain depressed if the speed is higher than about 10 m.p.h. This all depends upon the track condition. If a track is well constructed and well maintained, the depression is small and the recovery is slow - a heavy rail has great stiffness and its depression is small, and the stiffness tends to keep it down if depressed. Thus it can be shown by means of the above formula that on a good track the track resistance is less than one-tenth of a pound per ton for ordinary trains*, - a negligible amount for practical purposes. Resistance due to rolling friction, however, amounts to about 30 percent of the train resistance at low speed.

The foregoing analysis of rolling resistance leads to the conclusion that the resistance due to rolling friction can be expressed by an equation of the form,

$$R_r = \frac{A''}{P + c''}$$

and that the aggregate rolling resistance, too, can be expressed by an equation of that form, since the resistances due to the work done by the wheels on the rails and that done to rail joints are negligible quantities compared with other resistances.

D. Miscellaneous Resistances.

Due to the fact mentioned before, we have little information about resistances of this class. The following analysis

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* This is true when the number of cars of a train is considerable. For a single or double-car train track resistance may amount to 1 or 2 lbs. per ton, an amount which should not be ignored.

of the flange action due to oscillation of cars, however, may throw a little light on this subject.

1. Flange action.* - In Fig. 63, let DA represent one of the rails, and GA the wheel-base of a truck in its position of mean inclination to the rail,

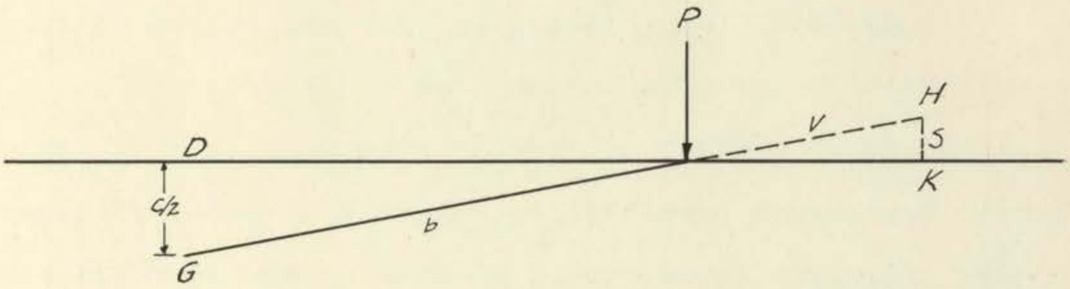

Fig. 63.

with the leading wheel touching the rail at A, the tread of the rear wheel at G being at a distance c/2 from the rail, where c is the maximum play between the flanges and the rails.

The truck tends to roll in the direction GA but is prevented from so doing by the action of the rail on the flange at A, which produces a force P at right angles to the rail, which acts on the truck and tends to turn it about the point G. The force P will depend upon the moment of inertia of the truck about G.

If the truck is moving with a velocity V it would roll

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* This analysis of flange action is due to C. A. Carus-Wilson, except that part in which the resistance due to flange action is considered as a function of pressure as well as of speed.

from A to H in unit time if $AH = V$. The force P brings the wheel to K over the distance $KH = S$ with an acceleration a, where $S = (1/2)a$. If M is the mass of the truck, and eM be the equivalent mass at A for rotation about G, then $P = eMa = 2eMS = eMVc/b$, and the corresponding tractive effort required is $\underline{m}eMVc/b$, where $\underline{m}$ is the coefficient of friction. Hence T, the tractive effort per ton, is proportional to eVc/b.

The quantity e will depend upon the distribution of the weight of the truck with respect to the wheel, and as this is nearly the same with trucks of different wheel-base, except in unusual cases where there is a very large overhang, this quantity may be taken to be constant. It follows that the tractive effort required to overcome flange-action is proportional to MVc/b.

In the case of "bogie-coach" the flange-action is proportional to MVc/b for each of the "bogie-trucks" of mass M' and wheel-base b. Hence if M is the mass of the whole coach, including the trucks, the tractive effort is proportional to $2M'Vc/Mb$.

In this analysis Carus-Wilson assumes, as we have seen, that the coefficient of friction, $\underline{m}$ is constant, consequently the resistance due to flange-action is independent of pressure. The coefficient of friction, however, is not independent of pressure but assumes the form,

$$\underline{m} = \frac{a'}{P + c'}$$

Then, substituting this value of $\underline{m}$, we have

$$R = \left(eV \frac{c}{b}\right) \frac{a'}{P + c'}$$

or $$= \frac{B}{P + k} V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (III).$$

in which R is the resistance due to flange action, V the speed, P the intensity of pressure between flange and rail, and B and k are certain constants.

The foregoing analysis of flange action covers, probably the whole external effect of oscillation and concussion. We know, however, nothing about their internal effect, except that they may act and react upon each other or, even if they do not so react, their resultant effect upon the tractive effort would be of insignificant value. Therefore, we assume, until their exact nature is known, that the miscellaneous resistance as a whole is a linear function of speed and varies inversely with the weight of cars as represented by formula (III).

2. Review of opinions on rolling resistance and miscellaneous resistances. - The following summary of the opinions of various authorities may serve to help us to an opinion concerning the validity of the foregoing conclusions about rolling resistance and miscellaneous resistances.

"Rolling friction", writes Armstrong, "is due to the friction of metal rolling on metal where the surfaces are not perfect, the bending of rails due to insufficient support, such as ties or ballast, or too light rail for the weight carried, flange friction between rail and wheel flange; all these factors being proportional to speed and hence represented by a straight line function .... f = A + BS".

Aspinall, in discussing his "miscellaneous resistances", which include all the resistances due to oscillation, concussion, flange friction, rolling friction, etc., states "these miscellaneous resistances have been plotted .... and their value approximates to

$$R_m = \frac{V}{5.4} - 1.$$

"R_m increases as speed increases. This may be due to the fact that oscillation and probably flange friction are greater at high speeds with a long train than with a short one." Referring particularly to the resistance due to rail depression, he stated that according to his personal observation, when an engine was moving slowly at about 2 m.p.h. the inclination was about 1/720, the deflection of the rails being about 0.2 inch, but when the engine was running at 45 m.p.h. the deflection was about 0.25 inch, that is deflection increased with speed; and that the deflection would also depend upon the kind of rails, tracks. etc.

Mr. Fowler, who made the famous train resistance tests with Aspinall, says that he "made an observation one whole morning lying down near a main line and watching every train. He did not remember any single instance where there was not recovery in some degree after the engine has passed and in case of bogies between the wheels of successive bodies." "At low speed (below 10 m.p.h.)", Carus-Wilson says, "it does recover and it", referring to the resistance due to the rail deflection, "is 2.8 lb. But above 10 m.p.h. it does not recover, so the value becomes about 0.14 lb., a negligible quantity for a train of a large number of cars". Dudley says, "in America, a test was made and it was found that 85 lb. rails, under 80,000 lb. capacity and 15 m.p.h., did not recover for the entire train."

Wellington says "Experiment indicates that their aggregate" - journal resistance and rolling friction - "varies somewhat, but not materially, with the velocity." Blood, in his "rational formula" seems to consider that the "rolling friction" which includes the effect of oscillation and concussion varies with the first power of velocity.

This review of the opinions and former analyses of rolling resistances leads to the conclusions that: first, the rolling resistance due to rolling friction may be expressed by an equation of the form,

$$R_r = \frac{A''}{P + c''} \qquad \text{.................. (II)}$$

second, the resistance due to the work done by the wheels in depressing the rails is a negligible quantity when the number of cars of a train is large and the speed is above 10 m.p.h.; and third, the sum of the miscellaneous resistances is a linear function of speed and it varies inversely with the weight per car, that is,

$$R_m = \frac{B}{P + k} V \qquad \text{.................. (III)}$$

E. Atmospheric Resistance.

1. Review of atmospheric resistance investigations. - As early as the seventeenth century, the subject of wind force or air resistance, which later was found to be an important element in the design of windmills, bridges, chimneys, projectiles,

and the operation of railway trains, received the attention of eminent investigators and philosophers. Newton in about 1690 deduced from his law of falling bodies a formula for the pressure on a flat plane in air current, which is said to be confirmed by his classical experiment on wind force. The formula is usually expressed as follows:

$$p = \frac{wv^2}{2g} ,$$

where p denotes the pressure per unit area of the flat plane, w the weight of the air per unit volume, v the velocity of the air, while g is the acceleration due to gravitation. From this it will be readily seen that the pressure p depends upon w, the weight of air, and w in turn depends upon the temperature, barometric pressure and quality of the air. In the actual field of train operation however, the influence of these elements on the weight of the air is so slight that we can safely assume that w has the constant value of 0.0763 lb. per cubic foot. The preceding equation then becomes

$$p = 0.0025V^2 \quad \text{..................} \quad (9)$$

in which p represents the pressure in lb. per sq. ft. of a flat plane perpendicular to the air current, the velocity of which is V in miles per hour.

In 1759, Smeaton published a table of velocities and wind pressures, which is fairly well expressed by the equation,

$$p = 0.005V^2 \quad \text{..................} \quad (10)*$$

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* All quoted in Kent's Mechanical Engineers' Pocket-book, 7th ed. (1907) page 492-494.

It may be noted that in this formula the coefficient of V^2 is twice as large as in that of Newton, and it was said that "the formula was never well established, and has floated chiefly on Smeaton's name and for lack of a better".*

As the results of later experiments, Professor Martin gives

$$p = 0.004V^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)*$$

and Whiple and Dives,

$$p = 0.0029V^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)*$$

Kent* gives exceedingly interesting information on this subject. The following is an extract therefrom: "Prof. H. Allen Hazen (Eng. News, July 5, 1890) says that experiments with whirling arms, by exposing plates to direct wind, and on locomotives with velocity running up to 40 miles per hour, have invariably shown the resistance to vary with V^2. In the formula $p = .005sV^2$, in which p = pressure in lbs., s = surface in sq. ft., V = velocity in m.p.h., the doubtful question is whether the factor .005 is correct or not. Perhaps some of the best experiments for determining this value were tried in France in 1886 by carrying flat boards on trains. The resulting formula in this case was, for 44.5 miles per hour, $p = .00535sV^2$."

"Prof. Kernot, of Melbourne (Eng. Rec., Feb. 20, 1894) states .... the pressure upon one side of a cube, or a block proportioned like an ordinary carriage, was found to be 0.9 of

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* All quoted in Kent's Mechanical Engineers' Pocket-book, 7th ed. (1907) p. 492-494.

that upon a thin plate of the same area. The same result was obtained for a square tower. A square pyramid, whose height was three times its base, experienced 0.8 of the pressure upon a thin plate equal to one of its sides, ... A bridge consisting of two plate girders connected by a deck at the top was found to experience 0.9 of the pressure on a thin plate equal in size to one girder, when the distance between the girders was equal to their depth; and this was increased by one fifth when the distance between the girders was double the depth. A lattice-work in which the area of the openings was 55% of the whole area experienced a pressure of 80% of that upon a plate of the same area. The pressure upon cylinders and cones was found to be equal to half that upon the diametral planes, and that upon an octagonal prism to be 20% greater than upon the circumscribing cylinder. A sphere was subjected to a pressure of 0.36 of that upon a thin circular plate of equal diameter. A hemispherical cup gave the same result as the sphere; when its concavity was turned to the wind the pressure was 1.15 of that on a flat plate of equal diameter. When a plane surface parallel to the direction of the wind was brought nearly into contact with a cylinder or sphere, the pressure on the latter bodies was augmented by 20%, owing to the lateral excape of the air being checked ...."

In 1900, Nipher* published the accounts of his experiment on air resistance. The result checks well the theoretical

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* F. E. Nipher. "The Frictional Effect of Railway Trains upon the Air", Trans. St. Louis Academy of Science, vol. 10, No. 10, (Nov. 11, 1900), and Railway & Engineering Review, vol. 41, (1900), p. 48.

formula $$p = 0.0025V^2 \quad \text{.................} \quad (13)$$

His data also indicate the effect of wind on the constant of the above formula, the variation being from 0.0020 to 0.003 for various wind conditions.

A long series of experiment was undertaken in 1898-1900 by Aspinall* and the result is very closely represented by the formula,

$$p = 0.003V^2 \quad \text{..................} \quad (14)$$

Perhaps the most elaborate experiment on air resistance was carried out in 1900-1903 on the experimental track between Marienfelde and Zossen, near Berlin. The report** of this experiment contains the following statement: "The tests of last year gave an excellent opportunity for measuring the air resistance at high speeds. The results of a great number of measurements, which, as in former years, were obtained by U-shaped tubes, check up well with one another. On Plate XI the values as observed during three runs from Marienfelde to Zossen and during two runs from Zossen to Marienfelde are given. The measurements were made on three different days, and in recording the values the direction of the wind and the strength of the wind were taken into consideration, as already described in the preceding report. The curve for the air pressure corresponds to the formula $p = 0.0052V^2$, wherein p is the air pressure for one

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* J.A.F. Aspinall: "Train-Resistance", Proc. I.C.E., vol. 142, (1901-1902); and Bulletin International Ry. Congress, 1902.

** "Report of the Berlin-Zossen Electric Railway Test", McGraw-Publishing Company, New York, (1905).

square meter (10.76 sq. ft.) of plane surface perpendicular to the direction in which the car is running, V is the speed in kilometers per hour. While it seems that the curve gives at lower speeds somewhat higher values than the values found in reality, and vice versa at higher speeds, it checks up well as a whole with the readings. If still a nearer approximation of the value is desired, it would be necessary to decrease the coefficient 0.0052 for speeds up to 100. kilometers (62 miles) and increase it for speeds above 100 (62) or to change the exponent for V. In considering the very slight inaccuracy of the given formula and its great simplicity, it seems to be justifiable to retain it in its present form and employ it regularly in railroad practice."

Another series of air resistance tests was made in 1905 in connection with the Universal Exposition at St. Louis by the Electric Railway Test Commission.* The result of the tests for the air resistance of a flat vestibule shows that the coefficient of V^2 varies from 0.0035 to 0.0023 within the test speeds of from 20 to 60 miles per hour, the higher value being for lower speeds. This is directly opposed to the conclusion arrived at during the Berlin-Zossen tests, that is, the curve representing $p = 0.0025V^2$ lies below the points obtained in the St. Louis test for lower speeds, and vice versa for higher speeds.

In the above review little has been said about the exponent of V in the air resistance formulas. Lieut. Crosby**, in 1890, published the result of his experiments, declaring that

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* Report of the Elec. Ry. Test Commission, McGraw Pub. Co., 1906.
** Kent's Mech. Eng. Pocket-book, p. 493.

the air resistance is a linear function of speed and not a second degree function, but all the later experiments as well as the earlier tests have proved the reverse. A. M. Wellington* says, "Air resistance for example, is known by observations on projectiles to vary more nearly as the cube of the velocity, when the latter is very great, but at all ordinary velocities it appears to vary very nearly as the square, ... although it may very easily be as $V^{1.9}$, or $V^{2.1}$, or even $V^{2.2}$, ..." Blood in his "rational" train resistance formula assumes the air resistance varies with 1.8 power of speed, but the correctness of this assumption is doubtful and it impairs the simplicity of the formula.

2. Formula for air resistance. - The foregoing review covers practically all the well-known experiments and investigations made on the air resistance to a flat plane. The results of different experiments, however, vary greatly, and it is somewhat unsafe to make any positive statement as to the numerical factor of the formula. It seems, however, after careful consideration of individual tests, that the head-end air resistance of railway trains can be very closely estimated by means of Newton's formula,

$$p = \frac{w}{2g} V^2 = 0.0025V^2,$$

which has been confirmed by Nipher, and by the Berlin-Zossen

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* The Economic Theory of Railway Location, (A. M. Wellington) page 517.

tests, and which differs little from the results of Dines, Aspinall, and of the St. Louis tests.

The "rear suction" and the skin friction of railway cars in motion have been investigated by Goss, Aspinall and Nipher and in the Berlin-Zossen and the St. Louis tests. Definite valuesfor these items, however, are not yet determined. In the St. Louis test the rear suction of a car with a flat end at a speed of 20 miles per hour was found to be about 10 percent of head air resistance. The skin friction is found by both Goss and Nipher to be also 10 percent of head-end air resistance. Goss gives the following data* which is very valuable for estimating, at least, the relative values of skin friction, rear suction, and head-end air resistance:

$p = 0.001V^2$ for the first car or locomotive of a train

$p = 0.00026V^2$ for the last car of a train

$p = 0.00008V^2$ for the second car of a train, and

$p = 0.0001V^2$ for any intermediate car between the second and the last of a train.

In actual train service the head-end air resistance of any car except the front car or the locomotive is insignificant in amount. The above data indicate that rear suction is also a very small quantity in all cases except that of a very high speed train composed of few cars. If P' represents the head-end air resistance of the first car or locomotive, and n the number of other cars of the train we deduce from the above data the following relation:

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* W.F.M.Goss, "Locomotive Performance", Wiley & Son, Pub., p.398.

Total air resistance of a train = $P'(1 + n/10)$.
Then, replacing P' by P which we found previously to be the most probable value of head-end air resistance, we get

$$R''_a = 0.0025AV^2(1 + n/10) \quad \dots\dots\dots\dots\dots \quad (15)*$$

where R''_a represents the total air resistance in pounds of the entire train. If we charge the head-end air resistance entirely to the locomotive at the head of a train, the total air resistance of the cars alone is, in pounds,

$$R'_a = 0.0025AV^2(n/10)$$

or simply,

$$R'_a = 0.00025AV^2n,$$

and if W represents the average weight per car in tons, the total weight of the train is nP, and the air resistance in pounds per ton is

$$R_a = 0.00025AV^2/W \quad \dots\dots\dots\dots\dots\dots\dots \quad (16)$$

Although there are no conclusive experimental data on the subject, it is obvious that the skin friction of a seventy-foot passenger car can not be the same as that of thirty-eight foot car. Experience shows and theory indicates that the skin friction is proportional to the surface or length of a car. The above formula has been derived from the relations determined for a model proportioned to a thirty-eight-foot car, and it applies consequently to a train composed of cars of that length, which is practically the length of all ordinary freight cars. To make the formula more general and applicable to passenger cars, we must multiply the formula by the ratio 1/38, where 1 is the average length of the car in feet. Further we have assumed that R_a is entirely due to skin friction, but skin friction does not increase directly with the cross-sectional area of the cars; it will vary with the perimeter, which is practically proportional to the square root of the cross section. So, assuming the area of ordinary freight cars to be 100 sq. ft., we replace A by 100 $\sqrt{A/100} = 10\sqrt{A}$, and finally get an air resistance formula as a function of speed, cross sectional area, length, and average weight of cars, as follows:

$$R_a = 0.0025\sqrt{A}\ l\ V^2/38'',$$

$$= \frac{0.000066\sqrt{A}\ l}{W}\ V^2 \quad \ldots\ldots\ldots\ldots\ (IV).$$

F. Train Resistance Formula.

1. A Characteristic equation of train resistance. - In the foregoing study and analysis of the elementary resistances which compose the entire inherent resistance to a train in motion, it is found that each of them may be closely represented by a formula of a certain form thus:

(A). Journal resistance = $\frac{A'}{P + c'}$

(B). Rolling resistance = $\frac{A''}{P + c''}$

(C). Atmospheric resistance = $\frac{0.000066\sqrt{A}l}{W}.V^2$

(D). Miscellaneous resistance = $\frac{B}{P + k}$ V.

The inherent resistance is the sum of these elementary resistances, therefore,

$$R = \frac{A'}{P + c'} + \frac{A''}{P + c''} + \frac{B}{P + k} V + \frac{0.000066\sqrt{A}l}{W}.$$

In this equation P represents the intensity of pressure on the rubbing or rolling surfaces. In the case of journal friction, P depends upon the dimensions of bearings, capacity of cars, extent of loading etc.; while for rolling and miscellaneous resistance it depends upon the design, dimensions, and quality of the wheel and rail and the determination of its precise value is very difficult. Practically speaking, however, the design and dimensions of both wheel and rail are very well standardized and the weight of the car is a sufficiently accurate measure of the intensity of pressure in all cases. The dimensions of bearings differ greatly with the capacity of cars, but the specific pressure is practically constant and differs only with the gross weight of car. Hence, the average weight per car, W, is the most convenient measure of P and may be given the place of P in the above equation.

The first two terms of the right member of the equation are the same in form, and they may be combined and assumed to have the same form as before, that is,

$$\frac{A'}{W + c'} + \frac{A''}{W + c''} = \frac{C}{W + c}$$

This particular process is not mathematically correct, but the error due to this assumption is very small and it makes the resulting formula very simple.

Then, as the final practical characteristic equation of inherent train resistance, we have

$$R = \frac{C}{W + c} + \frac{B}{W + k}V + \frac{0.000066\sqrt{A}l}{W}V^2 \quad \ldots\ldots\ldots\ldots \text{(V)}$$

Where V denotes the speed in miles per hour, W the average weight per car in tons, A the cross section of cars in square feet, l the average length of cars in feet, and C, B, c, and k are certain constants, which vary, however, with the design, dimensions, quality, maintenance, etc. of railway rolling stock, and track.

2. Determination of the constants. - There are at least two or three different ways of determining the values of the constants in the above characteristic equation. The first is to determine them by calculation with data obtained by special tests on bearing friction, air friction, rolling friction, etc., but the difficulty of making such tests corresponding exactly to actual railway conditions is apparent. The second method is to determine the constantsby means of the data obtained by actual train resistance tests made with a dynamometer car. There are, however, five distinct constants in the characteristic equation and the calculation is rather laborious. The third method is a combination of the first two, namely, as many as possible of the constants are determined by the first method, and the remaining constants, such as that for miscellaneous resistance, are then determined by the second method.

The constants in the following freight and passenger train resistance formulas, (VI) and (VII) have been determined by the third method just mentioned. The factor of the third term - air resistance - has been determined previously. There is reason to believe that practically the entire train resistance at low speeds - say below five miles per hour - is composed of only journal resistance and rolling resistance. With this point in view, the constants C and c of the first term have been determined from data for freight train resistance at zero speed for various average car weights.

$$R = \frac{250}{W + 20} + \frac{5}{W + 40}V + \frac{.025}{W}V^2 \quad \text{for freight trains} \;..(\text{VI})$$

In the following table (I), the values in column 2 are the first terms of the group of formulas for freight train resistance* exhibited on page 101. The expression, 250/(W + 20) has been found to represent these values closely. The exact values calculated from this expression are shown in column 3. A comparison of the values in columns 2 and 3 will readily justify the adoption of 250 and 20 for the constants, C and c respectively.

We have determined all the constants of the terms representing atmospheric, journal and rolling resistances. The next step is to determine the constants of the second term which represents miscellaneous resistance. The values of atmospheric resistance at 40 m.p.h. for various car weights are given in column 4, and the values in column 5, are the sums of the values in columns 3 and 4. The total inherent train resistance* at 40 m.p.h. for various car weights is shown in column 6. The difference of the values in columns 5 and 6 are given

---

* E.C.Schmidt: "Freight Train Resistance", Eng. Exp. Station University of Illinois, Bulletin No. 43.

TABLE I.

1.	2.	3.	4.	5.	6.	7.	8.	9.
Avg. Car Wt. W, tons	Jour.& Rolling Resis-tance from tests	$\frac{250}{W + 20}$	$\frac{.025}{W}(40)^2$	Col. 3 + 4	Resis-tance 40 m. tests	6 - 5	$\frac{5}{W + 40}(40)$	7 - 8
15	7.15	7.15	2.35	9.50	13.40	3.9	3.63	0.27
20	6.30	6.25	2.00	8.25	11.8	3.55	3.33	0.22
25	5.60	5.55	1.60	7.15	10.6	3.45	3.04	0.41
30	5.02	5.00	1.34	6.34	9.5	3.16	2.83	0.33
35	4.49	4.55	1.14	5.69	8.6	2.91	2.67	0.24
40	4.15	4.16	1.00	5.16	7.9	2.74	2.50	0.24
45	3.82	3.84	0.89	4.73	7.3	2.57	2.37	0.20
50	3.56	3.57	0.80	4.37	6.8	2.51	2.22	0.29
55	3.38	3.33	0.73	4.06	6.3	2.24	2.10	0.14
60	3.19	3.12	0.67	3.79	6.0	2.21	2.00	0.21
65	3.06	2.94	0.62	3.56	5.7	2.14	1.92	0.32
70	2.92	2.78	0.57	3.35	5.6	2.25	1.84	0.41
75	2.87	2.63	0.53	3.16	5.5	2.34	1.74	0.60

in column 7. It is clear that these values must be miscellaneous resistance at 40 m.p.h. The equation $5V/(W + 40)$, values of which appear in column 8, represents these values fairly well as will be seen by a comparison of the values in columns 7 and 8. There are slight disagreements between them but they are not important in actual applications of the formula. After calculations for the values at other speeds, it has been found that the constants, 5 and 40 for B and k respectively are very satisfactory round numbers.

Table II shows the results of the formula and the test data.* They generally agree within 1 or 2 percent, except those for 70, and 75 tons average car weights at 40 m.p.h. The data for these two points cannot be given as much weight as other points because no actual tests were made on these particular points. Further proof of the validity of our formula will be found when we compare the formula with the original points obtained in the tests as shown in Fig. 64.

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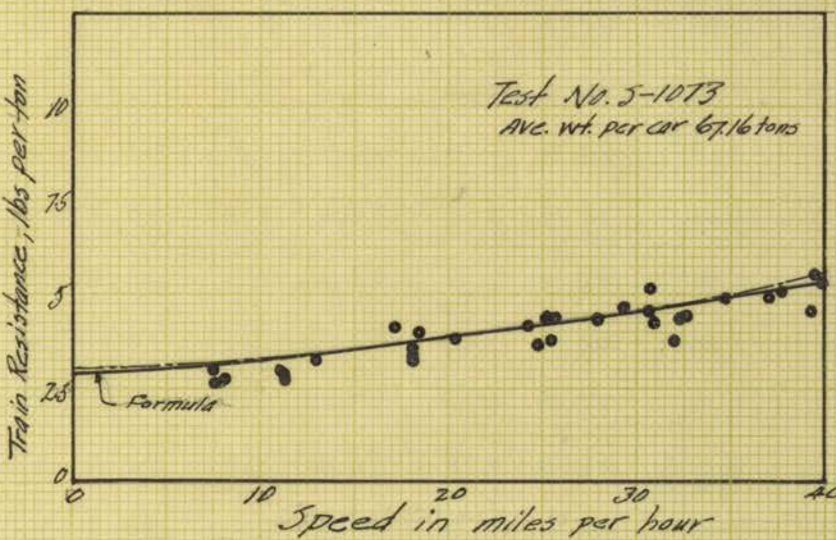
* Schmidt's "Freight Train Resistance". The values for 0 m.p.h. are obtained from his formulas page 34, and other values are taken from his Table 3 on page 35.


Test No. 5-1073
Ave. wt. per car 67.16 tons
Train Resistance, lbs per ton
10
7.5
5
2.5
0
Formula
0
10
20
30
40
Speed in miles per hour

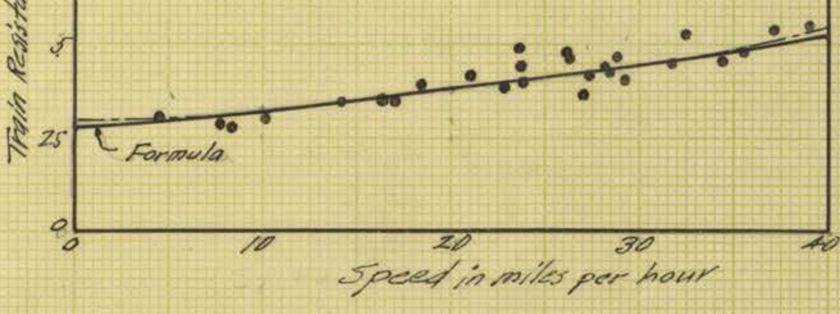

Test NO. 5-1076.
Ave. wt. per car 69.92 tons
Train Resistance, lbs per ton
10
7.5
5
2.5
0
Formula
0
10
20
30
40
Speed in miles per hour

Fig. 64.

TABLE II.

Avg. Car Weight, W.	0 m.p.h. by Test	by Formula	10 m.p.h. by Test	by Formula	20 m.p.h. by Test	by Formula	30 m.p.h. by Test	by Formula	40 m.p.h by Test	by Formula
15	7.15	7.15	8.2	8.56	9.6	9.56	11.3	11.52	13.4	13.13
20	6.30	6.25	7.3	7.28	8.5	8.42	10.0	9.89	11.8	11.58
25	5.60	5.55	6.5	6.41	7.6	7.47	9.0	8.73	10.6	10.19
30	5.02	5.00	5.8	5.79	6.8	6.76	8.0	7.88	9.5	9.17
35	4.49	4.55	5.2	5.29	6.1	6.22	7.3	7.29	8.6	8.36
40	4.15	4.16	4.7	4.84	5.5	5.66	6.6	6.59	7.6	7.36
45	3.82	3.84	4.3	4.48	5.0	5.24	6.0	6.11	7.3	7.10
50	3.56	3.57	4.0	4.17	4.6	4.88	5.5	5.68	6.8	6.51
55	3.38	3.33	3.7	3.89	4.3	4.56	5.1	5.31	6.3	6.16
60	3.19	3.12	3.5	3.66	4.0	4.29	4.9	5.02	6.0	5.79
65	3.06	2.94	3.3	3.46	3.9	4.05	4.7	4.73	5.7	5.38
70	2.92	2.78	3.2	3.28	3.8	3.84	4.5	4.48	5.6	5.19
75	2.87	2.63	3.2	3.10	3.7	3.63	4.5	4.23	5.5	4.90

Passenger cars are always of better construction than freight cars, and they are also more carefully maintained than the latter. Passenger and freight trains run on common tracks. There is no reason why passenger train resistance should materially differ from freight train resistance, except the following two.

Some passenger cars have six-wheel trucks whereas practically all freight cars are equipped with four-wheel trucks, and the specific bearing pressure of six-wheel trucks may be lower than that of four-wheel trucks. If this is the case, passenger train resistance must differ from freight train resistance. The conclusion* of a reliable passenger train resistance test, however, reads, "From the resistance figures obtained on the various trains no consistently greater increase in resistance was found for trains having the greater number of six-wheel truck cars. In fact, some 10 car trains made up of six four-wheel truck cars and four six-wheel truck cars showed lower resistance figures than when composed of eight four-wheel truck cars and two six-wheel truck cars. From the analysis of these and other tests made on the W. J. & S. R.R. it was concluded that the resistance of steel passenger cars at a given speed is a function of the weight and is practically independent of the arrangement of truck wheels ...."

Another reason why the specific resistance of passenger trains should be greater than that of freight trains is that passenger cars have generally greater area for skin friction than freight cars of equal weight, consequently the air resistance of passenger trains, which is the major part of total inherent train resistance at high speeds, will be much greater.

Keeping in mind the facts just mentioned, the following formula, (VII) has been developed, by simply changing the factor of the third term according to the theory previously laid down.

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* Pennsylvania Railroad Test Department Bulletin No. 26, page 25.

$$R = \frac{250}{W + 20} + \frac{5}{W + 40}V + \frac{.0425}{W}V^2 \quad \text{for passenger trains..(VII)*}$$

The results of the formula are graphically represented and compared with the results of other formulas and experiments, in Fig. 65. The curves labelled "Pa. R.R." are those representing the values equivalent to 85 percent of "average maximum" resistance expressed by the Pennsylvania Railroad passenger train resistance formula referred to in the preceding chapter**; those labelled "Schmidt and Dunn" are some of the results of the passenger train resistance tests recently made on the Illinois Central Railroad, and other curves are those representing the results of well known formulas which require no explanation. Although the formula has been developed entirely independent of any passenger train resistance test data, the results agree fairly well with more recent test results, namely the Pennsylvania Railroad tests and the Schmidt and Dunn tests. In view of this fair agreement and great divergence in the test data, the formula may be adopted for general use where no reliable data from dynamometer car tests on a particular road with particular equipment is available.

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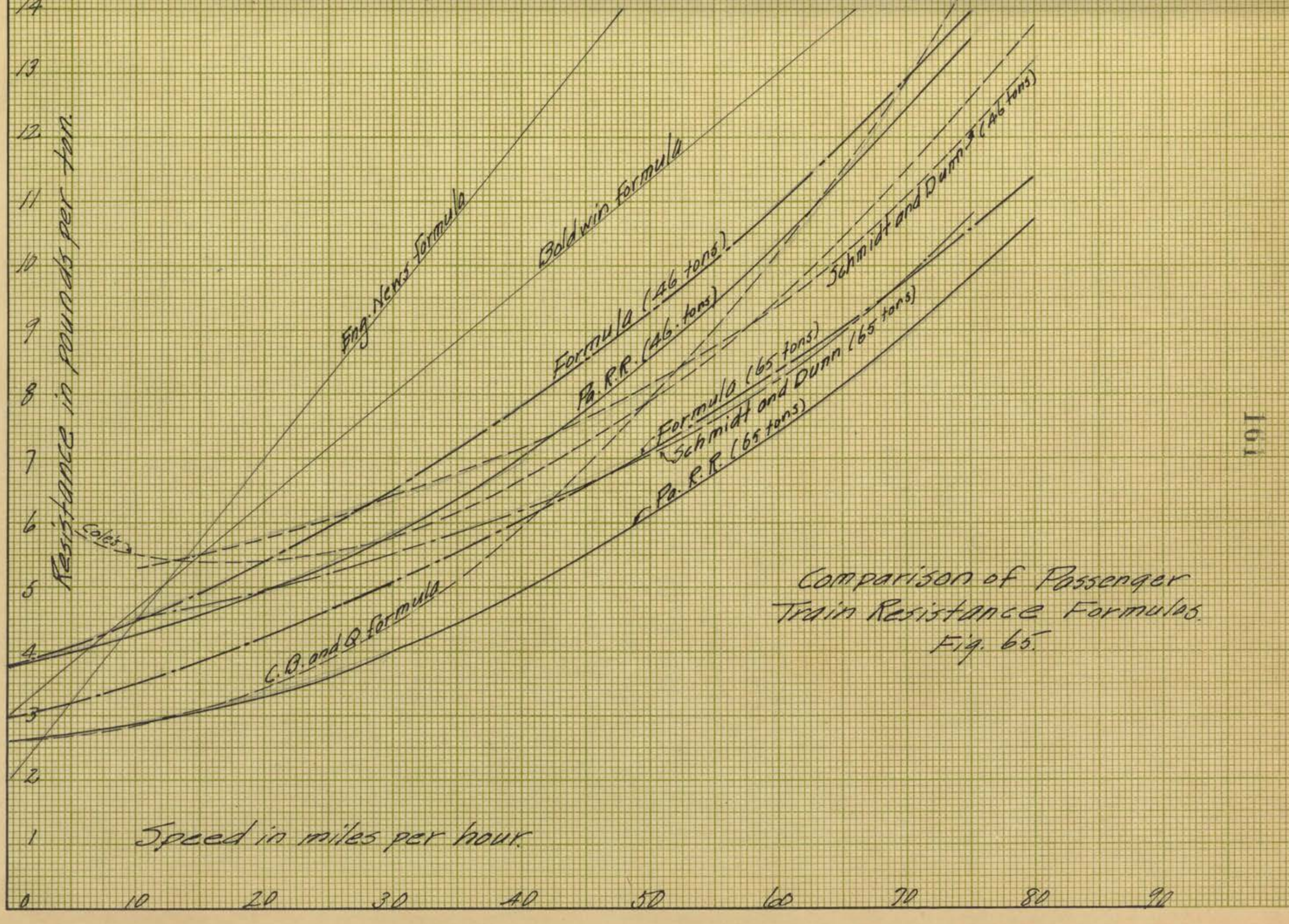
* Assuming A = 120 sq. ft. and 1 = 65 ft. long.

** See also Penna. R.R. Test Dept. Bul. No. 26.

ø E.C.Schmidt & H.H.Dunn: "Passenger Train Resistance", Bul. 194 American Railway Engineering Association, Feb. 1917.



Comparison of Passenger Train Resistance Formulas.
Fig. 65.

VIII. PERFORMANCE OF RAILWAY BRAKES.

1. Railway brakes. - The function of railway brakes is to check the speed of trains smoothly when necessary and to stop them at the desired place within a given time. The importance of the role the brakes play in railway train operation is apparent and it cannot be too strongly emphasized, especially in case of heavy high speed trains. There are several different systems of brakes, namely, the hand brakes, steam brakes, vacuum brakes, compressed air brakes and electro-magnetic brakes; some of them, however, are obsolete today while another is still in the experimental stage. The type most effective and most extensively used at present is the air brake. But the history of the air brake alone presents a long series of gradually improved types, the concise description of which requires volumes. Hence, no description of any system or type of the brakes will be attempted in this paper, but a brief review of their performance which has direct relation to the motion of railway trains will be attempted.

2. Retarding force of brakes. - The retarding force of brakes can easily be computed from the following formula,

$$R = fPe,$$

where R represents the retarding force per wheel in pounds, f the mean coefficient of friction, P the "braking power" or braking force per wheel computed from the actual air cylinder pressure in pounds, and e the efficiency of the brake rigging. The value of P can be calculated easily when the dimensions of the air cylinder, the air pressure and the lever ratio of the brake arms are known. The efficiency of the standard (single shoe) brake rigging* varies from 66 to 69 percent within the braking power range of 30 to 150 percent, and 68 percent is a fair average value for practical purposes. The coefficient of friction, however, varies with several important factors as in the case of the sliding friction of lubricated surfaces and of non-lubricated rolling surfaces, which have been discussed in the preceding chapter.

3. Review of brake tests. - The first and one of the most elaborate experimental investigations made on this subject was the famous Galton-Westinghouse railway brake test**, in which the coefficient of friction of cast-iron brake shoes on steel tired wheels is found as shown by the points in Fig. 66. From this data R. A. Parke*** derived the following formulas for the relation between the coeffieicnt of friction and speeds:

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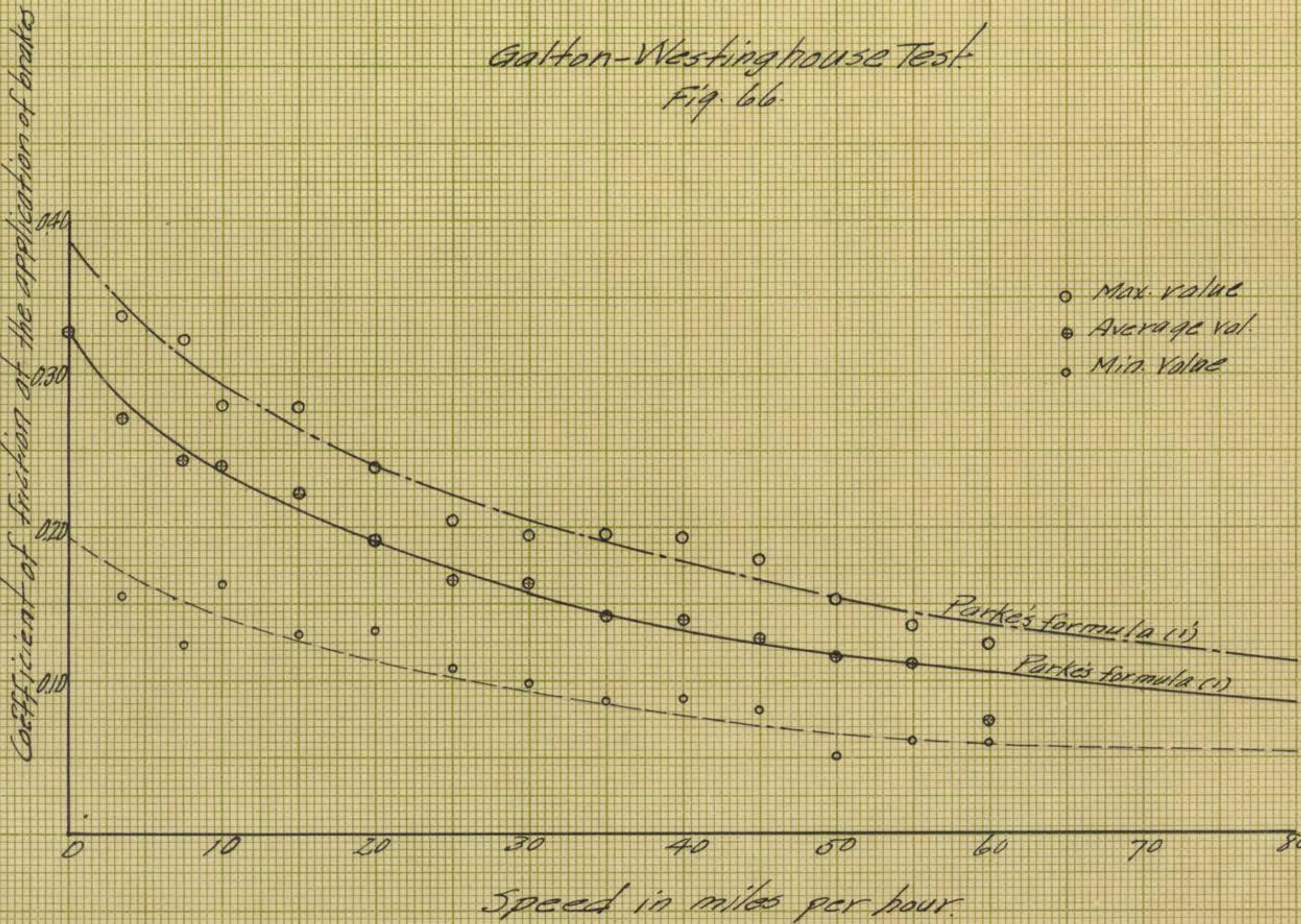
*"Brake Tests", Pennsylvania RR. Test Dept. bulletin No. 25 page 132.

** D. Galton: "Effect of Railway Brakes", Inst. of M.E., vol. p. 467, and p. 590, (1878), and vol. , p. 170, (1879).

*** R.A.Parke: "Railroad Car Braking", A.I.E.E., vol. 20, p. 235, (1902)

Galton-Westinghouse Test.
Fig. 66.



$$f = \frac{0.326}{1 + o.03532V} \text{ for the average values, ... (1)}$$

and $$f = \frac{0.382}{1 + 0.02933V} \text{ for the maximum values, ... (1')}$$

where f represents the coefficient of friction and V the speed. The curves showing the results of these formulas are shown in Fig. 66. In this test Galton found also that the coefficient was affected by the duration of the application of brakes as well as by the speed. After a careful study of the data Parke concluded that the coefficient decreases not only with the time but also with speed, that is, it varies with distance, which is the product of time and speed, and he derived the following formula for this relation:

$$f = \frac{1 + 0.000472L}{1 + 0.00239L} f', \text{ .... (2)}$$

where f is the coefficient at the beginning or at the instant of the brake application, which may be found from formula (1) or (1'), and f' the coefficient at the end of distance L.

The formulas (1) and (1') are regarded as very reliable and applicable to present practise, while (2) is accurate only in slightly less degree. In general the form of the presentation of both the original data and the formulas is of theoretical interest, but it is not convenient. For instance in formula (2), the value of f' is that which we need in order to find L, but f' cannot be found until we know L. A rational solution of (2) by means of an auxiliary equation of condition has been suggested by Parke but the process is so complicated that it is only of little value to practical engineers. Parke himself did not recomend the practical use of formula (2).

A number of special brake tests were made in the Berlin-Zossen Tests of 1903, and the average coefficient of friction shown by these experiments at various speeds is given as follows:

Speed, m.p.h. ----	109.0	105.2	99.2	92.9	80.3	62.0	31.0	15.5	6.2
Coef. of fric.,% -	6.9	6.6	6.6	6.4	5.8	6.1	8.4	10.0	13.

It may be noticed that the values are very low, agreeing with only the average minimum values of the Galton test. They are also very low compared with reliable data obtained in this country, which will be cited in the following paragraphs.

In 1909, an extended brake test was conducted by the Master Car Builders' Association*. Much important information on brake performance were disclosed, but no attempt was made to determine the coefficient of brake friction on the road. In connection with this test, however, tests on various kinds of shoes were made at the Purdue University and the American Brake Shoe and Foundry Company's laboratories. The results revealed great difference in the coefficient of friction of brake shoes of different materials and makes. Some of these data are shown compared with the results of later tests in Fig.68.

4. <u>Mean coefficient of friction computed from Pennsylvania Railroad Tests.</u>** - The results of the most elaborate brake tests extending over a period of five years were published in 1913. These tests were thorough in every respect and the results are conclusive. In this test the efficiency of the brake

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* Proc. M.C.B.Ass'n., vol. 44, p. 83-273. (1910)
** "Brake Tests", Penna. R.R. Test Dept. bul. No. 25, 1913.

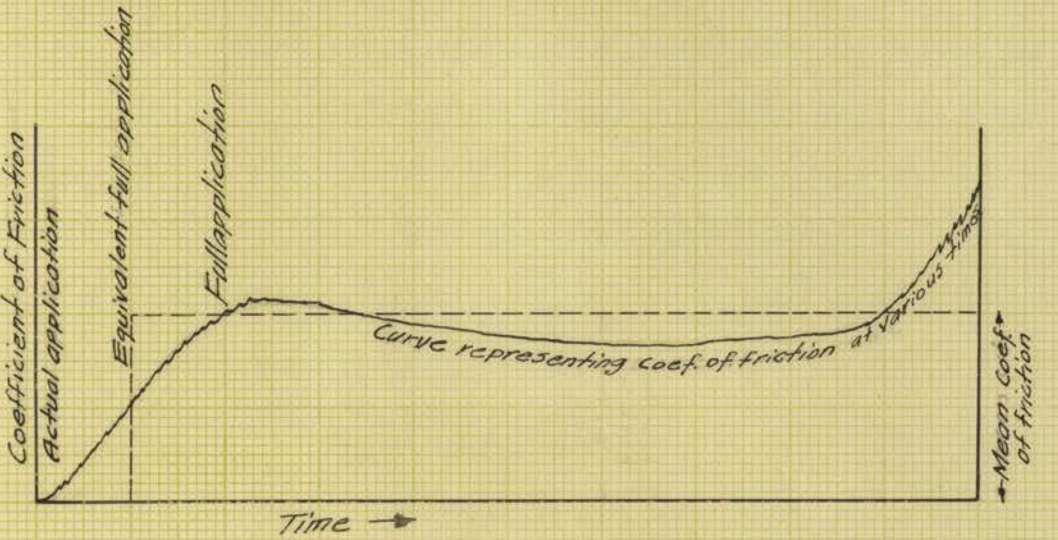
Coefficient of Friction
Actual application
Equivalent full application
Fullapplication
Curve representing coef. of friction at various time
Mean Coef. of friction
Time

Fig. 67.

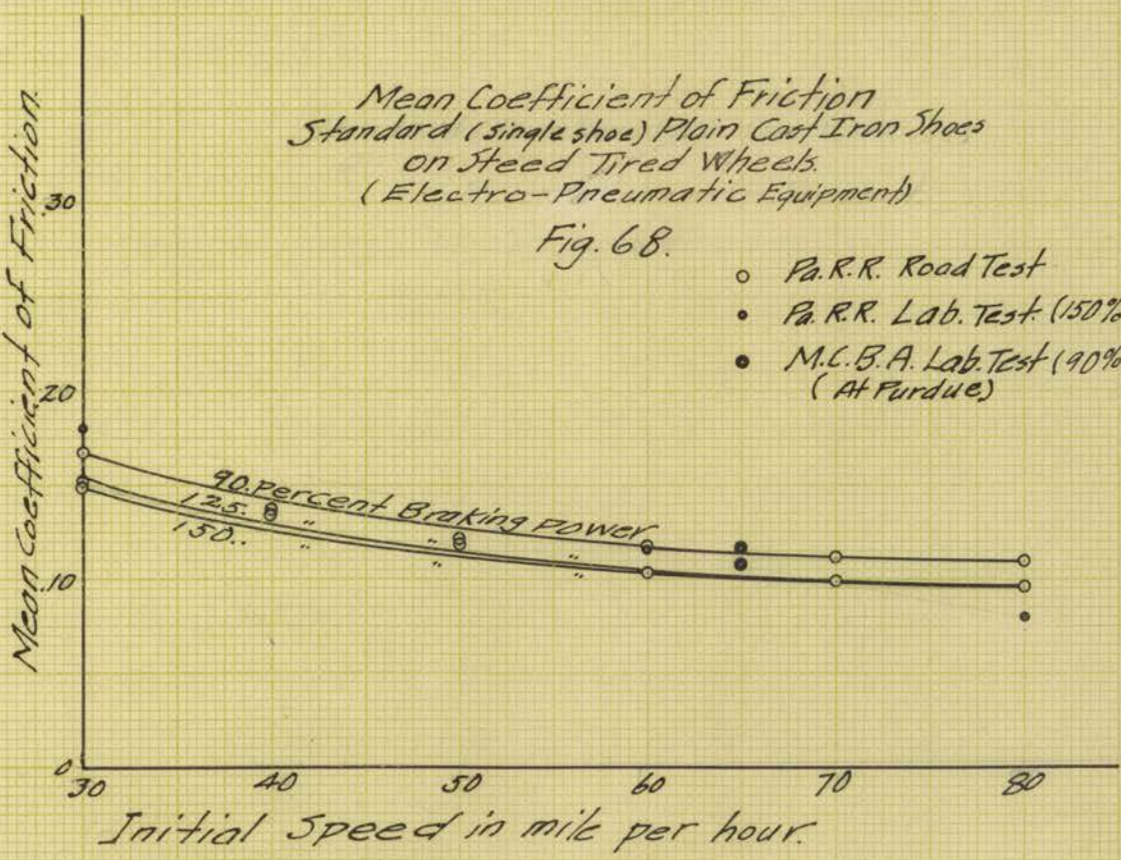
Mean Coefficient of Friction
Standard (single shoe) Plain Cast Iron Shoes
on Steed Tired Wheels.
(Electro-Pneumatic Equipment)
Pa. R.R. Road Test
Pa. R.R. Lab. Test (150%. B.P.)
M.C.B.A. Lab. Test (90%. B.P.)
(At Purdue)
90 Percent Braking Power
125
150
Mean Coefficient of Friction
0
.10
.20
.30
30
40
50
60
70
80
Initial Speed in mile per hour.

Fig. 68.

rigging was found to be about 68 percent, and the time required to get full pressure on the brake shoes after their application was about 2 seconds; the stopping distances for various initial speeds - 30, 60, and 80 m.p.h. - were also accurately determined. With these data the mean coefficient of plain solid cast iron brake shoes on steel tired wheels is computed by means of the following formulas:

$$F = \frac{0.0334V^2}{S} \quad \ldots\ldots\ldots\ldots\ldots\ (3)$$

and

$$\underline{m} = \frac{F}{P\ e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4)$$

where F represents the retarding force per shoe in pounds, V the speed in miles per hour, S the displacement in feet after the equivalent full application, m the mean coefficient of friction (see Fig. 67), P the braking force per shoe computed from the air pressure in the cylinders, and e the efficiency of brake rigging. In formula (3), it is assumed that the inherent train resistance is neutralized by the kinetic inertia force of rotation of the car wheels, axles, etc. as recomended by both the Master Car Builders' Association and the Pennsylvania Railroad Test Department. The values of m thus computed are given in the following table and are also plotted in Fig. 68. The mean coefficient of standard (single) solid cast iron shoes obtained by laboratory tests is also plotted on the same diagram. Among the numerous points obtained by laboratory tests made to supplement the Master Car Builders' Association test, only two are comparable since only two cast iron shoes were tested at the braking force of 11448 pounds per shoe, which corresponds to 90 percent braking force. These two points are plotted on the diagram and found to agree very well with the results computed from the Pennsylvania test. The mean coefficient of friction of plain solid cast iron shoes on steel tired wheels represented in Fig. 68, is derived from the most reliable tests by ~~the~~ approved methods, and is fairly well supported by laboratory tests; we may, therefore, use it when an estimate of stopping distance is to be made.

Mean Coefficient of Friction
Plain Cast Iron Shoes on Steel Tired Wheels.

Initial speed, m.p.h.	30.	40.	50.	60.	70.	80.
At 90% braking power,	.117	.137	.121	.118	.112	.11
At 125% " "	.151	.134	.118	.105	.099	.095
At 150% " "	.149	--	--	.106	.099	.096

It may be added here that the stopping distance used in the computation is not the average value but the maximum within which all the test trains stopped. Therefore, the stopping distance estimated by the coefficient will be safe for trains with electro-pneumatic equipment. To estimate the stopping distance of trains with regular pneumatic equipment extra distance corresponding to the displacement with the initial speed for one second should be added. The percent braking power referred to is that of cars and not of locomotive and trains, but it may be considered as that of the train when it is applied to trains of ordinary make-up, since the tests were made with trains of a make-up common in actual train service. Further, the incidental train resistances, such as grade and curve resistance should be properly considered, and if the weather is not good the friction of wheels on rails must be considered.

IX. SPEED-TIME, DISTANCE-TIME, AND SPEED-DISTANCE CURVES.

A. Representation of Speed, Time, and Distance.

The interrelations among the three important elements in the motion of a body, viz: speed, time, and distance, can be expressed at least in three different ways; first, they can be expressed by numbers in a tabular form, second, by mathematical equations, and third, by graphs or curves. Each of these has certain advantages and disadvantages when compared with others, but the third method is complete in representation, easy in production, and giving a very vivid conception of the fact it represents, is the most practical way. The speed-time curves, distance-time curves, and speed-distance curves, which are self-explanatory and require no explanation of their meaning, are the curves just mentioned.

The uses of these curves are numerous. Speed-time and distance-time curves are very useful, if not indispensable, in the accurate determination of running time of trains, which in turn is a very important element in several railway problems, as will be explained in detail later; in predetermination or estimation of electric energy consumption and fuel and water consumption, and also in determination of capacity of electric traction motors. These curves are also very useful in connection with tonnage rating, railway location and re-location. Speed-distance curves lack certain important qualities possessed by speed-time and distance-time curves, and can not be employed in certain problems in which the latter curves are effective; but they have the distance as their argument, which is more readily conceivable than time, and is a more direct expression of the facts. This peculiar quality makes them preferable in certain problems.

There are several different methods for constructing these curves, namely analytical methods, chart methods, graphical methods and mechanical methods. As will be seen in the following description the first two methods are superseded by the last two methods on account of their accuracy, ease in construction, and hence their practical application. In the paragraphs to follow we will study all these methods for the construction of these useful curves.

B. Analytical Methods of Determining the Speed, Time and Distance Involved in the Motion of Trains.

If the acceleration or the net tractive force of a train is constant as in the case of a falling body, the speed, time, and distance involved in the motion of the train can be easily computed by means of the simple kinematic formulas,

$$V = at + Vo, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$S = \frac{1}{2}at^2 + So, \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

where V denotes the speed, a the acceleration, t the time, Vo the initial speed, S the distance and So the initial distance. In the case of railway train motion, however, acceleration is seldom a constant value* but one which varies with the speed of the train, that is, $a = f(V)$, and the computation is not usually so simple. Since, however, $a = dV/dt$, we get,

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* Acceleration with resistance (electric) of all motors is assumed to be constant. Also the tractive effort of three-phase motors can be considered as constant so long as the number of poles in action is constant. See Fig. 5.

$$\frac{dV}{dt} = f(V)$$

$$dt = \frac{dV}{f(V)}$$

or $$t = \int \frac{dV}{f(V)} \quad .... \quad (3)$$

Similarly, since $dS = Vdt$, we have

$$dS = V\frac{dV}{f(V)}$$

or $$S = \int \frac{VdV}{f(V)}. \quad .... \quad (4)$$

Hence, if $a = f(V)$ is some simple function, equations (3) and (4) may be solved and the speed, time and distance can be calculated. The relation between acceleration and speed of most railway motive powers, however, is not quite simple and a precise representation of the relation by means of a mathematical expression is very difficult.* Even if we assume such an expression can be produced without trouble, the resulting formulas for distance, time, etc., would be in forms too complicated for practical use. On the other hand, if we adopt a simple expression, in order to avoid the resulting complication, it would introduce serious errors in the final results, a fact which makes the entire work of little value. Nevertheless, for certain kinds of investigation, in which great accuracy is not important, the methods may be of some use.

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* F. W. Carter considers such representation for certain electric motors is not difficult.

1. Carter's method. - In his article, "Predetermination in Railway Work", F. W. Carter presented an analytical method for determination of time and distance of electric railway motors whose speed-torque relation is simple and can be expressed with simple equations. He presumes that the relation for a particular motor can be closely expressed by an equation of the form,

$$(F + F_o)(V - V_o) = KF_oV_o,$$

where F represents the gross tractive effort, V the speed and Fo, Vo, and K are certain constants for a particular motor. Combining this relation with certain principles in mechanics he has developed the following formulas:

$$t = (q' - 1)\left\{ - (q - 1) - \log_e \frac{q' - q}{q' - 1} \right\}$$

and

$$S = q't - \frac{q' - 1}{2}(q - 1)^2,$$

where t denotes the time in units of wV_o/KF_og, a constant quantity for a particular motor and load; S the distance in units of wV_o^2/KF_og; q the ratio V:Vo; and q' a similar ratio which varies with the motor, train resistance, etc. Besides these formulas, he gives six more formulas for the determination of time and distance on the "acceleration with resistance", coasting, and braking curves. For further details, reference may be made to his original article with discussion in Trans. Am. Inst. Elec. Eng., vol. 22, (1903) pp. 133 - 174.

2. Isaacs and Adams' method. - In 1910, Isaacs and Adams published an article in which they described an analytical

method for plotting speed-time curves, etc.* In this method the speed-tractive effort relation of steam locomotives is assumed to be represented by a straight line as shown in Fig. 67, and train resistance by a broken line as shown in Fig. 68. Based upon these rough but simple assumptions, they have derived the following formulas for the acceleration, distance, and time of steam train movement:

Symbols: A = a certain constant.

B = boiler pressure in lbs. per sq. in.

C = L/D

D = diameter of drivers in in.

d = diameter of cylinders in in.

G = percent of grade

F = acceleration force in lbs.

V = train speed in m.p.h.

S = distance in feet.

t = time in seconds.

W = weight of train including locomotive and tender, lbs.

For acceleration,

$$F = \frac{1}{11,000}(A - KV)$$

where $A = (10,450d^2CB - 110WG - 8.25W)$

$K = (392\ C^2d^2B + 1.65W)$.

For distance,

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* J. D. Isaacs and Adams: "Effect of physical characteristics of a railway upon the operation of trains", Proc. Am. Ry. Eng. Assoc., vol. 11, part 2, p. 1327. (1910).

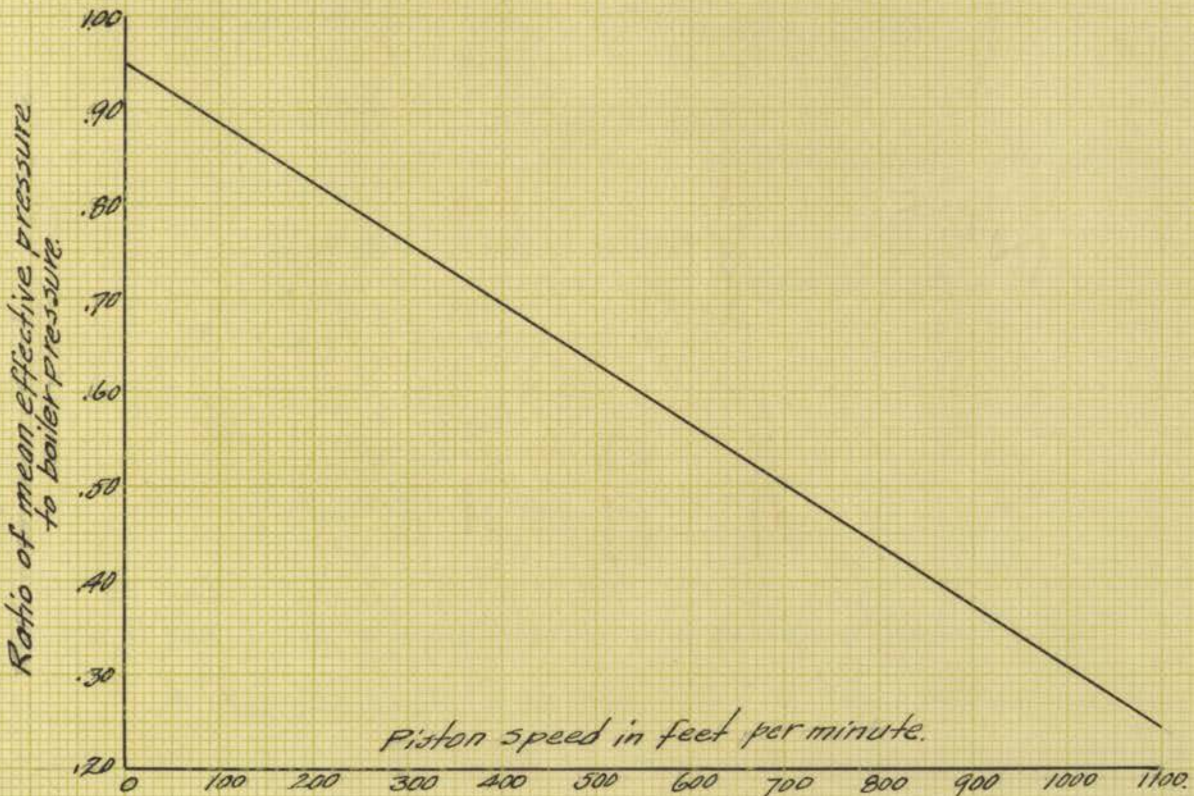
Ratio of mean effective pressure to boiler pressure.
1.00
.90
.80
.70
.60
.50
.40
.30
.20
Piston speed in feet per minute.
0
100
200
300
400
500
600
700
800
900
1000
1100.

Fig. 67.

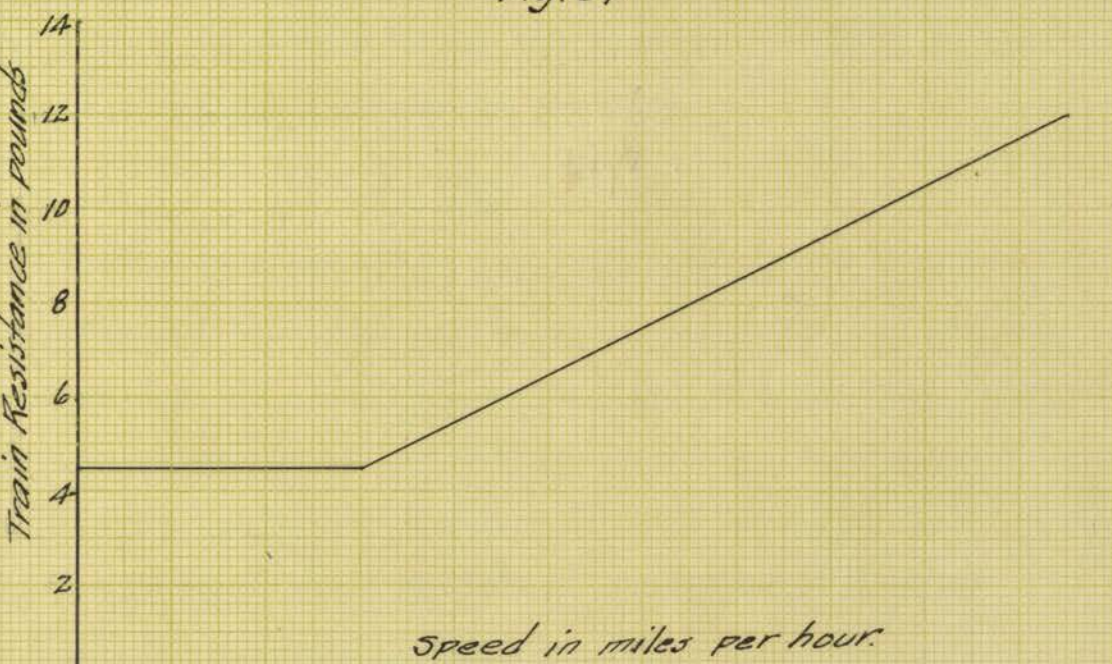
Train Resistance in pounds
14
12
10
8
6
4
2
0
Speed in miles per hour.
10
20
30

Fig. 68.

$$S = \frac{-734.8W}{K}\left\{Vo - V_1 + 2.3\frac{A}{K}\log_{10}\left\{\frac{\frac{A}{K} - Vo}{\frac{A}{K} - V_1}\right\}\right\}$$

where $\left.\begin{array}{l} A = 10{,}450d^2CB - 110WG - 24.75W \\ K = 392C^2d^2B \end{array}\right\}$ when V = 0 - 10 m.p.h.

$\left.\begin{array}{l} A = 10{,}450d^2CB - 110WG - 8.25W \\ K = 392C^2d^2B + 1.65W \end{array}\right\}$ when V - 10 - 35 m.p.h

For time,

$$t = \frac{-19.2W}{K}\left\{\log_{10}\left\{\frac{\frac{A}{K} - Vo}{\frac{A}{K} - V_1}\right\}\right\}$$

where K and A are the same as in the distance (S) formula.

3. Freudenberger's method. - L. A. Freudenberger published an article* in which is described his method of computing the time required by a train or car to attain a given speed. This method is substantially the same as Carter's, except that he determines the function in the form,

$$\frac{1}{a} = \frac{1}{f(V)} = \phi(V),$$

from a diagram showing the relation between the speed and the reciprocal of acceleration, which he prepares from a diagram representing the relation of speed and acceleration. Another point in which his method differs slightly from Carter's is the use of a train resistance equation as a function of speed while Carter considers train resistance independent of speed.

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* "Plotting of speed-time curves from the acceleration-speed curve". Electrical World and Engineer, vol. 42., pp. 96 and 219. (1903).

4. <u>"Point to point" method</u>. - Another method which is worthy of mentioning is the "point to point" method. In this method the tractive effort-speed relation is not expressed in any mathematical equation, but the curve is divided into small segments and the average value of each segment is taken independently as the accelerating force between the two speeds which correspond to the two ends of the segment. For example, let curve AB in Fig. 71 represent the speed-tractive effort (net) relation of a given locomotive. The times required to reach the speeds for instance, 10, 20, 30 m.p.h., etc., are to be found in order to plot the speed-time curve. Then dividing AB into segments we can find the average accelerating forces for various speed increments; and dividing these values by a certain constant which will be explained in detail later, say k, we get corresponding accelerating forces. Then, with these quantities and the relation,

$$\frac{F}{M} = \frac{\Delta V}{\Delta t} \quad \text{or} \quad t = \frac{k}{F}(V - V_{,}) + t_{1},$$

we can calculate t for any speed, since V, the speed increment is chosen at will and t_1 the initial time is a known value if we proceed from zero speed. The speed increment should be so small that the averaged acceleration forces will not introduce into the final results any objectional error. If there is any incidental resistance the net tractive effort should be properly adjusted in the calculations. The time in coasting and braking motion can be computed substituting the train resistance - inherent and incidental - and braking force respectively, for the tractive effort.

C. "Chart Methods" for Plotting of Speed-Time and Speed-Distance Curves.

1. Chart methods for speed-time curves. - In 1902, C.O. Mailloux presented to the American Institute of Electrical Engineers a very valuable paper entitled "Notes on the Plotting of Speed-Time Curves", which is still regarded as one of the best papers, if not the best, on this subject. His method is based upon the fundamental relation,

$$t = \frac{V}{f(V)} \quad \text{or} \quad t = \frac{1}{f((V - V_i)/2)}(V - V_i) + t_i,$$

in which t represents the time required to attain the speed V from initial speed V_i, and t_i the time at V_i. In this method the function $a = f(V)$ is not to be expressed in an equation as in the cases of Carter's, etc., but the value $a = f((V-V_i)/2)$ is to be found from the "acceleration-speed chart" which differs from the speed-tractive effort curve only in the scale of ordinates. In order to minimize calculations he uses a "reciprocal chart" which consists of equilateral hyperbolas corresponding to different values of ΔV, and from which the product,

$$\frac{1}{f((V - V_i)/2)}(V - V_i),$$

which is the time increment or $(t - t_i)$ can be readily found as soon as the value $f((V - V_o)/2)$ for the proper velocity increment is found from the "speed-acceleration chart". This is simply a modification of the ordinary "point to point" method mentioned before, hence the accuracy depends upon the magnitudes of speed increments chosen in the actual process. If we take

ΔV as small as dV the result of the method is absolutely correct - of course within the errors of scales. In practise, however, the velocity increment can not be taken infinitely small. Mailloux proved that even if we take the increment as large as 1 to 5 m.p.h. the resulting answer is as accurate as that of the analytical methods.*

The chart method just described is satisfactory in every respect, except that it requires a number of separate charts the production and use of which are laborious and cumbersome. With this fact in view, E. C. Woodruff** has devised another chart method in which one of the charts necessary in the former method is eliminated. In this method the speed-current-tractive effort diagram is used without any alteration. The "service curves", which he produces on the sheet on which the speed-current-tractive effort diagrams are plotted - thus eliminating one sheet of paper - are of practically the same nature as Mailloux's reciprocal chart. The only difference in them is that the argument of the former is force while that of the latter is acceleration. Practically speaking, the only departure Woodruff made fromMailloux's method, as he claims, is that he plotted the service curves and speed-current-tractive effort curves on one sheet of paper while the latter put them on two separate sheets. This scheme has a certain advantage but also great disadvantages. The first objection is that the chart would be

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* Trans. Am. Inst. Elec. Eng., vol. 19, O. 988; vol. 22, p. 174.

** Trans Am. Inst. Elec. Eng., vol. 33, p. 1673; and Elec. Ry. Journal, vol. 44, p. 1155, (1914).

too crowded and the second is that the service curves can apply only to one particular characteristic of motors, while the reciprocal chart once made can be used for any characteristic curves of any motors or steam locomotives.

2. Chart methods for plotting distance-time curves. - Methods for plotting distance-time curves from speed-time curves by means of a chart have been described by several writers.* They are, however, essentially the same and employ the sort of chart shown in Fig.69, which is nothing but a series of straight lines representing the equation,

$$S = V\left(\frac{5280}{3600}\right)t,$$

where S is the distance in feet, V the speed in miles per hour, and t the time in seconds. The method of using this chart would be clear without explanation.

D. Graphical Methods for Constructing Speed-Time, Distance-Time and Speed Distance Curves.**

1. Speed-time curves. - In the previous sections, we

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* Dawson: Electric Traction on Railways. B. J. Arnold: A paper read before A.I.E.E., June 19, 1902.
E. C. Woodruff: "A graphic method for speed-time and distance-time curves". Proc. A.I.E.E. Nov. 1914.

** All the methods, except those otherwise stated, to be discussed in this section, are those independently developed at the University of Illinois. They are not found, so far as the writer is aware, in any English, French, or German publication. It was, however, found later that one of the pamphlets sent to Prof. E.C. Schmidt by Mr. Lipetz, a prominent Russian railway Engineer, УПРОЩЕННЫЕ ПРІЕМЫ РАЗСЧЕТА ВРЕМНЬ ХОДА ПОѢЗДОВЪ, contains discussions of similar methods.

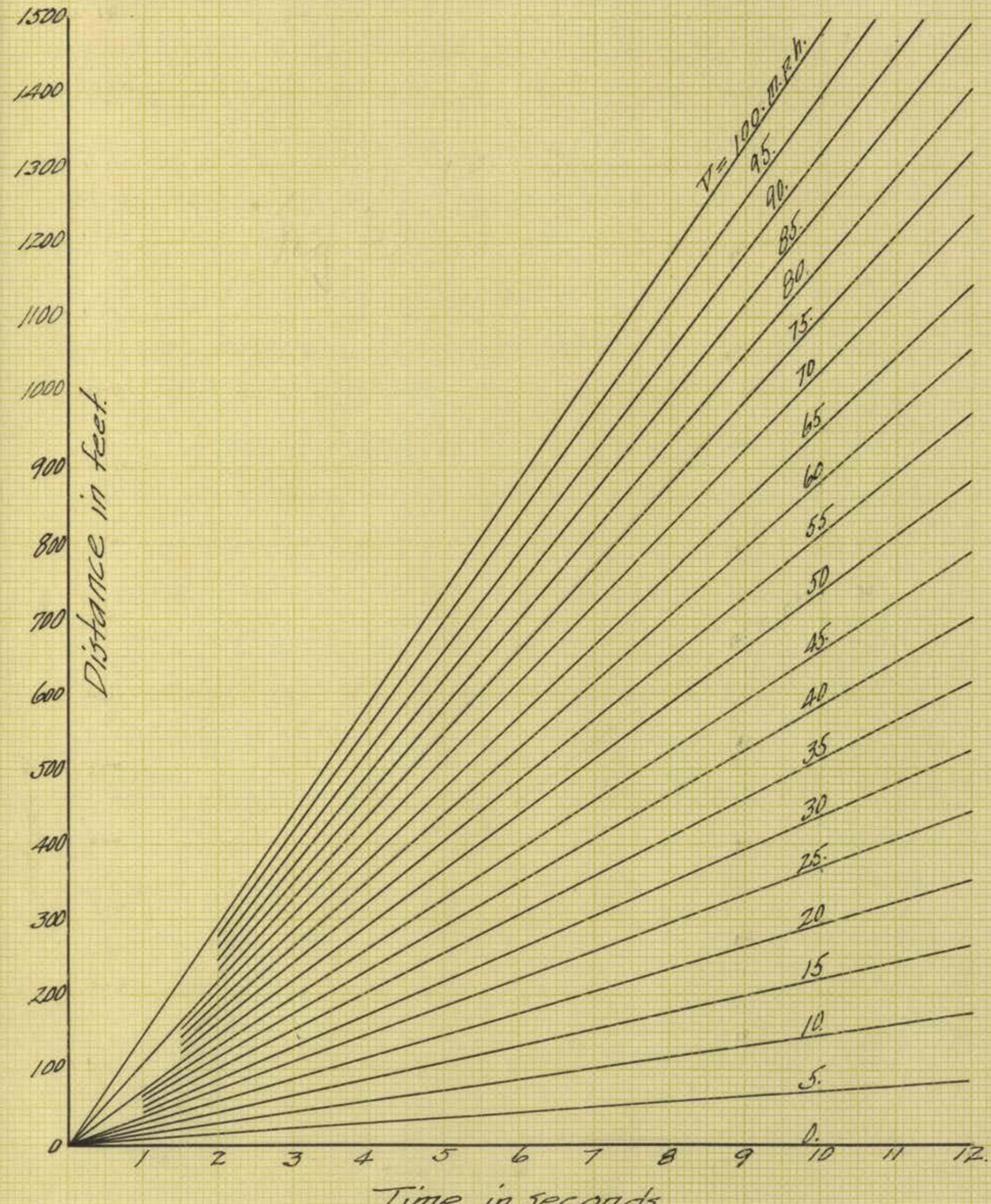
V = 100. m.p.h.
95.
90.
85.
80.
75.
70
65
60
55
50
45.
40
35
30
25.
20
15
10.
5.
0.
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
Distance in feet.
1
2
3
4
5
6
7
8
9
10
11
12.
Time in seconds.

Fig. 69.

reviewed briefly the methods for plotting speed-time curves with the values calculated analytically or found from charts. A description will be given in this section of a graphical method for constructing speed-time curves. In this method several intermediate steps followed in other methods, are eliminated. Consequently it has great advantages in its actual application; but the explanation of the method is very difficult and hardly understood unless all the steps, even those unnecessary in actual application, are carefully considered in its explanation. It should be borne in mind that the description is long and tedious, but it does not follow that the actual application is so.

It has been shown that the effective force of train motion at any instant is the algebraic sum of the motive forces, train resistances, and braking forces, the sign of each force being positive when it acts in the direction of the motion and negative when opposite, and that when the sum is positive the train accelerates, when negative retards, and when zero the motion is uniform. This algebraic summation is effected geometrically or graphically as follows: Suppose in Fig. 70 the ordinates of the curve AB represent the drawbar pull behind the tender of a locomotive, in pounds per ton, that is, the amount obtained by dividing the drawbar pull into the total weight of cars in the train, and curve CD the inherent train resistance in lbs. per ton, which is determined by a dynamometer car test or adequate train resistance formula. Then, the vertical distance between the curves AB and CD represents the net tractive effort on a level tangent track at various speeds of the train.

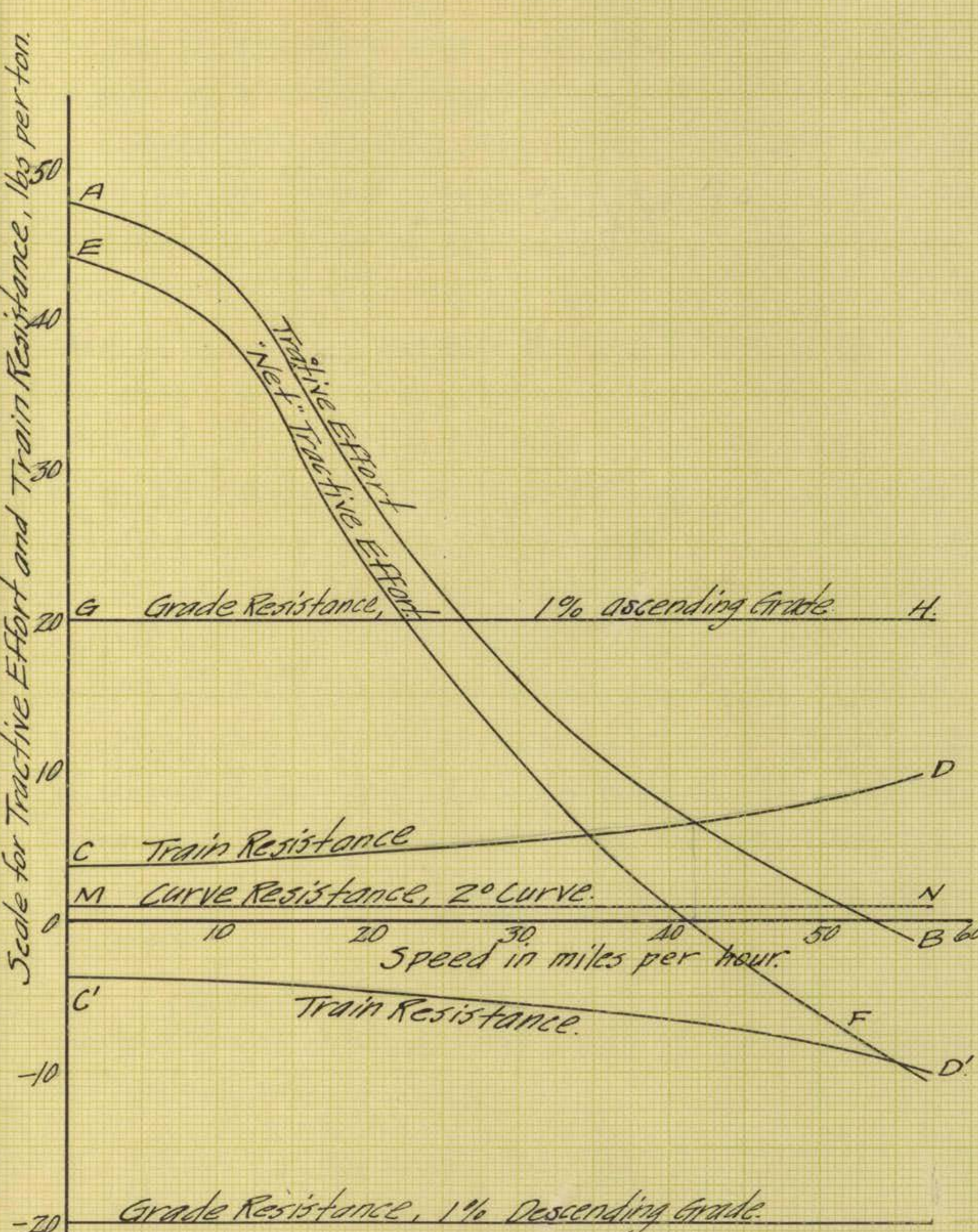
Scale for Tractive Effort and Train Resistance, lbs per ton.
50
40
30
20
10
0
-10
-20
A
E
Tractive Effort
"Net" Tractive Effort
G
Grade Resistance, 1% ascending Grade
H.
D
C
Train Resistance
M
Curve Resistance, 2° Curve.
N
10
20
30
40
50
60
B
Speed in miles per hour.
C'
Train Resistance.
F
D'
Grade Resistance, 1% Descending Grade.

Fig. 70.

The available tractive force on an inclined or curved track or on both may be found in a similar way. For instance a train is on a one percent ascending grade. Then, since grade resistance is 20 lbs. per ton and independent of speed, if we draw a line parallel to and above the speed-axis with a proper scale, as shown by the line GH in Fig. 70, the difference in ordinate between EF and GH will represent the force at various speeds on the grade. If a train is running on a level track of two degrees curvature, the additional resistance is 1.0 pound per ton, and a line like MN may be drawn as in the case of grade resistance and the available tractive effort on that track at various speeds may be found. If a train is coasting or drifting on a level tangent track, the effective retarding force is only the train resistance, which can be represented below the axis like C'D'. If a train is moving with the brakes applied the retarding force is the sum of the braking force and train resistance and can be represented in a similar manner. Thus, the effective force under any conditions of running and of track can be very easily represented.

It was also shown that in order to accelerate a train at the rate of A miles per hour per second it is necessary to apply the effective tractive force,

$$F = (91.1 + 145.5 \frac{N}{W})A, \text{ lbs. per ton.}$$

Then,
$$A = \frac{F}{(91.1 + 145.5N/W)}.$$

But, if V is the speed in miles per hour, and t the time in seconds, $A = \Delta V/\Delta t$, then

$$\frac{\Delta V}{\Delta t} = \frac{F}{(91.1 + 145.5N/W)}$$

Since the value of $(91.1 + 145.5N/W)$ is constant for a particular train, let us call this k, then the above equation becomes

$$\frac{\Delta V}{\Delta t} = \frac{F}{k}.$$

This is the fundamental relation on which the method is based. Suppose a speed-time curve has been constructed, then the slope of a secant drawn through two points on the curve at any speed V and $V + \Delta V$ is equal to $\Delta V/\Delta t$, which is the right member of the above equation. Suppose again that a right triangle whose base is equal to k in a certain scale and its height equal to F in the same scale, has been constructed, the value of F being the mean value of the effective tractive forces between V and $V + \Delta V$ on the speed-tractive effort curve like one shown in Fig. 71. Then, the above equation of condition indicates that the slope of the secant of the speed-time curve is equal to the slope of the hypotenuse of the right triangle. Since V represents any speed it is general, hence any secant on the speed-time curve is always parallel to the hypotenuse of the right triangle whose base is k and the height is the mean value of the tractive efforts corresponding to the speeds at which the secant intersects the speed-time curve. The construction of such triangles is very easily effected when the speed-pull curve of a locomotive is given, and the hypotenuse of the triangles determines the slope of the secant between any two speeds. Thus, any speed-time curve can be easily constructed.

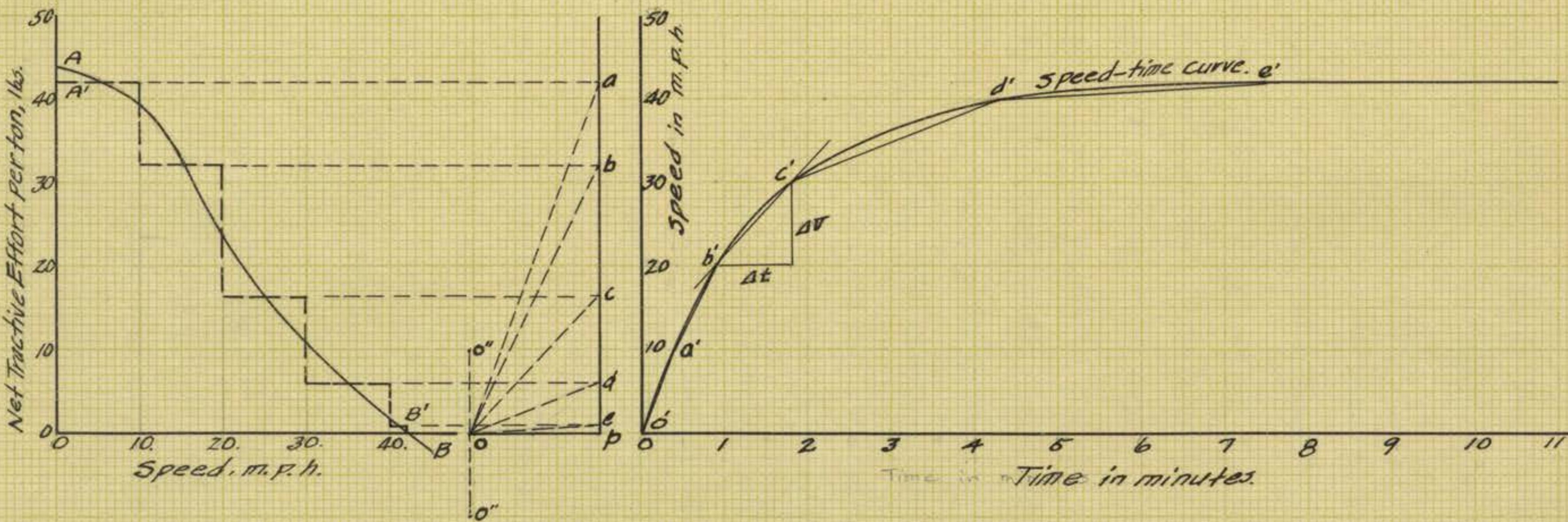
Net Tractive Effort per ton, lbs.
Speed, m.p.h.
Speed in m.p.h.
Time in minutes.
speed-time curve.
A
A'
B
B'
a
b
c
d
e
p
o
o''
a'
b'
c'
d'
e'
o'
Δv
Δt

Fig. 71.

As an elementary example of the method, let us suppose first, the effective tractive effort of a locomotive or an electric traction motor is represented by the broken line A'B' instead of AB in Fig. 71. Then, the effective tractive force between 0 and 10 m.p.h. is equal to ap, and F/k ; tangent of the angle aop. Draw o'a' parallel to oa from o' up to the 10 mile-line as shown in Fig. 71. Between 10 and 20 m.p.h. the effective force is equal to pb, and F/k = tangent of the angle bop. Draw a'b' parallel to ob from a' until it meets the 20 mile-line at b'. In a similar way draw b'c', c'd', d'e', etc. parallel to bc, cd, de, etc. respectively. In each case the value of Δt is automatically found and summed up on the abscissa or the time axis of the speed-time curve. Then, the broken line o'a'b'c'd'e' ..... is the speed-time curve which corresponds to the tractive effort represented by A'B'. If we take 1 or 2 miles per hour for ΔV instead of 10 miles per hour as we did, the speed-time polygon would be very close to the actual smooth speed-time curve. At any rate the points o', a', b', c', ... are on the real speed-time curve, and a smooth curve ~~which~~ drawn through these points must be very close to the real curve, and for practical purposes, it may be taken as the real curve without any objectionable error. Theoretically speaking, if we take ΔV as small as dV, we get the exact curve.

Next, suppose a train is to start and run on a 1/2% ascending grade. Then the tractive effort will be reduced by 10 lbs. per ton. The corresponding speed-time curve can be constructed in exactly the same way after locating the point o"

10 lbs. per ton above the point o and using o" instead of o. If the train is on a descending grade the grade resistance acts in the opposite way and becomes a motive force, and the tractive force will be increased proportionally. The speed-time curve can be constructed in a similar way simply by locating 1" below o with a proper scale. The resistance due to curvature of track can be treated the same as grade resistance, assuming 20 degree curvature offers same resistance as 1 percent ascending grade. In case a train is coasting on a level track or on an ascending grade the force acting on the train is opposite to the motion, or it is in the negative direction. In this case the force, F should be laid out below p, and o" should be located on o if it is on the level, above o if it is on an ascending grade, and below if it is on a descending grade. A train running with brakes applied may be similarly treated.

So far nothing has been said about the scales by which k, F, V and t are to be laid down or measured. This is certainly a very important element in any graphical method. In practise the scales of F and V are fixed and the problem is to determine to what scale k should be laid out in order to represent t to a desired scale.

Let (ΔV) be the value of ΔV per inch of the scale (of speed-time curve).

(Δt) be the value of Δt per inch of the scale (of speed-time curve).

(F) be the value of F per inch of the scale

(k) be the value of k per inch of the scale

Then from the relation, (5) we get

$$\frac{\dfrac{\Delta V}{(\Delta V)}}{\dfrac{\Delta t}{(\Delta t)}} = \frac{\dfrac{F}{(F)}}{\dfrac{k}{(k)}}$$

or
$$\frac{\Delta V}{\Delta t} \cdot \frac{(\Delta t)}{(\Delta V)} = \frac{F}{k} \cdot \frac{(k)}{(F)}.$$

But $\Delta V/\Delta t = F/k$, hence

$$\frac{(\Delta t)}{(\Delta V)} = \frac{(k)}{(F)}$$

or
$$(k) = \frac{(F)}{(\Delta V)}(\Delta t).$$

For example, if $(F) = 20$ lbs. per ton/inch, $(\Delta V) = 20$ miles per hour/inch and (Δt) is desired to be 120 seconds/ inch, then

$$(k) = \frac{10}{10} \times 120 = 120 \text{ lbs. per inch},$$

and if k is 95 lbs., $k/(k) = 0.791$ inch.

Mr. Lipetz' method differs from the method just described at least in the following two points: (1) He lays out unit length on each speed line and the perpendicular lines are erected on the lines on which the values of F taken from the speed-tractive effort curve are laid out, as shown in Fig. 72. The principle is exactly the same; but this is a rather cumbersome method. (2). He considers that the polygons thus constructed constitute the envelope of the speed-time curve, and the curve should be drawn tangent to this polygon. Practically speaking, this is not a very important difference. Theoretically, however,

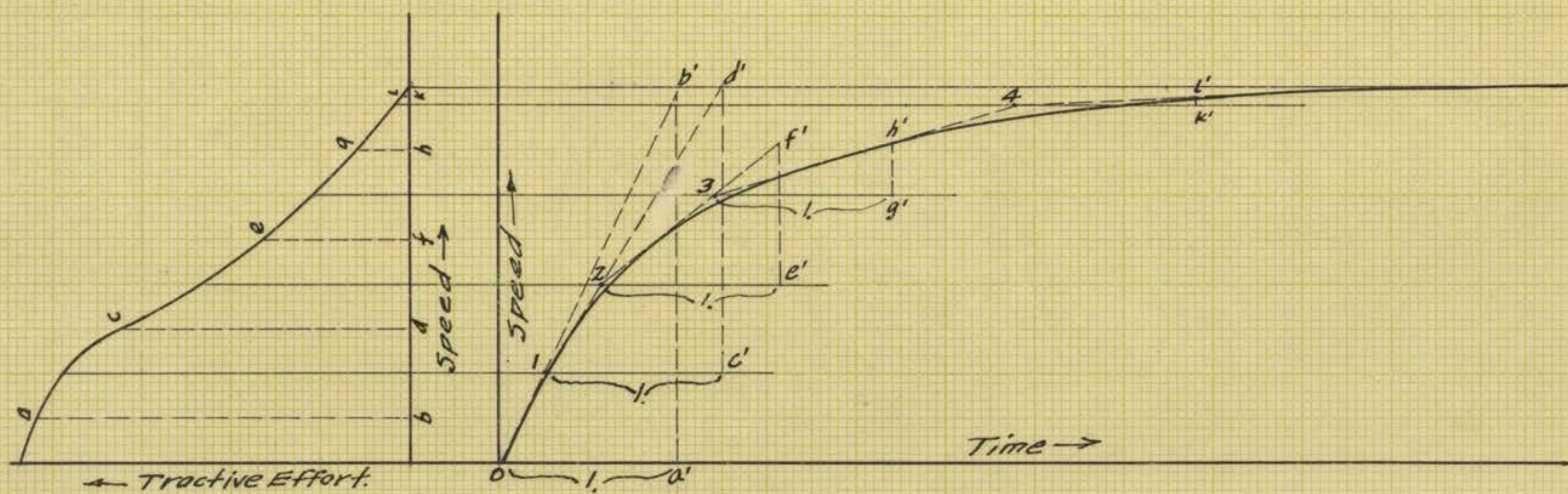
← Tractive Effort.
Speed →
Speed →
Time →
a
b
c
d
e
f
g
h
i
k
0
1.
a'
b'
c'
d'
e'
f'
g'
h'
k'
l'
1
2
3
4

Fig. 72.

it is not the envelope but secants to the curve.

2. Distance-time curves. - It is a well known fact that, whether acceleration is a constant or a variable, the speed-time curve is the first derivative curve of distance-time curve, and vice versa. The distance-time curve, then, may be obtained directly from the speed-time curve by a certain graphic integration.* Let us first suppose that the broken line curve, A'B' in Fig. 73 is the speed-time curve instead of the actual speed-time curve AB, from which the distance-time curve is to be produced, and that the area of each section under the broken line is equal to that of the section under the actual curve.

If ΔS is measured in miles, V in miles per hour, and t in seconds, we have the following relation from the definition of velocity:

$$\frac{\Delta S}{\Delta t} = \frac{V}{3600}, \quad \text{or} = \frac{V}{k'}$$

This relation indicates that the slope of the secant which passes through two points on the distance-time curve at t and $t + \Delta t$, is parallel to the hypotenuse of the right triangle whose base is 3600 to a certain scale and the height is V to the same scale, V being the mean velocity between t and $t + \Delta t$. Then, it is clear that we can construct the distance-time curve from the speed-time in a similar way to that by which we constructed

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* The following method has been developed without any knowledge in graphic integration, although it was found later that the method is the general graphical integration applied to this particular problem, as Hütte applied it to finding area of steam indicator cards. See Hütte, vol. 1. page 1012.

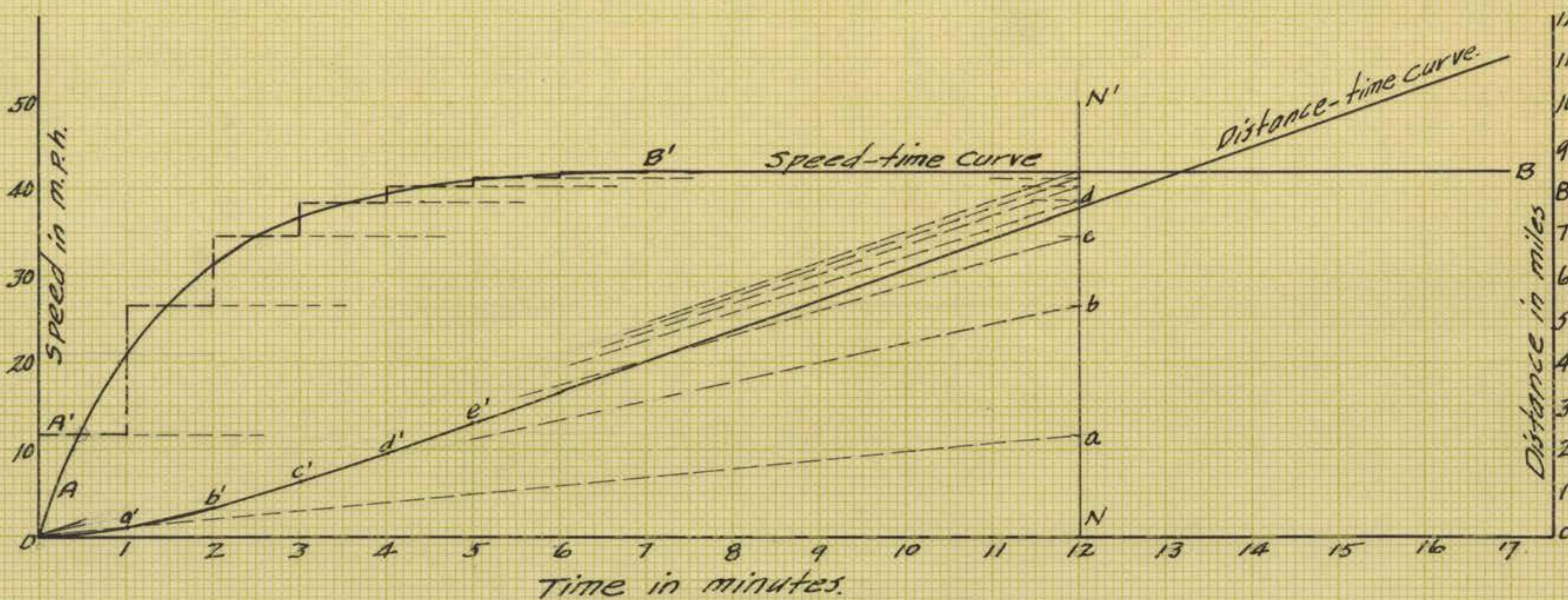
Speed-time curve
Distance-time curve.
Speed in m.p.h.
Distance in miles
Time in minutes.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
10
20
30
40
50
A
A'
B
B'
N
N'
a
b
c
d
a'
b'
c'
d'
e'

Fig. 73.

speed-time curves from speed-tractive effort curves, i.e., simply taking V instead of F and k' instead of k.

As an illustration of the method, in Fig. 73, the line NN' is drawn perpendicular to the time axis at a distance k' from the origin. The point a is located on NN' so that Na is equal to the mean velocity between 0 and 60 seconds. Connect o and a; then oa is the hypotenuse of the right triangle whose height is Na = V, and the base k', which is parallel to the secant cutting through the distance-time curve at t = 0 and t = 60 seconds. Then the point designated by a' in the figure must be on the distance-time curve. The point b is located on NN' so that Nb represents the mean velocity between t = 60 and 120 seconds. The line a'b' is drawn from a' parallel to ob. Similarly b'c', c'd', etc. are drawn. The polygon o a' b' c' d' ... represents the distance-time curve corresponding to the broken line curve A'B'. Since, however, the points o, a', b' ..... are those on the distance-time curve corresponding to AB, the smooth curve drawn through these points may be taken as the distance-time curve corresponding to AB, at least for practical purposes. Taking $\Delta t = dt$, however, we get the curve theoretically correct.

A proper scale of k' can be found from the following formula:

$$(k') = (\Delta t)(V)\frac{1}{(\Delta S)}$$

For instance, if (Δt) = 120 seconds/inch, (V) = 20 miles per hour, and (ΔS) = 4 miles/inch,

$$(k') = \frac{120 \times 20}{4} = 600.$$

and since k' = 3600 always,

$$\frac{k}{k'} = \frac{3600}{600} = 6.0 \text{ inches.}$$

A similar method has been described by Mr. J.G.Pertsch in the Sibley Journal of Engineering in 1910. * According to Richey his method is exactly the same except that he considers the distance-time curve is tangent to the polygon. As pointed out previously a similar notion of Lipet's on the speed-time curve, if we chose ΔV and Δt small enough there is no appreciable error to adopt either idea, but theoretically the polygon is not an envelope but is a group of secants or chords which intersect on the curve.

3. <u>Speed-distance curves.</u> - If speed-time and distance-time curves have been made, the speed-distance curve can be plotted with values found on these curves. This, however, is a very laborious method when only the speed-distance curve is required. In such a case the following method for construction of the speed-distance curve would be of great value.

Since, in any case

$$A = \frac{\Delta V}{\Delta t} \quad \text{and} \quad V = \frac{\Delta S}{\Delta t}\,3600,$$

we have, on elimination of Δt,

$$\frac{\Delta V}{\Delta S} = \frac{A}{V}3600$$

But, $A = F/k$, then

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* See, A. S. Richey's Electric Railway Handbook, p. 180, 1915 Ed.

$$\frac{\Delta V}{\Delta S} = \frac{F}{V \frac{k}{3600}}$$

$$\frac{\Delta V}{\Delta S} = \frac{F}{(Vk'')}, \text{ where } k'' = k/3600.$$

This equation indicates that the slope of the speed-distance curve is parallel to the hypotenuse of the right triangle whose height is F and the base (Vk"). Then it is clear that if we take (Vk") instead of k we can construct the speed-distance curve exactly the same way as the speed-time curves. In this case, however, the base of the triangles is not constant but varies with V.

The values of (Vk"), however, can be easily handled by means of a uniform scale for (Vk") at various speeds laid out on the base of the triangles as shown in Fig. 74. There are at least two other schemes for handling this value, but the method just mentioned will be found the most practical one.

Another method is to consider F/(Vk") as (F/V)/k", i.e. keeping the denominator k" constant in the case of constructing the speed-time curve, vary the numerator instead. Then if we make a curve representing the relation between F/V and V, which can be made without much trouble from speed-tractive effort curves, we can find the value of F/V for the heights of the various triangles and the speed-distance curve can be constructed exactly the same way as the speed-time curves. Fig. 75 shows the diagram to be used in this method.

The principles employed by Lipetz in developing his method are exactly the same. In finding the values of F/V, which are

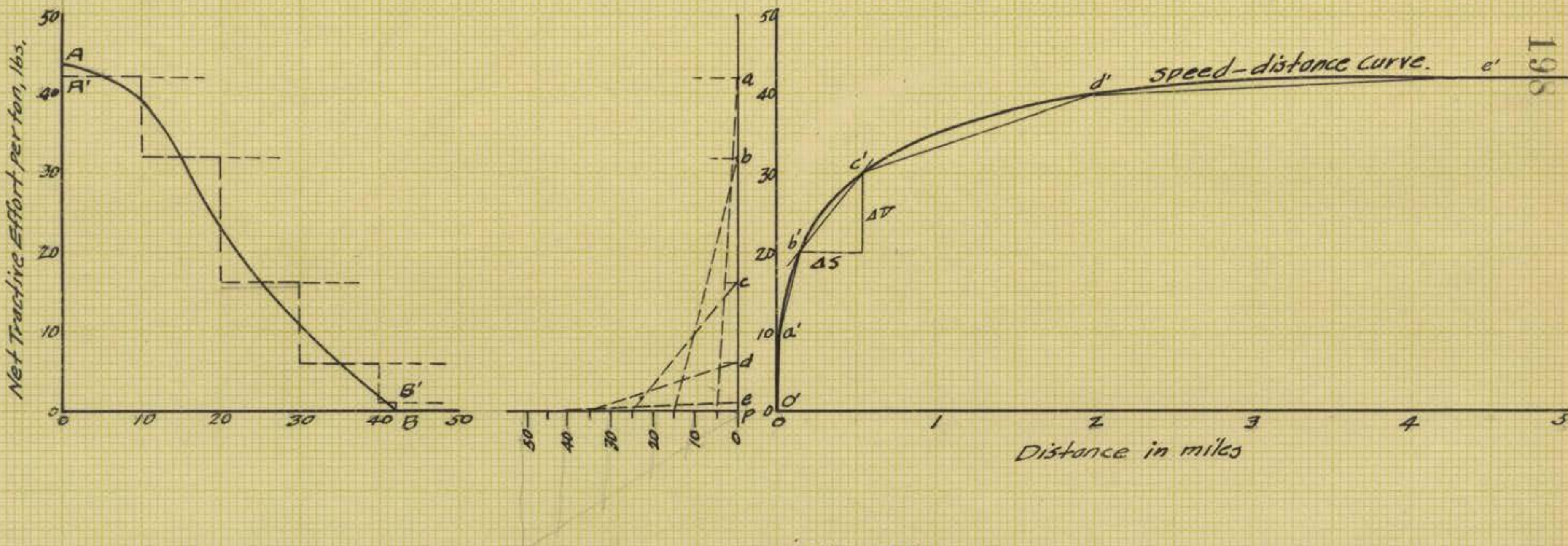
Net Tractive Effort per ton, lbs.
A
A'
B'
B
a
b
c
d
e
p
a'
b'
c'
d'
e'
o'
ΔV
ΔS
speed-distance curve.
Distance in miles

Fig. 74.

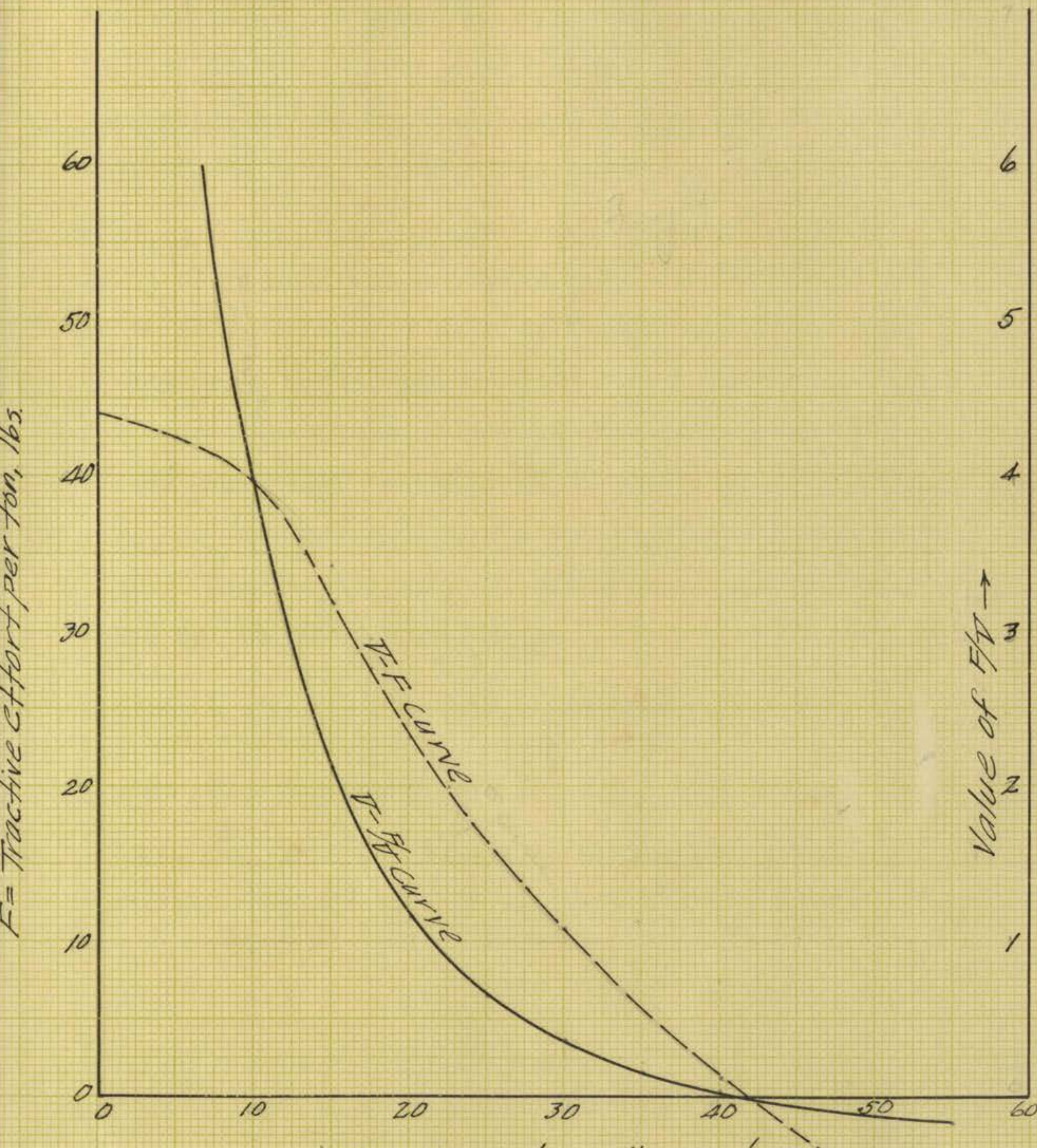
F= Tractive effort per ton, lbs.
Value of F/V →
V-F curve
V-F/V curve
V = Speed in miles per hour
0
10
20
30
40
50
60
1
2
3
4
5
6

Fig. 75.

proportional to F/(Vk"), he connects the mean values of F with the origin of the speed tractive effort curve as shown in Fig. 76. It will be easily seen that the slopes of these lines are proportional to F/(Vk") or equal to F/V, so, the slope of speed-distance curve at any point is proportional to these slopes. This is an excellent method if any scale of distance, for instance 1.724 miles per inch, is acceptable. If, however, some integer or round number like 1 mile or 2 miles equal one inch, is required, this method may be found very laborious because in order to get it in such a convenient scale the speed-tractive effort curve must be replotted to such a certain scale as will give the desired scale to the final speed-distance curve.

The proper scale for (Vk") in the first method, will be found by the following formula:

$$(Vk'') = \frac{(F)(\Delta S)}{(\Delta V)}.$$

For instance, if (F) = 20 lbs. per ton/inch, (ΔV) = 20 m.p.h. / inch, and (ΔS) is desired to be 1 mile/inch in scales, the scale of the bases of the triangles,

$$(Vk'') = \frac{20 \times 1}{20} = 1.0$$

and actual length = (95./3600)/1.0 = .0264V inch, if k = 95 lbs. and it is .264 inch when the mean velocity is 10 m.p.h. and 1.32 inch when the velocity is 50 m.p.h.

It may be noted here that if we replot the speed-tractive effort curve with the scale for the abscissa found from the above formula, speed-distance curve of the desired scale can be

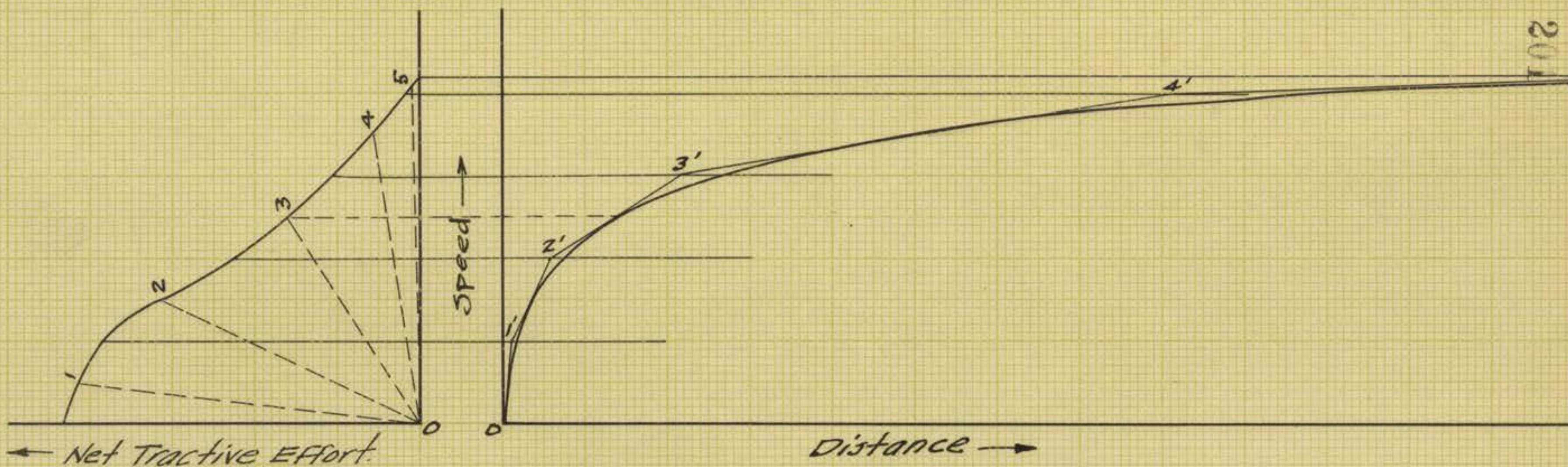

Fig. 76.

constructed according to Lipetz' method.

The convenient scale for the second method for constructing speed-distance curves can be found from the following formula:

$$(\underline{k}) = \frac{(F/V)(\Delta S)}{(\Delta V)}$$

4. <u>Time-distance curves.</u> - When a speed-distance curve has been constructed, the corresponding time-distance curve can be constructed directly from the former by the following method:

From the definition of velocity, we have

$$Vm = \frac{S}{t}\, 3600,$$

or

$$\frac{\Delta t}{\Delta S} = \frac{\underline{k}'}{Vm}$$

where t is the time in seconds, S the distance in miles, $\underline{k}'$ the constant (3600), and Vm the mean velocity during the time increment or the distance increment. This equation indicates that the slope of the secant which passes through the two points on a time-distance curve at S and S + ΔS is always parallel to the hypotenuse of a right triangle whose <u>height</u> is $\underline{k}'$ and the <u>base</u> Vm. The construction of such a triangle is not convenient, although triangles whose height is Vm and base $\underline{k}'$, can be constructed very easily from a speed-distance curve. The slope of the secant of the time-distance curve can be easily found since the hypotenuse of the former triangle is always perpendicular to that of the latter, - see Fig. 77a. Then, if we take a convenient length to represent $\underline{k}'$, for instance, .5 inch, the time-

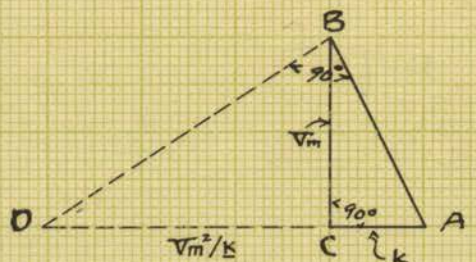

Fig. 77a.

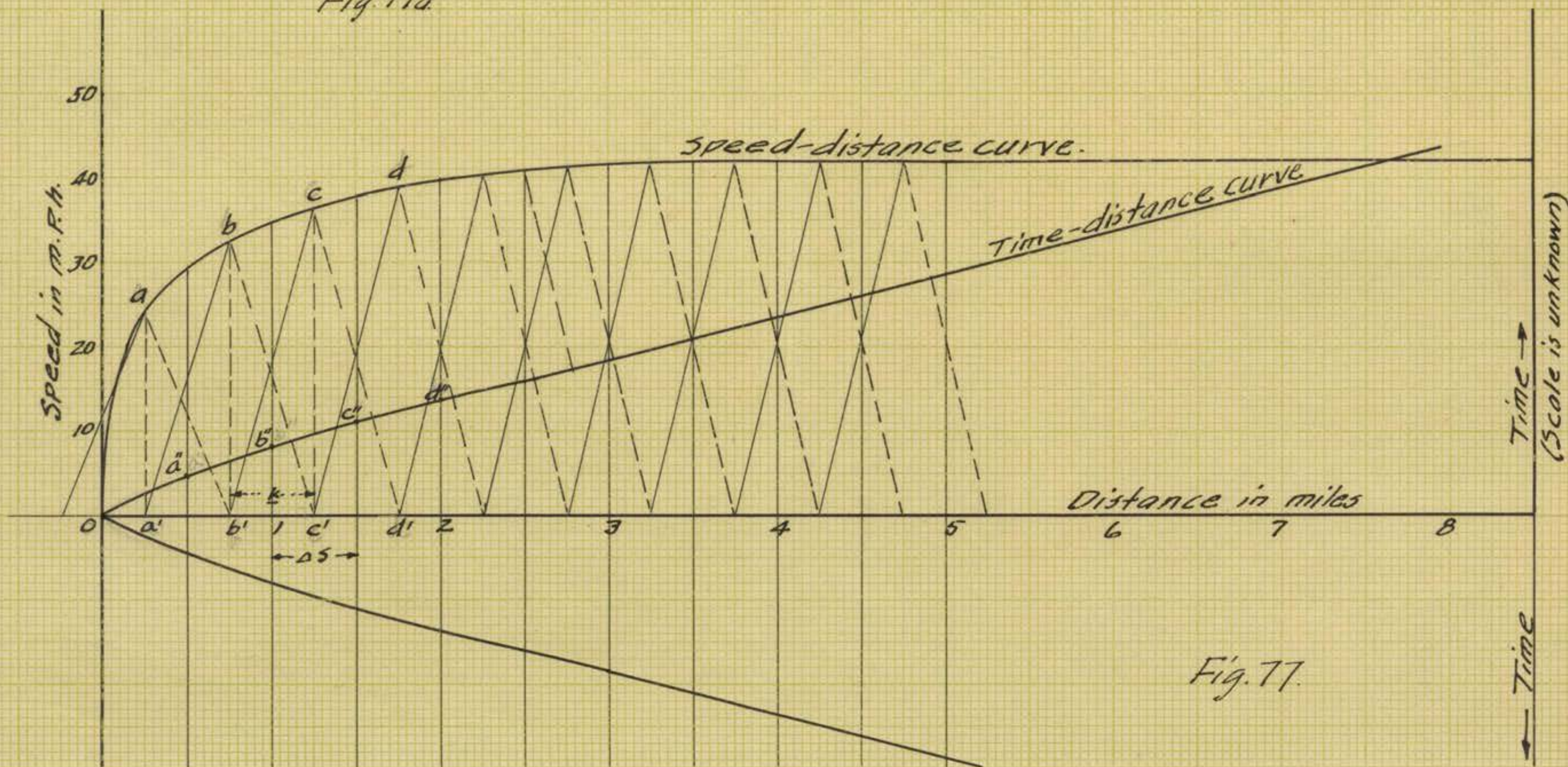

Fig. 77.

distance curve can be easily constructed as shown in Fig. 77, in which the lines, oa", a"b", b"c", etc., have been drawn perpendicular to ab', bc', cd', etc. respectively. If we take the distance increment infinitesimal the curve o a" b"c"d"... is the time-distance curve required. Mr. Lipetz' method is exactly the same as the above except that in order to produce the time-distance curve below the distance axis he used inclined lines drawn on the opposite side, for instance cb' instead of cd' in Fig. 77.

The method just described is entirely satisfactory if no particular scale is required, but when a convenient scale for time is necessary, the base of the triangles, aa'b', bb'c', ... should be a certain definite length, which may be found in the following formula:

$$(\underline{k}') = \frac{(Vm)}{(\Delta S)}(\Delta t)$$

where $(\underline{k}')$ is the value of $\underline{k}'$ in one inch of the scale

(Vm) is the value of VM in one inch of the scale

(ΔS) is the value of S in one inch of the scale

and (Δt) is the value of t in one inch of the scale

For example, if (Vm) = 20 m.p.h./inch, (ΔS) = 1 mile/inch, and the scale of time is desired to be 4 minutes or 240 seconds /inch, we get

$$(\underline{k}') = \frac{20}{1} \times 240 = 4800 \text{ per inch,}$$

but $\underline{k}'$ = 3600, hence the length of $\underline{k}'$ = 3600/4800 = 0.75 inch.

To lay out the base, 0.75 inch for instance, for each triangle is laborious and also involves error. It is much simpler to lay it out once as shown in Fig. 78, and locating a, b, c, ... for the values of Vm on a common line, draw the perpendiculars to the hypotenuses of the triangles thus constructed.

E. Mechanical Methods for Speed-time, Distance-time, and Speed-distance Curves.

1. Integraph. - As mentioned before, a distance-time curve is the first integral curve of the speed-time curve. Hence the former can be drawn by means of an integraph, a special form of planimeter which records or represents the area under the curve as the pointer traces the curve. The use of the integraph for this purpose has undergone practical test,* and has been found entirely satisfactory when a number of such curves are to be produced.

The integraph can also be employed in producing speed-time curves. It has been shown that

$$t = \frac{dV}{f(V)}$$

then it is apparent that by means of an integraph any speed-time curve can be produced if a curve representing the relation between the speed and the reciprocal of the accelerating force is available. Such curves can be produced from a diagram of speed-tractive force relation, but it would be very laborious;

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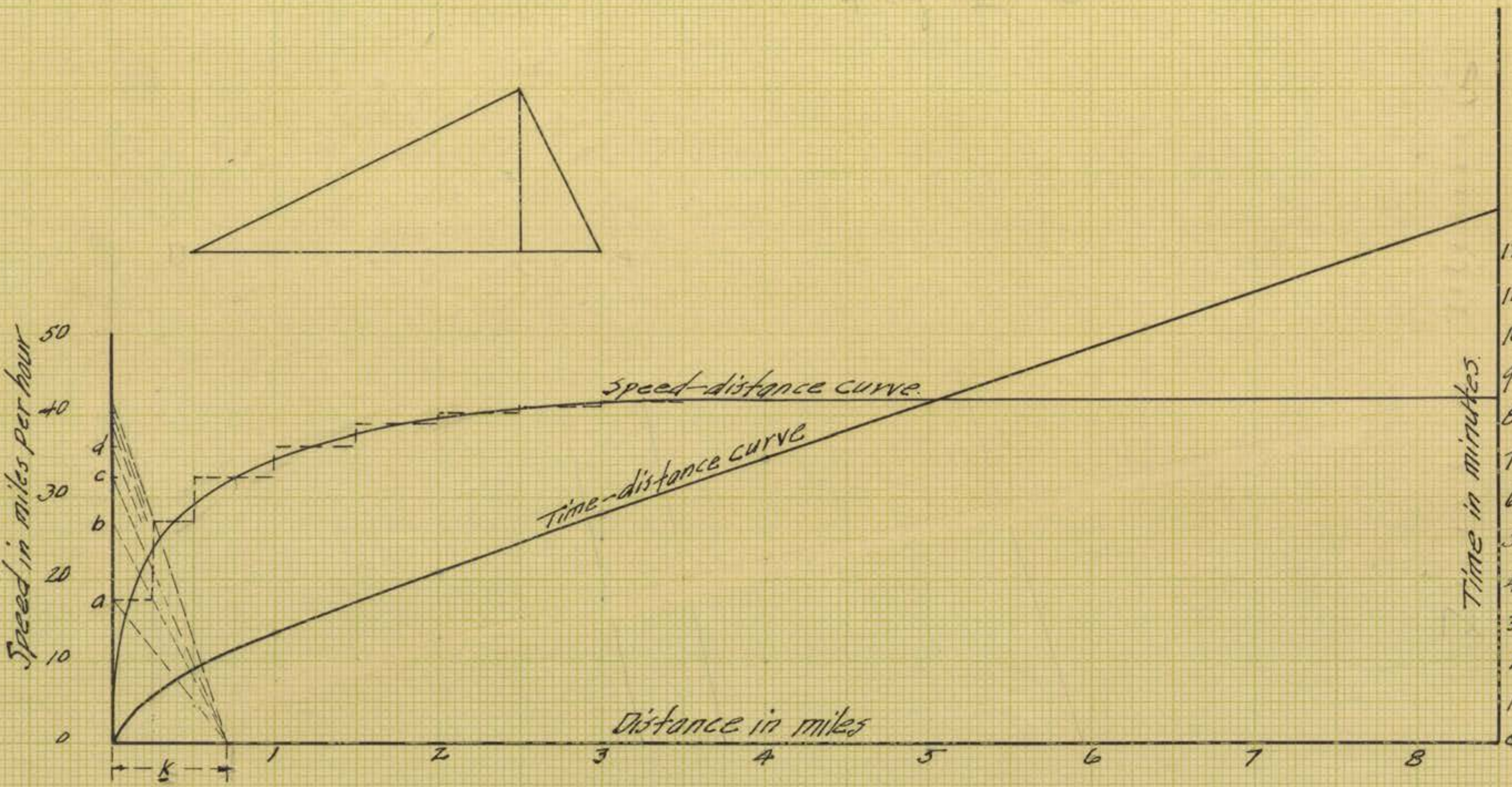
* C. O. Mailloux: "Notes on plotting of speed-time curves", Trans. A.I.E.E., vol. 19, p. 901. (1902)



Speed in miles per hour
0
10
20
30
40
50
a
b
c
d
Speed-distance curve.
Time-distance curve
Distance in miles
1
2
3
4
5
6
7
8
k
Time in minutes.
0
1
2
3
4
5
6
7
8
9
10
11
12

Fig. 78.

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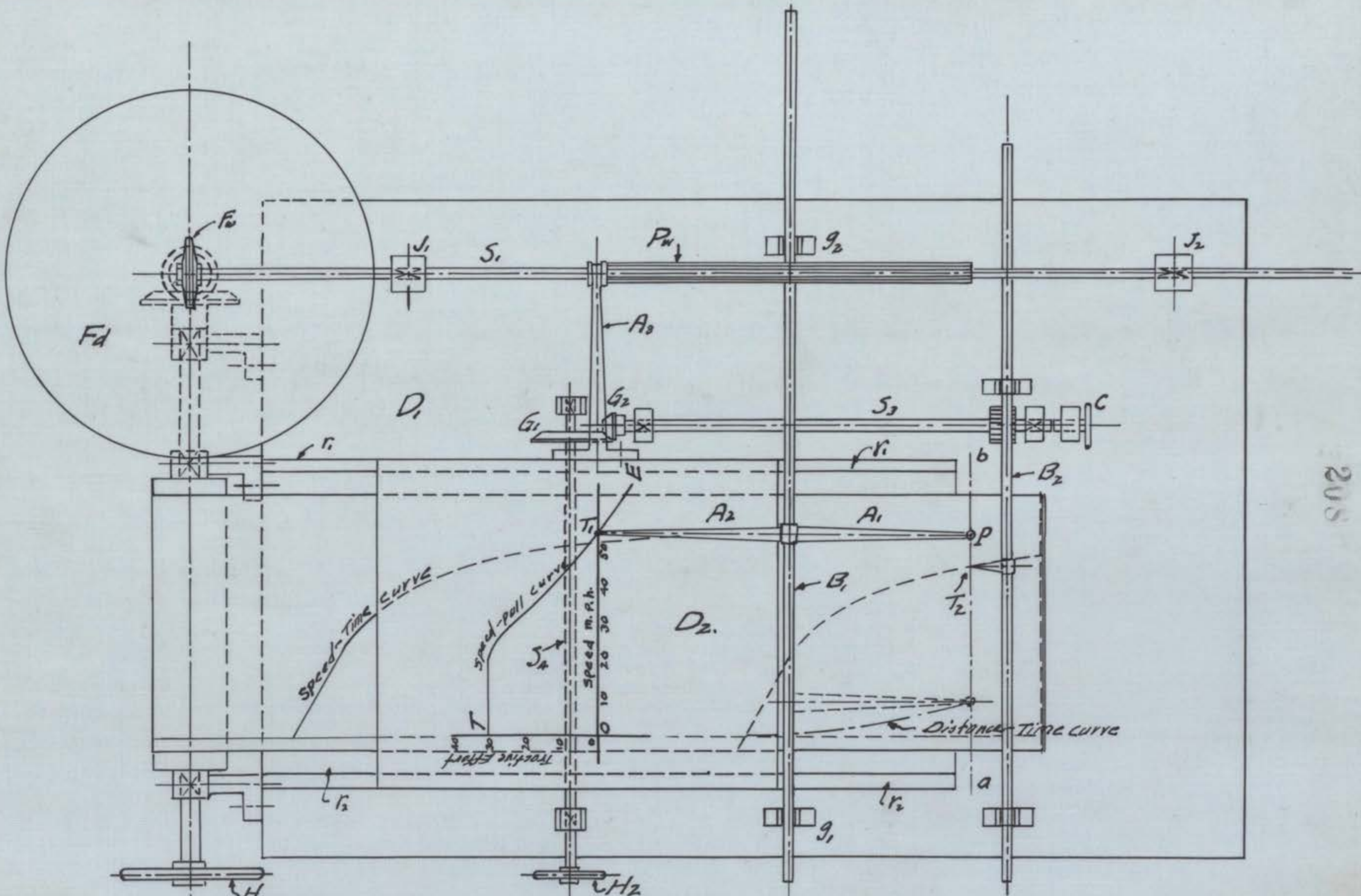
moreover the integraph has a certain mechanical defect so that the entire speed-time curve can not be drawn with an integraph of ordinary design.

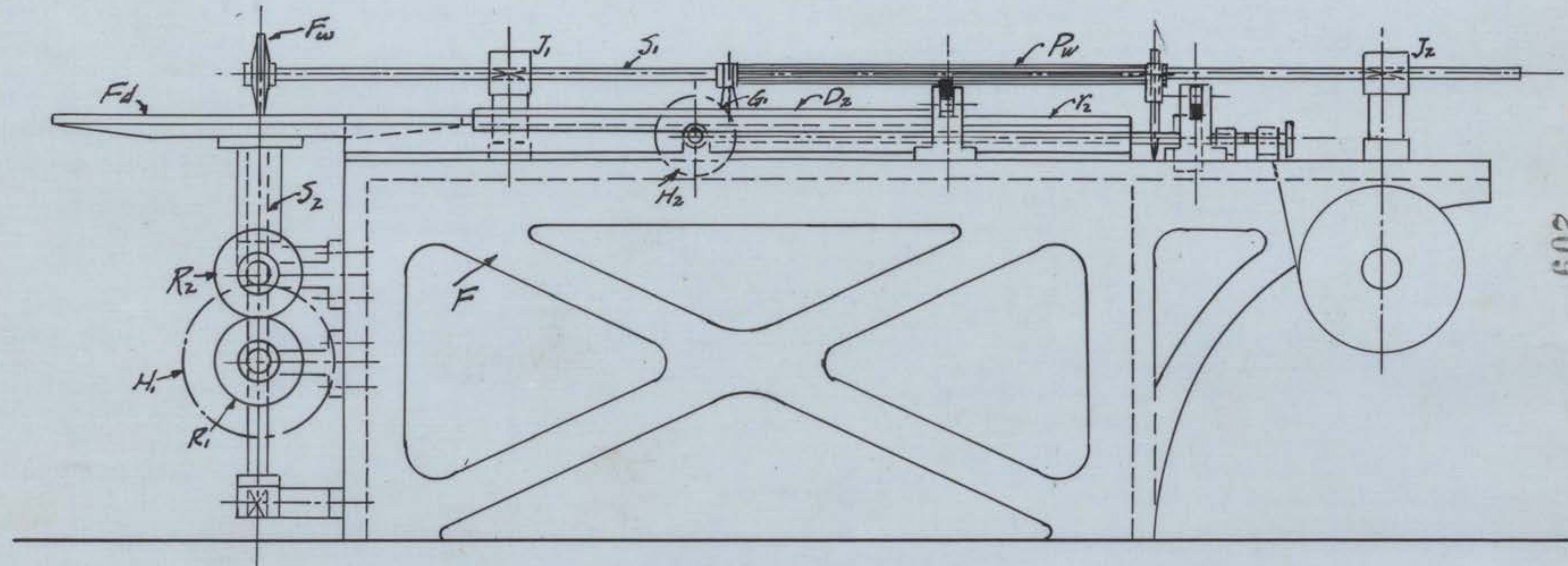
Professor A. M. Buck devised a certain method by which the speed-time curve can be drawn by means of an ordinary integraph with certain special attachments, directly from the speed-tractive force curves. The integraph has, however, the same mechanical limitation, that is, the recording pencil cannot travel in the vertical direction further than 45 degrees from the base line, and the complete speed-time curve thus produced would have too great a length, which is not convenient for practical use.

2. "Kineograph", an Instrument for Generating Speed-time, Speed-Distance, and Distance-Time Curves. - The graphical methods described in the foregoing paragraphs are simple and accurate, and have been proved highly practical, especially when the curves to be drawn are not numerous and in great variety. In the case of a mass of work which requires a variety of curves for ever-varying profile and track curvature, an instrument specially designed for this purpose, and which generates the graphs accurately and easily with minimum time and labour, will be found a very profitable machine. The following is a brief description of such an instrument.

a. Description of kineograph and its use for generation of speed-time curves. - The general layout of the instrument is shown in Fig. 79. The instrument consists of three principal arrangements. The first is the arrangement by which a long sheet of paper travels on the surface of the drawing board D1

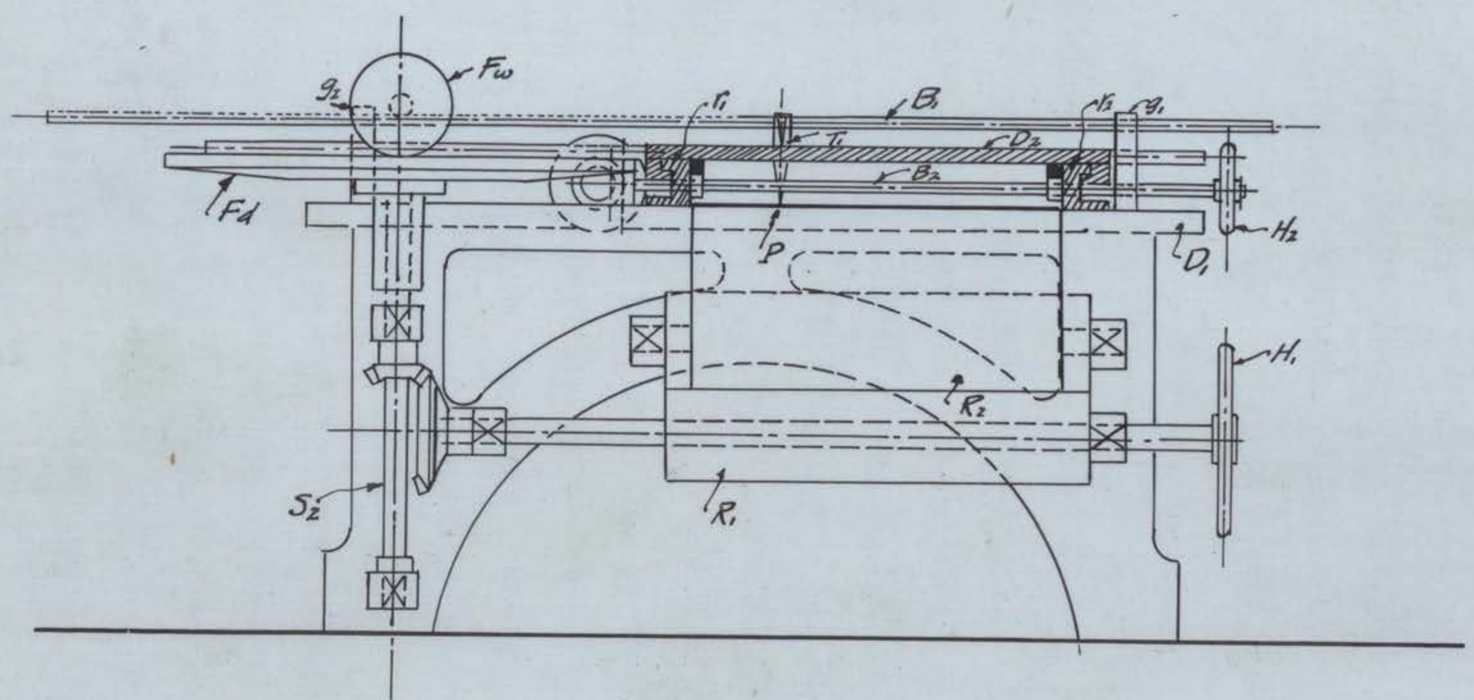


Top View
Fig. 79a.



Front View

Fig 79.b.



Side View

Fig. 79c

from right to left as the paper on the recording table of a dynamometer car. This movement of the paper is effected by the rollers, R1 and R2, located at the left side of the frame, F. One end of the paper, rolled up on the spool located at the right side of the frame, is caught between the rollers and when R1 is turned by the hand wheel, H1, the paper travels from right to left.

The second arrangement is a mechanism by which the recording pen or pencil, P travels on the paper in the direction perpendicular to that of the paper. The pencil arm, A1 is rigidly fixed to a beam, B1. This beam, which is supported and guided by two guides, G1 and G2, has a rack on top which is engaged with the pinion wire, Pw. This pinion wire is an integral part of the shaft, S1, which turns on small journals J1 and J2 when the small friction wheel, Fw, which is keyed to the shaft, is turned by a certain means. This shaft can be shifted along its axis by means of the arm, A3. The friction wheel is in contact with the upper surface of the large friction disc, Fd, which rotates on a horizontal plane when the vertical shaft, S2, which has a gear connection with handle H1, is turned. (The handle turns the friction disc and the paper rollers at the same time). Suppose that the friction wheel is shifted one inch toward the right from the center of the friction disc, and the hand wheel is being turned counter-clockwise. Then it is clear that the paper is traveling to the left and the beam B1 moves with the pencil away from us. The curve thus drawn by the pencil on the paper is an inclined straight line, since the ratio of the speed of the pencil to that of the paper is constant no matter how fast the paper travels so long as the position of the friction wheel on the friction disc is the same. By shifting the position of the friction wheel, however, we can get straight lines with any inclination, or a curve, by continuously shifting the friction wheel.

The third arrangement which constitutes a part of the instrument is a means by which the friction wheel is shifted according to any definite rule. A small board D2, on which we place a speed-tractive effort curve with thumb tacks or some other convenient means, is mounted on the rails, r1 and r2, and the board can be shifted on the rails by means of a certain mechanism when the small hand wheel, H2 is turned. The mechanism consists simply of two pinions and two racks located under the board. For the time being let us suppose the bevel gears, G_1 and G_2 are disengaged. (The use of these gears will be explained later.) This board and the shaft S1 is connected by A3, so that the board and the friction wheel move the same distance in the same direction when the handle H2 is turned. To generate a definite curve the friction wheel should not be shifted at random but according to the speed-tractive effort curve. For this purpose the tracer arm, A2, which has a tracing point, T1 at its end, is fixed to the beam B1 at a place right opposite to the pencil arm, so that when the tracing point indicates a certain speed on the speed-tractive effort curve the recording pencil is always at that speed on the speed-time curve to be generated, even though the pencil may not be in the correct position along the time-axis. The pencil is kept always at a correct position along the time-axis, as well as along the speed-axis by properly moving the board D2 as follows: Shift D2 with H2 so that the friction wheel is at the center of the friction disc, Place the speed-tractive effort curve on D2 such way that the tracer point is always on the speed-axis of the curve when B1 is moved along its axis. Bring the tracing point to zero speed. Shift the board with H2 until T1 comes to the point indicating the starting tractive effort, like T. Now the friction wheel is at a distance equal to the tractive effort represented by TO. Turn H1 with the left hand and H2 with the right hand in such a way that the tracing point is always on the tractive effort curve like TE. Then the paper travels toward the left, the pencil moves along the speed-axis, and the curve drawn by the pencil on the travelling paper is the speed-time curve corresponding to the speed-tractive effort curve used.

The proof that the curve thus drawn is a speed-time curve is as follows: For simplicity's sake let us suppose the tractive effort curve is represented by the broken line A'B' instead of AB as shown in Fig. 80, then we get a polygon a', b', c', ... instead of the smooth curve, a'b'c' .... Further, in this instrument the speed of the paper, V', and the speed of the pencil, V", have the following relation:

$$V'' = cdV',$$

where c is a certain constant which depends upon the gear ratios used in the design of the instrument, and d is the distance of the friction wheel from the center of the friction disc which is always equal to the tractive effort at various speeds, F. Then,

$$cd = cF = \frac{V''}{V'}$$

But,

$$\frac{b'b''}{a'b''} = \frac{V''}{V'}$$

then $= cF$

but, $b'b'' = \quad V$, therefore,

$$cF = \frac{\Delta V}{a'b''},$$

and the comparison of this equation with $a = \Delta V/\Delta t$, shows that if the acceleration, a is equal to cF, a'b" must be equal to t. The value of cF may not be equal to a but is always proportional to a. Therefore, a'b' in Fig. 80 is the speed-time curve to a certain scale, corresponding to the tractive effort, F, in Fig. 80. Similarly, b'c', c'd', etc., are those, in the same scale, corresponding to the tractive efforts, F_2, F_3, etc. respectively. It is clear that if we take the increment of speed infinitesmal, A'B' becomes AB, and the curve produced with AB is a smooth curve and the required speed-time curve.

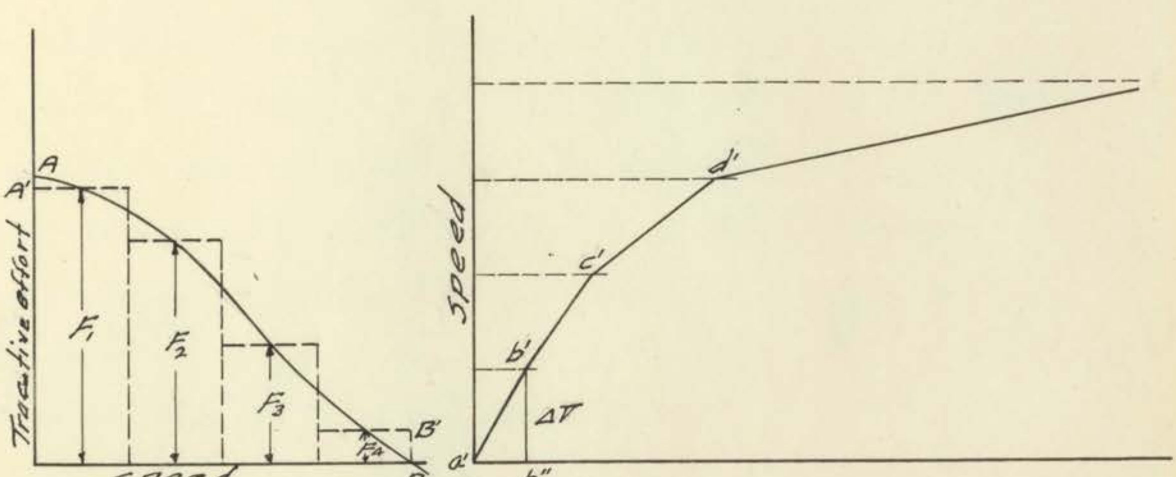


Fig. 80.

Further proof of the theory and the mechanical possibility of the instrument is as follows: In order to check the mathematics previously developed and demonstrate the mechanical possibility of such instrument, a working model as illustrated

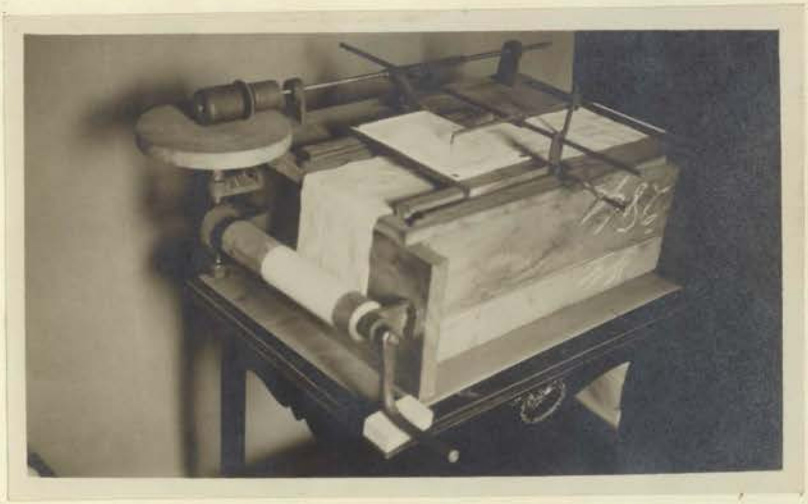


Fig. 81.

in Fig. 81, was constructed. The model being made in a hurry and with tools not suitable for the production of a mathematical instrument, is necessarily very crude. The result obtained was, however, rather surprising. The mechanical possibility has been fully demonstrated, and the curve generated with this instrument when checked with the result of the graphical method was found to agree perfectly at all points except one where the paper slipped or wrinkled in the operation. The model was crude, as mentioned before, and it lacked several attachments which ought to be in actual instruments.

Kineograph in generation of speed-distance curves. - In the discussion on the graphical method for speed-distance curves, we have seen that when we use a curve representing the tractive effort divided by corresponding speeds instead of the speed-tractive effort curve, we can get speed-distance curves exactly the same way as for the construction of speed-time curves. This is entirely true in the case of mechanical generation with Kineograph.

Kineograph in drawing distance-time curves. - This instrument with a few simple attachments becomes a device for drawing integral curves similar to the integraph or it may be used as a planimeter in finding the area of any plane surface; hence as explained before, such an instrument can be conveniently employed in the generation of distance-time curves, when speed-time curves have been constructed. The principal parts of the attachments are the beam, B2 (see Fig. 79) to which a tracer point, T2 is fixed, and the shaft, S3, which has a bevel gear connection with S4 at its left end and has a pinion at the other end which engages with the rack on the beam B2. When the instrument is to be used as an integraph the thumb screw, C, is tightened so that G2 engages G1. The method of drawing the distance-time curve is as follows: First bring the initial point of speed-time curve on the line ab. Bring the friction wheel to the center of the friction disc and both the pencil and the tracer T2 on the initial point of the curve. Then turn the hand wheels H1 and H2 in such a way that T2 is always on the speed-time curve. The curve thus drawn by the pencil is the required distance-time curve. Why the curve thus generated can be a distance-time curve will be easily seen when we recognize the fact that by the operation the friction wheel is placed always at a distance from the center of the friction disc equal to the speed indicated by the tracer T2 and the pencil moves with proportional speed while the paper travels toward the left.

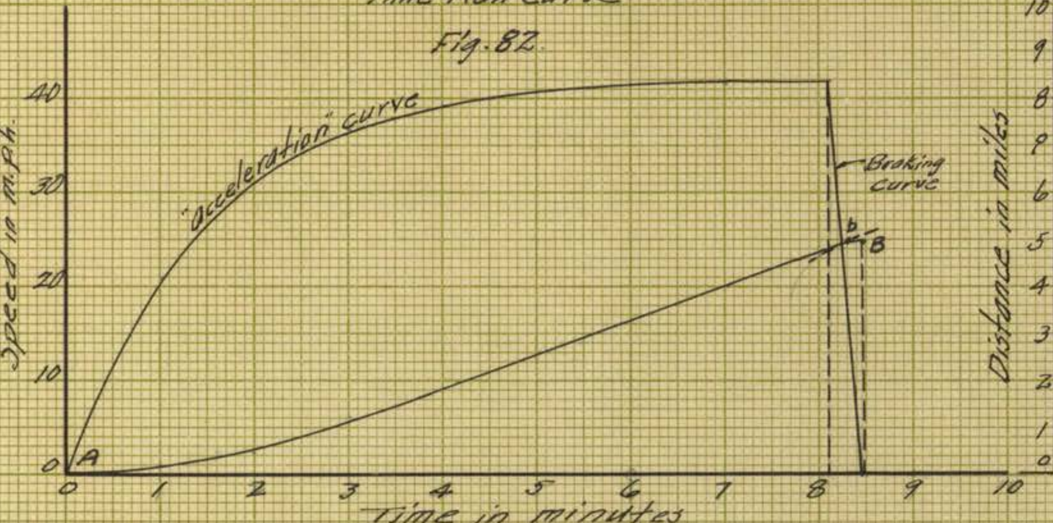
In concluding of this section it may be said that the description of the instrument is lengthy but the operation is very simple and a great number of desired curves will be generated with little labor and time. As to the probable cost of the instrument no estimation has been made, but it may be constructed with little expense in a railroad whop when it is not operated in full capacity, although an instrument made by a standard manufacturer of mathematical instruments like Mr. Coradi, Zurich, Switzerland, is to be recommended. It may be added here ~~the fact~~ that the instrument just described is useful not only in generation of speed-time, speed-distance, distance-time curves, but also in drawing any first derivative curve as well as first integral curves. The proof is omitted here since no application of derivative curves is found in this thesis.

F. Run Curves.

In applications of speed-time, speed-distance curves, etc., it is often convenient to have produced a run curve, which is a diagramatical representation of the relation among speed, distance, and time involved in a complete run of a train. The run-curves may be classified as (1) "time" run-curves, which being produced from speed-time and distance-time curves

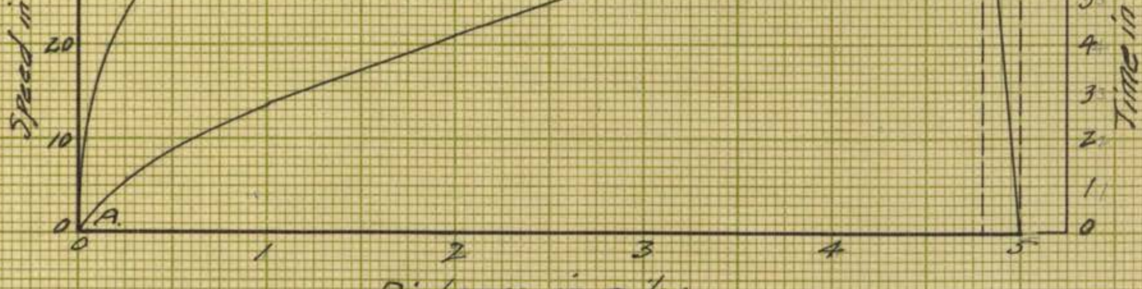
Time Run Curve

Fig. 82



Distance Run Curve

Fig. 83



has its time-axis as abscissa, and (2) "distance" run-curves, which being produced from speed-distance and time-distance curves, has its distance-axis as abscissa. Two typical run-curves are exhibited in Figs. 82 and 83. In both classes of run curves the part of curves corresponding to the acceleration period is called "acceleration" curve, and similarly the parts corresponding to the coasting and braking periods are called "coasting" and "braking" curves respectively.

A "time" run curve is produced as follows: Suppose by one of the methods described previously the speed-time and distance-time curves for "acceleration", coasting, and braking on various grades and curves have been constructed. Then, the construction of the run curve is merely an assembling these curves according to some operating condition. For instance, the train is to run a distance AB, say 5 miles, starting at A and stopping at B. Then, produce the "acceleration" curve as shown by Aa in Fig. 82. Shift the braking curve until it becomes tangent to the "acceleration" curve and the 5-mile-line at b and B respectively, (the curve may be shifted in any direction but should not be rotated). Now AB is the required distance-time run curve. Draw the "acceleration" and "braking" speed-time curves corresponding to Ab and bB, as shown by A'B'. This is the required speed-time run curve, and curves, AB and A'B' together constitute the "time" run curve. If the train coasted, the lines Ab and bB could not become tangent at any point, and "coasting" distance-time curve should become tangent to Ab and bB, when it is inserted between them. A "distance" run curve can be produced in a similar way.

X. REVIEW OF SOME RAILWAY PROBLEMS, AND APPLICATIONS OF SPEED-TIME, DISTANCE-TIME AND SPEED-DISTANCE CURVES.

A. Running Time of Railway Trains, and Train Schedules.
   1. Train schedule and running time.
   2. Review of the proposed method.
   3. Determination of running time, speed-time curves, etc.

B. Energy Consumption in Railway Motive Powers.
   1. Coal consumption of steam locomotives.
      a). Henderson's method.
      b). Houston's method.
      c). A method of estimating coal consumption.
   2. Energy consumption in electric locomotives.

C. Tonnage Rating.
   1. Nature of tonnage rating.
   2. Tonnage rating for maximum tonnage per train.
      a). A brief sketch of the development of tonnage rating.
      b). Adjusted tonnage method.
      c). Equated tonnage method.
      d). Drawbar pull method.
      e). Variable car-factor method.
   3. Tonnage rating for maximum ton-miles per locomotive per day.
   4. Tonnage rating for minimum ton-mile cost.

D. Selection of Locomotives for a Particular Service.

E. Railway Location and Relocation Problems.

A. Running Time of Railway Trains, and Train Schedules.

1. Train schedules and Running time. - The fundamental purpose of train schedules is to designate the time and place of train movements. The time and place, however, are functions of the tractive effort of the motive power, of train load, and of train resistance, etc. Many other important factors such as volume of traffic, competition in speed, meeting points, branch line connections, etc. on the one hand, and operating expenses and government regulations on the other hand, must be carefully considered in the construction of passenger train schedules and freight train schedules as well. The first, if not the most important, factor to be considered is the time required by a train to cover a distance, for instance, between one station and another. This running time of a train with a certain make-up and on a certain track can be determined by actually running the train. In fact, this is the method commonly employed in making train schedules. But, such a method is inapplicable unless the motive power and the train are actually in existence. This is not always the case. For instance, a train schedule is desired for a class of powerful locomotives to be purchased in order to surpass the schedule of a competing line or lines. Further, there are numerous railway problems which require an accurate knowledge of running time for their scientific solution. In this section, therefore, we will review some methods proposed for the predetermination of running time, primarily for the purpose of constructing passenger train schedules; and then study the practicability of the graphical and mechanical methods of constructing

speed-time curves, etc. when applied to this problem.

2. Review of the method proposed. - Although the problem of determining the running time is important in many respects and its value is being gradually recognized by many railway engineers, very few publications have been made and the following method is the only one which has come to our notice.

In 1902, all the railways in Germany* reported to the Administration as to their methods for schedule construction. Among the forty-seven railways, according to the report, twelve did not consider the influence of the grades and curves, seventeen considered them but without any definite ground, and the other eighteen roads took them into consideration in various ways. Among these eighteen railways, some classified the grades into three groups and increased schedule time on these groups of grades was added to the schedule of the trains on level tangent track. Some roads considered the influence of relative elevation of two stations without any regard to the details of the profile between them. Some roads increased the schedule only for curves regardless of the gradient. The Frankfurt-Bebroer Railway, however, reported that their train schedules had been constructed by means of the table next shown.

It may be easily seen that the time required by a limited train or express train can be readily found by multiplying the coefficient found in this table into the time required by the train on level track.

---

* E. Spirgatis: "Berechnung der Fahrzeiten aus den Zugkraften der Dampflokomotiven", R. Haupt, Leipzig, publisher. 1902.

Gradient between	Coefficient. Limited Trains (75 - 60 km./h)	Passenger Trains (60 - 45km./h)
1:600 --- 1:450	1.00	1.00
1:449 --- 1:350	1.05	1.00
1:349 --- 1:275	1.10	1.00
1:274 --- 1:225	1.15	1.05
1:224 --- 1:175	1.20	1.15
1:174 --- 1:135	1.30	1.30
1:134 --- 1:110	1.40	1.45
1:109 --- 1: 95	1.50	1.60
1: 94 --- 1: 85	1.65	1.70
1: 84 --- 1: 75	1.75	1.80

This table was developed on the following theory:

According to Clark, the total train resistance, R at speed V on a grade i is expressed as

$$R = (2.25 + \frac{(0.278V)^2}{80} + 100i)(\text{weight of train in tons}).$$

The tractive effort, T.E. at various speeds

$$T.E. = \frac{340 \times 75}{0.278V} = \frac{91726}{V}$$

Then, equating R and T.E., we have

$$199 = \frac{91726.}{(2.25 + 0.000996V^2 + 1000i)V}.$$

Solving for V at various i, we get

$$\begin{aligned} V &= 55 \text{ km.} \quad \text{when } i = 1:315 \\ &= 52 \qquad\qquad\quad = 1:250 \\ &= 48.5 \qquad\qquad = 1:200 \\ &= 43 \qquad\qquad\quad = 1:150 \\ &= 34.5 \qquad\qquad = 1:100. \end{aligned}$$

Then dividing 60 km. (speed on a level track) by V at various values of i, we get the coefficient as given in the table.

Now, let: t = the time required on a uniform grade.

L = the distance of the grade in km.

V = the velocity in km. per hr.

Then,
$$t = \frac{60L}{V},$$

and, if t_1, t_2, t_3 ...... are the times required over a series of different grades, and V_1, V_2, ... are the corresponding speeds, we have for the total time,

$$\underline{t} = \frac{60L_1}{V_1} + \frac{60L_2}{V_2} + \frac{60L_3}{V_3} + \ldots\ldots\ldots\ldots$$

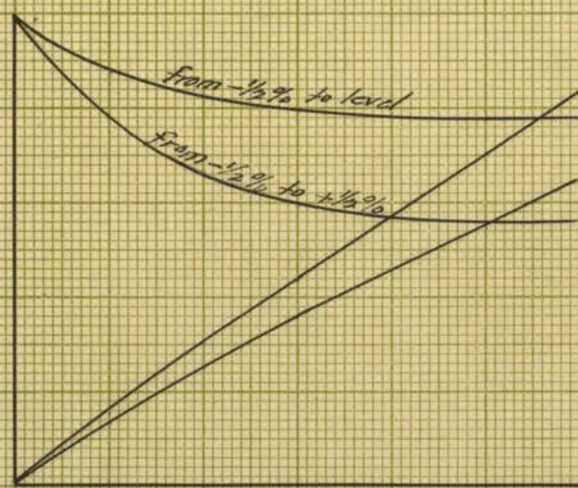
$$= \frac{60}{S}\left(L_1\frac{S}{V_1} + L_2\frac{S}{V_2} + L_3\frac{S}{V_3} + \ldots\ldots\right),$$

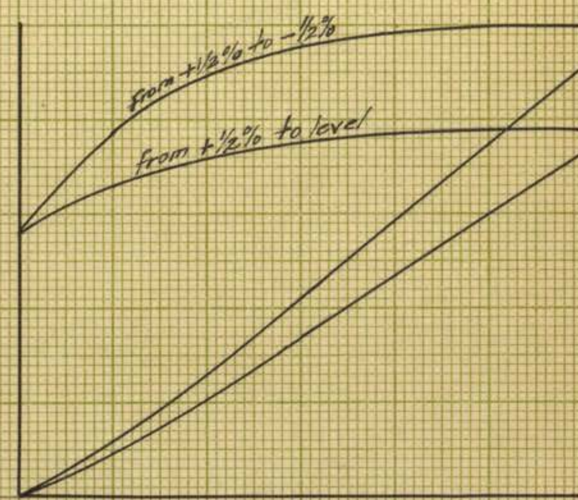
where S is the standard speed of a train on a level track and the value of the quantity within the parenthesis is called "virtual length". The values S/V_1, ... can be found in the table and L1, L_2, ... from the map of the division, therefore, the total running time of a train between stations can be computed by this formula.

The increase in schedule time due to slowing down the speed without stop at a station was assumed to be 1 minute; and slowing down with a stop 2 minutes. An allowance of 3 minutes each hour was made for freight trains.

In 1902, the General Railway Administration instructed all the railways to make their passenger schedules on this basis.

The method just described is rational and quite simple in its application. Clark's formula for train resistance, however, is not applicable to present railway train operation, because it gives almost 100 percent error in estimation, and the tractive effort used by Spirgatis (a hyperbolic formula) does





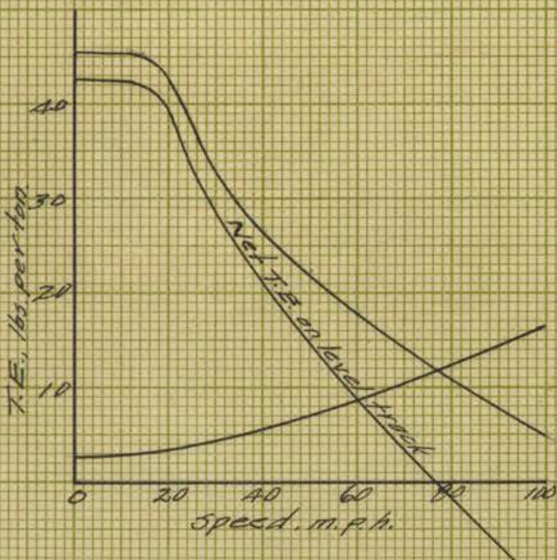


Fig. 84a.

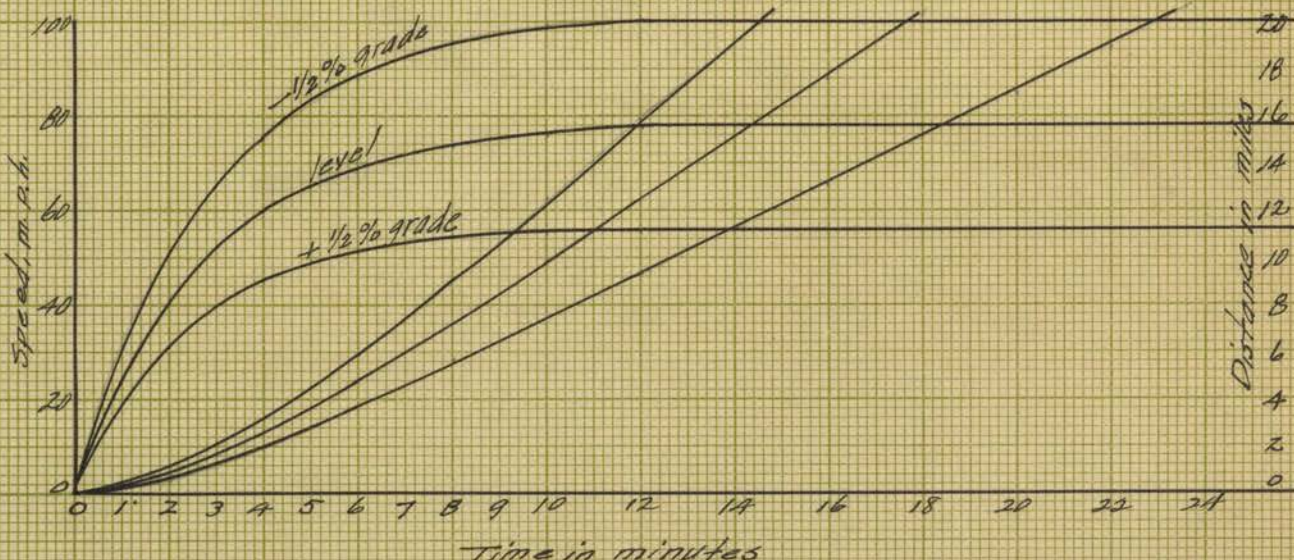
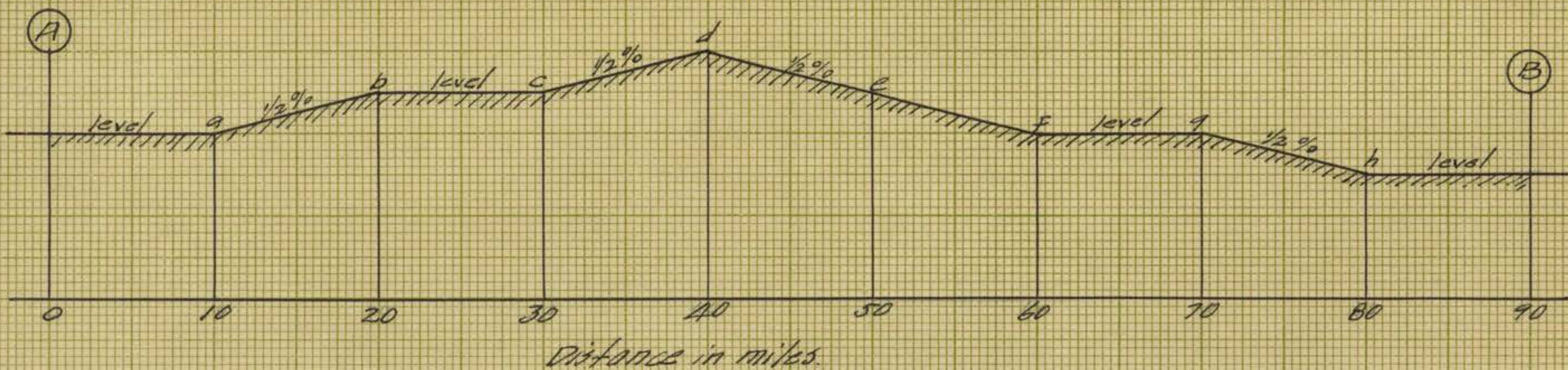


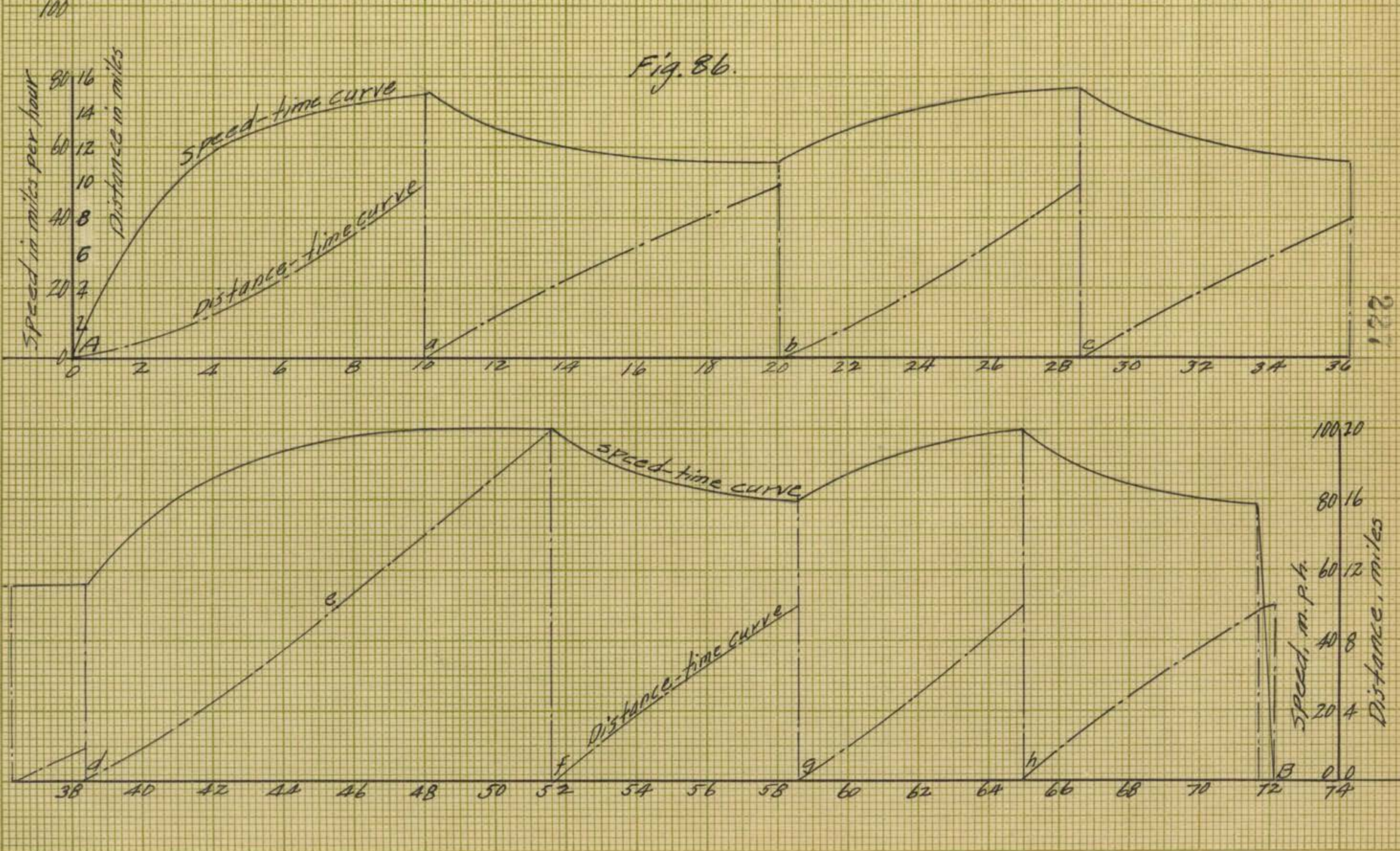
Fig. 84.



A
B
level
a
1/2 %
b
level
c
1/2 %
d
1/2 %
e
f
level
g
1/2 %
h
level
0
10
20
30
40
50
60
70
80
90
Distance in miles.

Fig. 85.

Fig. 86.


Speed in miles per hour
Distance in miles
Speed-time curve
Distance-time curve
A
a
b
c
d
e
f
g
h
B
Speed, m.p.h.
Distance, miles
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
60
62
64
66
68
70
72
74
100

not agree well with recent test results. Therefore, if this method is to be employed the formulas for train resistance and tractive effort should be replaced by more reliable ones.

3. Determination of running time by speed-time, distance-time curves, etc. - In the previous chapter we have seen that a run curve represents the relation between the speed, distance, and time during a complete run of a train. Then the time required in the run can be readily found when the run curve is constructed.

For example, let us suppose a train, whose speed curves are shown in Fig. 84a, is to run on a track whose profile is as shown in Fig. 85. Then the run curve as shown in Fig. 86 can be produced from the speed curves and the running time can be accurately determined.

B. Energy Consumption in Railway Motive Powers.

1. Coal consumption of steam locomotives. - As shown in the preceding chapter the drawbar pull of steam locomotives depends upon the rate of evaporation, which in turn depends on the rate of firing coal per hour. If the profile of the road is uniform and the rate of firing is practically constant over a division, the estimation of coal consumption is very simple; but if on the contrary, the profile is irregular, with heavy grades and sharp curves, the rate of firing will vary from time to time and an accurate estimate of coal consumption will require careful consideration of the profile and of the resulting speed, which is itself a direct function of coal consumption.

a). <u>Henderson's method.</u>* - G. R. Henderson has published an article on the "Fuel Consumption of Locomotives", in which he took a consolidation locomotive for example, and described a method for the predetermination of fuel consumption. The method is based upon the diagram shown in Fig. 87. A series of heavy lines constructed with his "hyperbolic" formula to which we referred in Chapter III,** represents the tractive effort of the locomotive at various rates of coal consumption per hour. Another series of light solid lines represents the total train resistance on various grades. The series of dotted lines, obtained by dividing the coal consumptions corresponding to the various tractive effort curves by the different speeds, represents the coal consumption per mile. The diagram will, therefore, enable us to estimate coal consumption as soon as we know the length of each of the uniform grades on the division and the speed of train on each of the different grades. But this can be done only when we have made a number of such diagrams for various loadings of trains for each of numerous grades. Further, in this method the speed of a train over a grade is assumed to be uniform, or more definitely speaking the speed is assumed to be always equal to the balancing speed on each grade. This assumption would be fair enough for trains running long distances without stops, like "limited" passenger trains; but it is not so for heavy freight trains which stop every

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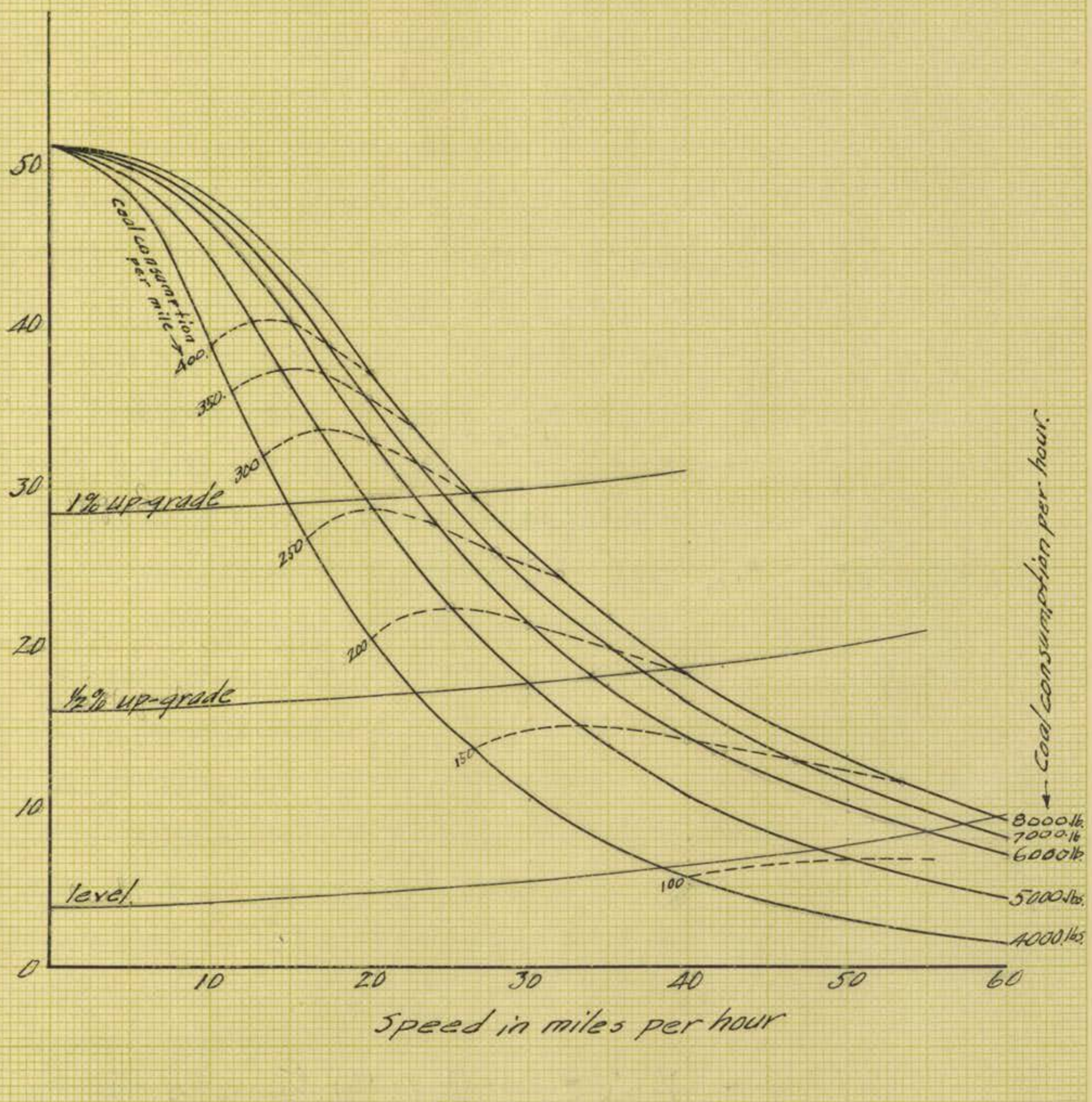
* American Engineer and Railroad Journal, vol. 79, p. 57 (1905); also "The Cost of Locomotive Operation", (1906), The Railroad Gazette, publisher.

**. Fig. 4 is similar to his diagram, but the tractive effort curves have been produced for Illinois Central Railroad locomotive No. 1748, by the formula in Chapter V.



50
40
30
20
10
0
10
20
30
40
50
60
Speed in miles per hour
Coal consumption per mile
400.
350.
300
250
200
150
100
1% up-grade
½% up-grade
level
Coal consumption per hour.
8000 lb.
7000 lb
6000 lb.
5000 lbs.
4000 lbs.

Fig. 87.

few miles, and which run most of the time at less than full speed. The magnitude of the error resulting from this method will be shown in connection with the new method discussed in the following paragraphs.

b). Houston's method. - Another method of estimating the coal consumption in steam locomotives has been described by Mr. Houston*. He assumes that if S denotes the speed below which a boiler can always generate sufficient steam for the engines working at full cut-off without drop in boiler pressure, the coal consumption of a locomotive is proportional to the speed for all speeds below S, and that it is constant or independent of speed for all speeds above S when the locomotive is working at its maximum power. He gives a table showing the amount of coal which can be economically consumed at the maximum power of locomotives with various heating surfaces. He also gives a diagram showing the relation between the evaporating power of one pound of coal at the various rates of working of a locomotive below maximum power. The rate of working is assumed by him to be directly proportional to the ratio of actual tractive effort at a given speed to the maximum tractive effort at that speed. With these data and these assumptions the coal consumption per unit time can be found for a locomotive working at maximum power or at any fraction thereof. To find the time during which a locomotive consumes coal while running he employes speed-time curves. Generally speaking, the method seems to be sound although the data he uses in developing the table and diagram are not so reliable

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* Thesis for M.S. in Railway M.E., University of Illinois, 1913.

as more recent experimental data.

c). <u>A method of estimating coal consumption in steam locomotives</u>. - With the formulas developed in Chapter V, the speed-pull relation at various firing rates or coal consumption per unit time can be determined for any locomotive, and from the speed-pull relations, by means of the method described in Chapter IX, the run curves for the locomotive with various train loads can be produced for any profile to be considered. When the run curves are made the coal consumption can be easily calculated, since the firing rates for different speed-pull relations are known and the duration of the period during which the locomotive consumes coal can be found from the run curves.

It may be mentioned here that with a certain locomotive and a given train load a number of series of run curves can be produced for a given profile, and the estimation of the coal consumption for any series can be made by the above method if desired. In practice, however, when a series of run curves is to be constructed it is governed not only by physical laws but also by operating and economic conditions, such as length of run, time, speed limit, passing points, maximum train tonnage, minimum coal consumption, etc., and practically speaking there is only one particular series of run curves which satisfies these conditions. Thus, although it is equally possible to estimate the coal consumption of a train run without restrictions, the method naturally leads to the estimation of coal consumption under the conditions of most economical train operation, and as a by-product of the method we have a series of run curves which is useful in constructing train schedules for freight or passenger service. It may not be out of place to direct here the attention of operating officials to the use of such schedules with instructions in freight train service, which will result in economy in fuel and in increased capacity of tracks.

Aside from this fact it may be said that the method outlined above is a logical procedure for the accurate estimation of coal consumption. We have sufficient experimental data for coal consumption <u>per unit time</u> but coal consumption can not be determined unless we know the duration of the period in which coal is consumed. This duration can only be determined by means of run curves produced either graphically or otherwise. As an elementary example, we will take the following case and compare the result with that obtained by a method in which the duration of the period of coal consumption is figured out only approximately.

Locomotive; Ill. Central R.R. locomotive No. 1748, weighing 225 tons.

Train load; 1000 tons behind the tender drawbar.

Firing rate; 6000 lbs. per hour, or 100 lbs. per minute.

Length of runs; 5 miles each.

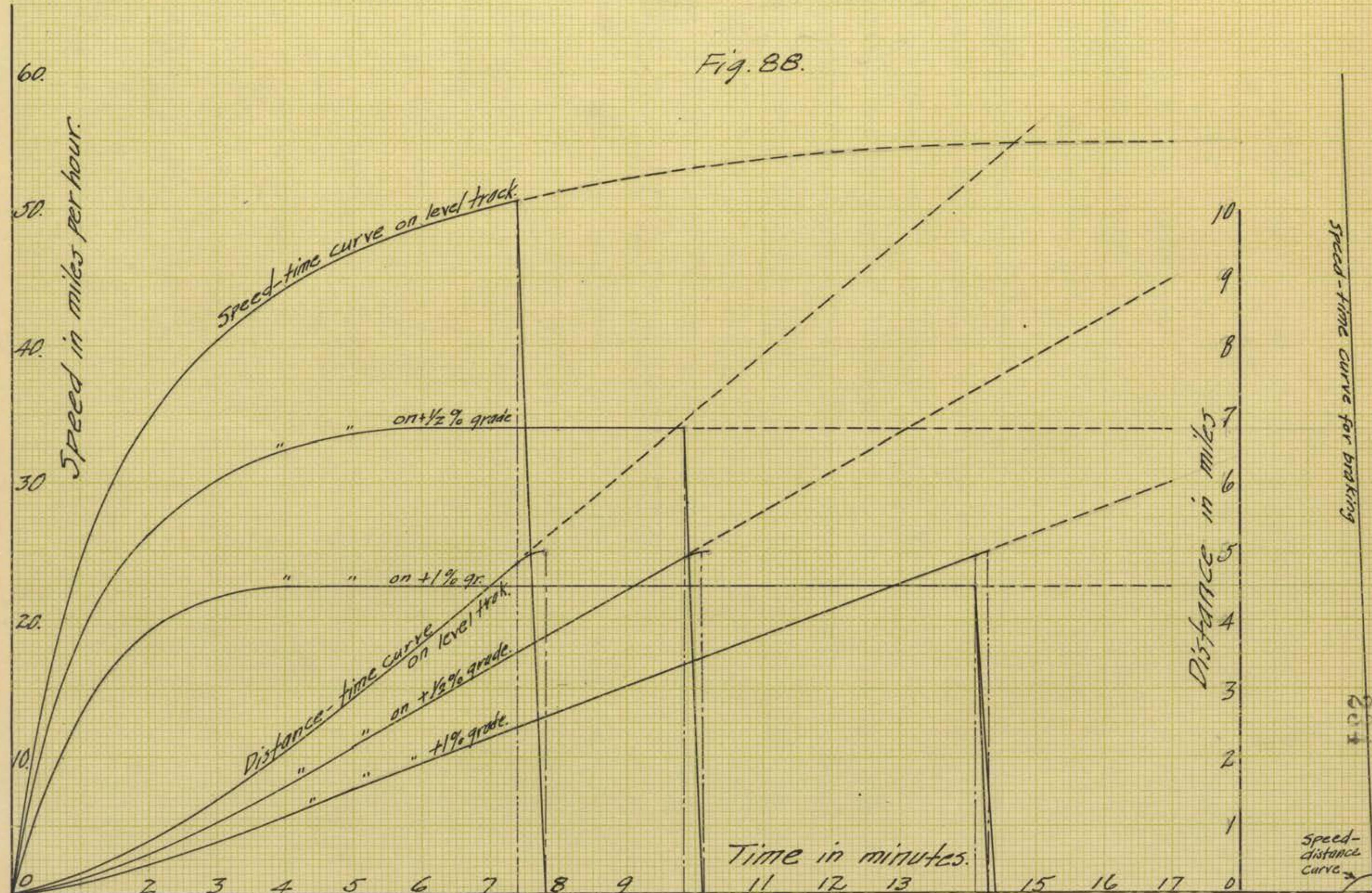
Profile; level, 1/2 percent, and 1 percent grades.

Braking power, 100 percent; Coefficient friction, 10 percent,

No curve on tracks; No coasting in runs; other conditions normal.

The run curves shown in Fig.88 have been produced to fulfill all these conditions. From them we can readily find the running times

Fig. 88.



Speed in miles per hour.
60.
50.
40.
30
20.
10.
0
Speed-time curve on level track.
" " on +1/2 % grade
" " on +1% gr.
Distance-time curve on level track.
" " on +1/2% grade.
" " +1% grade.
Time in minutes.
2
3
4
5
6
7
8
9
11
12
13
15
16
17
Distance in miles
10
9
8
7
6
5
4
3
2
1
0
Speed-time curve for braking
Speed-distance curve

as in the first line in the following table:

Item	Level track	½% Grade	1% Grade
Total running time, min.	7.8	10.1	14.3
Running time under steam	7.4	9.82	14.1
Coal consumption, lbs.	740.	982.	1410.

The coal consumption for each run of 5 miles can be easily calculated since the locomotive consumes 100 lbs. per minute under the assumption made. As assumed by Mr. Houston, the coal consumption of a locomotive below a certain speed (which is no doubt modified by gradient) is proportional to the speed. Adjustment for such speed variation may be made but the period during which a train runs under such conditions is very short, amounting to only 0.3 minute (10 lbs. in coal) and for practical purposes is unnecessary to make such fine adjustments.

The coal consumption of the locomotive under exactly the same conditions has been estimated by Henderson's method, the first described in this section, and the following results were obtained:

On level track; 5 miles at 55 miles an hour,
109 lbs. per mile = 545 lbs.

On 1/2% grade; 5 miles at 34 miles an hour,
177 lbs. per mile = 885 lbs.

On 1% grade; 5 miles at 22.5 miles an hour,
267 lbs. per mile = 1335 lbs.

This estimation was made by means of the diagram specially produced according to the original writer's directions for the locomotive and the conditions cited above. Comparison of the results of the two methods reveals that Henderson's method leads to errors in estimated coal consumption of from 5.3 to 26.4 percent.

	Level	1/2% Grade	1% Grade
Coal consumption by old method	545.	885.	1335.
Coal consumption by new method	740.	982.	1410.
Difference,	196.	97.	75.
Percent difference	26.4	9.8	5.3

This great error is entirely due to the fact that in the old method the running time was not determined correctly. It was roughly assumed that a train runs at a uniform speed throughout the run, or in other words that the acceleration and braking forces are infinite. The percent error, however, reduces as the length of run increases. For instance, if the length of the run is 10 instead of 5 miles the percent error on level track will be 13.2%; and the error becomes insignificant for a train running a long distance without stops. For a heavy freight train which stops almost every ten or twenty miles, however, the error is serious enough to warrant careful calculation of the running time by some rational method in order to obtain correct results in estimating coal consumption.

The coal consumption for firing up and for waiting on passing tracks is estimated to be from 12 to 20 percent of the total coal consumption for various operating conditions and 15 percent is considered to be a fair average value.

2. Energy consumption in electric locomotives. - Practically speaking a steam locomotive is a constant power machine

and its energy consumption is little affected by speed. Electric motors commonly used in railway traction, however, are variable power machines and their energy consumption has an intimate relation to the running speed. Consequently, speed-time run curves are indispensable for an accurate estimation of the energy consumption of electric motors.

The method of estimating energy consumption is as follows: Suppose that the speed-torque-current relation of the motor under consideration is as shown in Fig. 89 and that the corresponding run curve as shown in Fig. 90 has been produced. Then from the speed-torque-current relation we can construct a current-time* diagram corresponding to the particular run as shown in the same figure. The area under the current-time curve is

$$IT = \int_{t_1}^{t_2} i\,dt,$$

where I represents the average current used during the run, T the running time, i the current at various speeds, t_1 the time at the start and t_2 the time at the cut-off of the current. The value, $\int_{t_1}^{t_2} i\,dt$ can be determined by means of a planimeter, integraph, or kineograph, or by any other convenient graphical or analytical means. Then, since the ~~kilo~~-watt-hour of electric energy is equal to

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* For construction of current-time curves, see Professor A. M. Buck's Electric Railway Traction, or any other text-book on the subject.

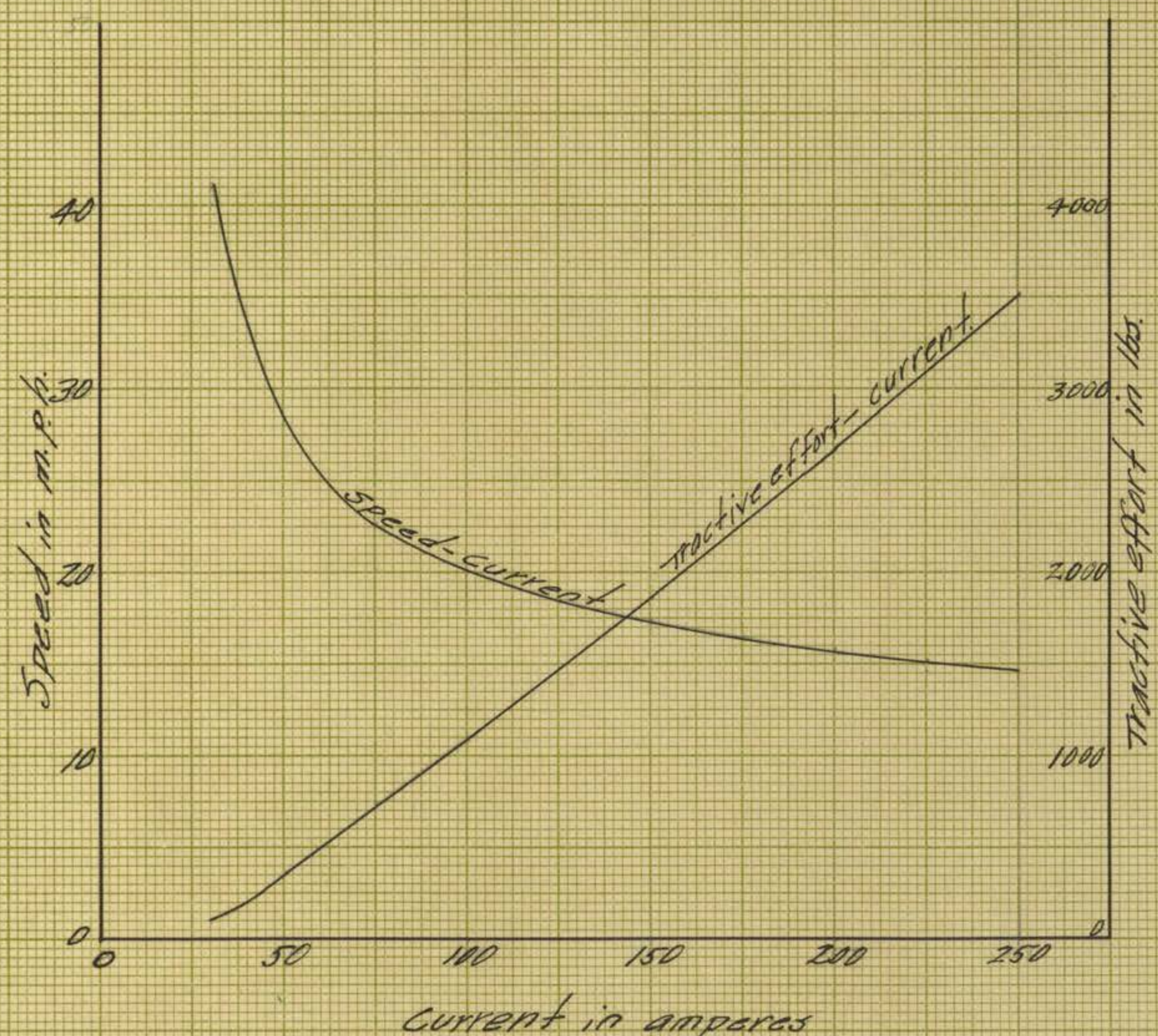

Speed in m.p.h.
Tractive effort in lbs.
Speed-current
Tractive effort-current
40
30
20
10
0
4000
3000
2000
1000
0
0
50
100
150
200
250
Current in amperes

Fig. 89.

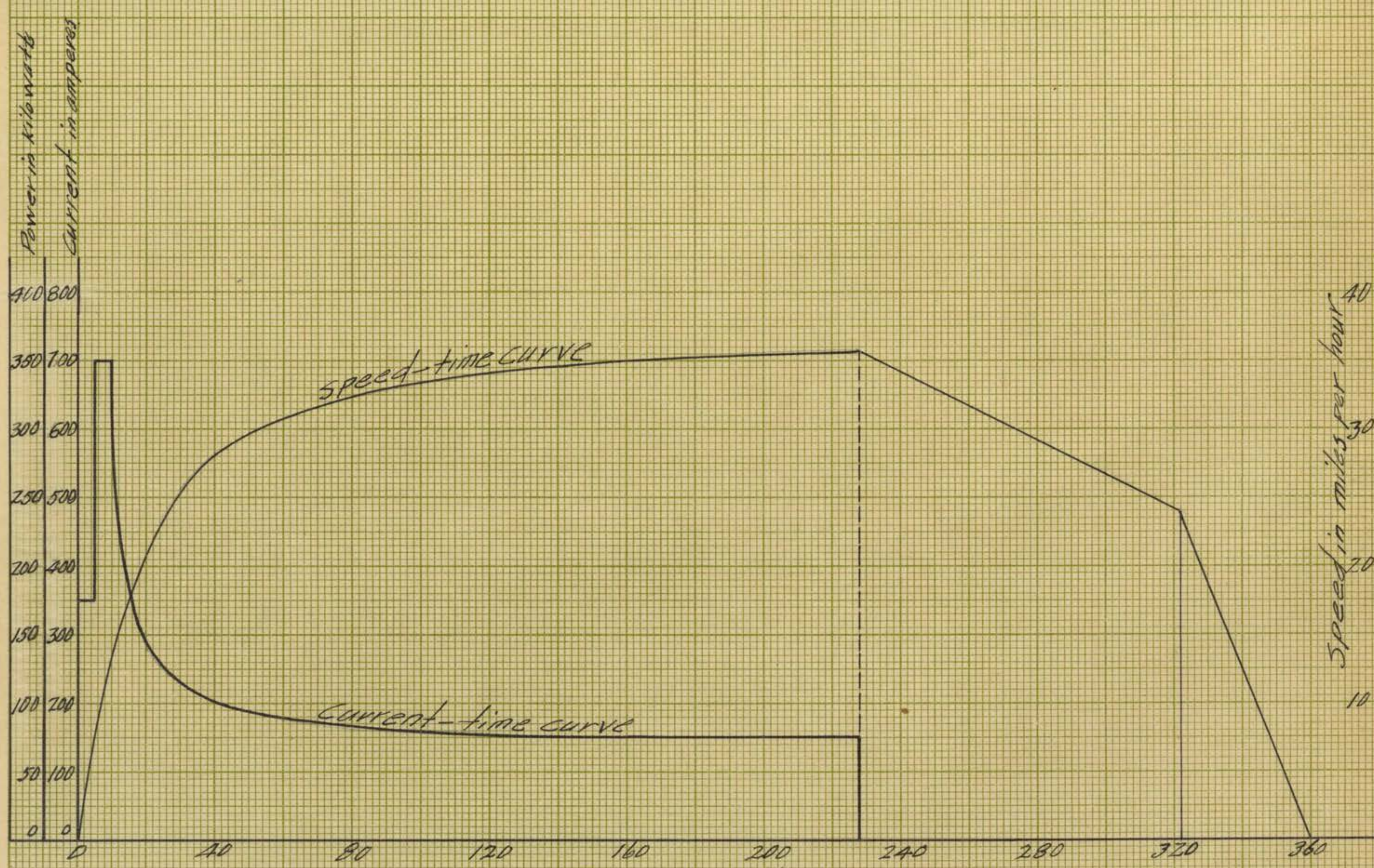
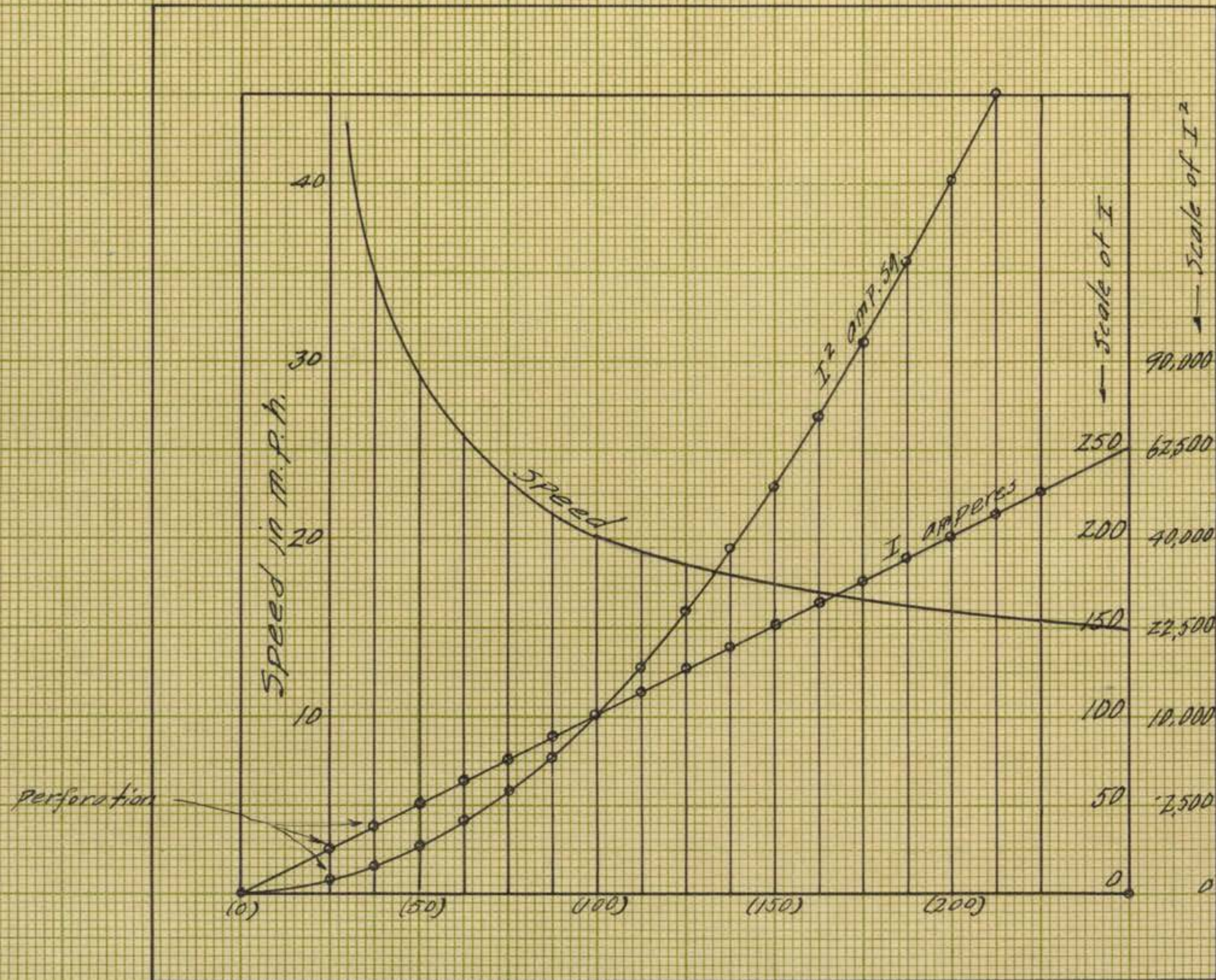

Power in kilowatts
Current in amperes
400 800
350 700
300 600
250 500
200 400
150 300
100 200
50 100
0 0
Speed-time curve
Current-time curve
Speed in miles per hour
40
30
20
10
0
40
80
120
160
200
240
280
320
360
Time in seconds

Fig. 90.

260



Speed in m.p.h.
40
30
20
10
Speed
I^2 amp. sq.
I amperes
Perforation
(0)
(50)
(100)
(150)
(200)
← Scale of I
← Scale of I^2
250
200
150
100
50
0
90,000
62,500
40,000
22,500
10,000
2,500
0

Fig. 91.

$$EIT = E\int_{t_1}^{t_2} i\,dt,$$

and the voltage E is a constant of known value*, the energy consumption can be accurately determined.

The construction of the current-time curves can be conveniently and quickly effected by the aid of a templet** such as is shown in Fig. 91.

C. Tonnage Rating.

1. Nature of tonnage rating. - Tonnage rating or locomotive rating is the determination of the proper tonnage to be hauled by a locomotive from one terminal to another under certain conditions. It implies the assignment of the rated tonnage to the locomotive in the yards. Tonnage ratings may be classified, according to the different purposes which it may be desired to serve, namely: (1) Determination of the maximum tonnage a locomotive can haul over a division within a certain time; (2) Tonnage rating for the purpose of transporting maximum tonnage per locomotive per day over a given division; and (3) Determination of the rating that will give the minimum cost per ton-mile.

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* There is a slight drop in potential toward the end of trolly wires. For a graphical method of finding corresponding drop in speed of motors, see Eng. Exp. Sta., University of Illinois Bulletin No. 90, page 7; or Electric Railway Journal, vol. 46 page 595. (1915).

** For the use of a similar device, see Engineering Experiment Station, University of Illinois, Bulletin No. 90, page. 33.

In any class of tonnage rating, it is necessary that the drawbar pull exerted by the locomotive be utilized with the utmost efficiency. Suppose that T represents the tractive effort or drawbar pull at a certain speed, R the total train resistance, c a certain constant and f the efficiency of tonnage rating, then in any case, we have

$$T = R + c,$$

and

$$f = R/T$$

or

$$f = (T - c)/T.$$

The quantity c is virtually reserve tractive effort and it is possible to make the value of c so small as to make f approach 100%, but this practice does not serve the ultimate purpose of tonnage rating since a train thus made up may be stalled on a ruling grade when the weather is adverse and the result would be traffic congestion. It is, however, the sole purpose of tonnage rating to keep the value of c as small as possible, and utilize all the potential capacity of the motive power and of the track. As will be seen later, in the first class of tonnage ratings, we consider T as a fixed value and try to find out what tonnage will give a value of R equal to T or nearly so, so that c becomes as small as possible, i.e., to have 100% for f. In the second class, both T and R vary with speed and we endeavour to find such a speed and corresponding tonnage as will give the maximum ton-mileage per locomotive per day when f is 100 percent. In the third class of ratings, T and R vary as in the second, and we try to find the corresponding speed, tonnage, and operating expense, from which we may determine the rating which will give minimum cost per ton-mile.

2. Tonnage rating for maximum tonnage per train. - In this class of ratings the controlling factor of the problem is the "ruling point" - usually the ruling grade of the division under consideration. If the ruling grade can be passed by the locomotive hauling a certain tonnage at a given speed, the locomotive is usually able to fulfil its assigned duty within the given time. If an accurate determination of the effect on speed or time of the rise and fall on other parts of the division is required, it may be computed by certain formulas or by means of speed-time curves. A ruling grade may, according to the character of the adjacent profile, be a momentum grade on which the momentum of trains aids them in surmounting the grade; or it may be a ~~momentum~~ grade which the trains surmount by means of the drawbar pull of the locomotive only.

a). A brief sketch of the development of tonnage rating.- Up to the early eighties, locomotives were generally assigned their loads by the number of cars without much reference to weight or tonnage. Experience, however, showed that a given locomotive could haul a certain number of lighter cars easier than it could haul an equal number of heavily loaded cars, and "tonnage rating", in which a locomotive was rated with a certain number of tons instead of cars, was inaugurated. After several years experience with this method, it was discovered that a given locomotive could haul à greater tonnage of loaded cars than of empty cars. Basing upon this fact, the "adjusted" tonnage method was devised and soon adopted by many roads. In this method, as it was originally introduced, the difference in car resistance between loaded and empty cars only was recognized; but later it was found that this method, under a certain admissible assumption, could be applied also to the adjustment of tonnage for cars of any weight and capacity, i.e., cars loaded to any fraction of their full capacity and cars of different capacity fully loaded as well as empty. A slight objection to this method is found in its use of a fictitious "adjusted tonnage", which frequently mystifies the men who make up trains in the yards. In view of this fact the equated tonnage method in which the rating in yards is effected in terms of a natural instead of a fictitious tonnage was introduced. It is generally expressed in the form of a table showing the tonnage which can be hauled by locomotives of various classes, according to different car weights. This table is constructed by equating the train resistance of trains of various car weights to the drawbar pull of different locomotives. This method is sometimes identified with the rational drawbar method to be described later. These methods, i.e., adjusted and equated tonnage methods, are based on the assumption that the relation between the resistance per car and the gross weight of the car is represented by a straight line. In certain cases, however, this assumption is not correct and the error it involves makes the efficiency of the rating sometimes as low as 90% and sometimes as high as 105%. To remedy this defect, two other methods were proposed. One of them is the so-called variable car factor method in which the relation of car resistance to the car weight is used in just the form in which it results from train resistance tests, and the different car factors for various car weights are expressed in terms of tons as in the case of the adjusted tonnage rating method. These car factors are added to the natural tonnage of the cars. This may be done by ordinary calculation and a table may be made for ready application in yards; sometimes, however, the addition is automatically performed by means of a special machine, in which when a key marked for the actual tonnage of a car is struck it prints the adjusted tonnage, and also the sum of the tonnages as in an ordinary adding machine. Another method is the drawbar pull method, in which the resistances of cars of various weights are found - as in the case of the variable car factor method - from the relation between car resistance and car weight, and they are added until their sum equals the drawbar pull of the locomotive. A table may be constructed which will give actual tonnages of trains for various classes of locomotives and for trains of uniform car weight. If the data showing the relation of car resistance to average car weight is used in the construction of this table, it will be applicable to trains of cars with uniform weight as well as to trains of cars with non-uniform weight.

The method of rating locomotives by the number of cars and the "straight" tonnage methods are obsolete today, although they are still used on small feeder and local lines where the volume of traffic is not sufficient to warrant more scientific methods. The method most extensively employed in this country at present is the single car factor adjusted tonnage method; but the tendency is toward the universal adoption of the drawbar pull method based upon actual resistances per car determined by tests.

b). Rating locomotives by the adjusted tonnage method. - The general procedure is as follows: First an experimental train is run with a locomotive over the ruling grade, the train being composed of a number of cars which can be hauled by the locomotive at a certain speed, say 10 miles per hour. Then another experimental train of empties or lightly loaded cars is run with the same locomotive over the grade at the same speed, and the tonnage and the number of the cars are determined. Let us suppose that by these tests the following data is obtained:

Loaded car train,	30 cars,	1830 tons.
Empty car train,	90 cars,	1350 tons.
Difference	60 cars,	480 tons.

Then, $480 \div 60 = 8$ tons ..... car factor,

and $30 \text{ cars} \times 8 + 1830 = 240 + 1830 = 2070$, adjusted tons.

Similarly the car factor and the adjusted tonnage can be determined for any locomotive over any ruling grade. The car factor depends only upon the ruling grade, while the adjusted tonnage depends only upon the locomotive. The yard master whose duty it is to make up the train is advised of the car factor and of the adjusted tons for each class of locomotives, and is instructed so to make up trains that the actual tonnage of the cars plus the number of cars multiplied by the car factor equals the adjusted tonnage. If, for instance, he has coupled 52 cars weighing 1580 tons to a locomotive rated at 2070 adjusted tons, and he wants to find out whether this tonnage is correct or not, he makes the following calculation

$$1580 + 52 \times 8 = 1996 \text{ adjusted tons},$$

74

and finds that the train is under-rated by 64 adjusted tons. He then adds another car weighing about 56 tons (64 - 8).

The validity of this method* may be demonstrated as follows: The results of dynamometer car tests show that the relation between the resistance per car in pounds and the gross weight of cars can be quite closely represented by a straight line as shown in Fig. 92. If R denotes the total resistance of one car on level tangent track in pounds, f_o the slope of the straight line, c the intercept of the line on the resistance-axis, and w the weight of car in tons, we have

$$R = f_o w + c,$$

and on a grade of g percent, the resistance per car is

$$R_g = f_o w + c + 20\ gw,$$

$$R_g = (f_o + 20g)w + c.$$

For a particular grade 20g is a constant, then representing $(f_o + 20g)$ by f, we get

$$R_g = fw + c. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Next, suppose that two experimental trains have been run in the same way as described before, and that the results are as follows:

	First Train	Second Train
Number of cars,	N_1	N_2
Weight per car, in tons,	w_1	w_2
Cross weight, in tons	W_1	W_2

Then, $$R_g N_1 = (fw_1 + c)N_1$$

and $$R_g N_2 = (fw_2 + c)N_2.$$

But the drawbar pulls in the two cases are equal, hence

$$(fw_1 + c)N_1 = (fw_2 + c)N_2.$$

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*L.G.Hass: "Car factor", R.R. Gazette, Nov. 1, 1895; also Penna. Railroad test dept. Bulletin No. 26.

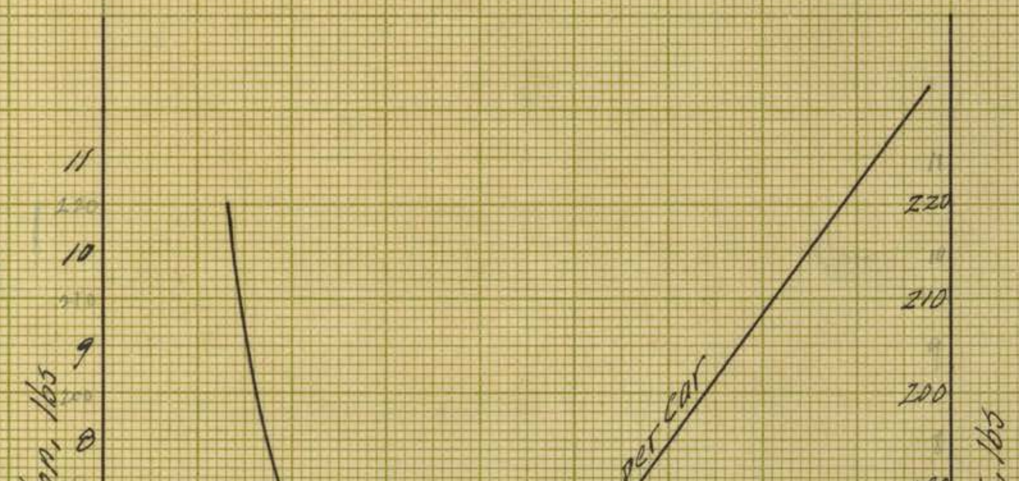

Resistance per ton, lbs
Resistance per car, lbs
Resistance per car
Resistance per ton
11
10
9
8
7
6
5
4
3
2
220
210
200
190
180
170
160
150
140
0
10
20
30
40
50
60
70
80

Fig. 92.

Dividing both sides by f,

$$w_1N_1 + \frac{c}{f}N_1 = w_2N_2 + \frac{c}{f}N_2$$

Then, representing c/f by K, we have

$$W_1 + KN_1 = W_2 + KN_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

and

$$K = \frac{W_2 - W_1}{N_1 - N_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

In equation (3) K is recognized to be the car factor, and the value of each member of equation (2) is the adjusted tonnage of the locomotive. It follows, then, the trains whose adjusted tonnage is the same offer the same total train resistance and require the same drawbar pull in order to be hauled over a grade at the same speed.

c). Equated tonnage rating. - This method was first developed by Mr. Wickhorst, Engineer of Tests, Chicago, Burlington & Quincy Railroad. This method is sometimes confused with the adjusted tonnage method, but they are entirely different. In this method the resistance for cars of different weights is calculated from a diagram showing the relation of train resistance in lbs. per ton to various car weights under different weather conditions. The number of cars that can be hauled over a grade is obtained by simply dividing the drawbar pull of the locomotive at the grade by the resistance per car, that is, if w represents the weight of the car in tons, g the percent of grade, N the number of cars, and T the drawbar pull of the locomotive over the grade, then

$$T = N(r + 20g)w$$

or

$$N = \frac{T}{w(r \mp 20g)}$$

and the tonnage the locomotive can haul is equal to Nw. The yard master and conductors are provided with tonnage tables of the form shown in Fig. 93.

d). Drawbar pull method. - This method was first introduced by M. H. Fergusson. From the results "of dynamometer car tests covering a period of six months, and including some 10,000 cars", he found that the curve representing the relation between the car resistance and car weight is not a straight line but a curve which is shown in Fig. 94. From this curve he found the resistance for cars of various weights, and by adding a proper grade resistance he constructed tables to be used in loading locomotives in yards.

e). Variable car-factor method, - is an adjusted tonnage method in which the car-factor is considered to vary with average car weight as indicated by results of dynamometer car test, whereas in the single car-factor method it is considered to be a constant. The rating by this method is effected usually by means of a special form of adding machine as mentioned before. This method has an advantage over the single car-factor method in its theory, but is more complicated in application. It involves the use of fictitious "adjusted" tonnage as in the case of the single car-factor method.

3. Tonnage rating for maximum ton-miles per day per locomotive. - There are many reasons for traffic congestion, for some of which operating departments must be responsible; but it is the railway engineer's duty to study this problem thoughtfully in order to devise a means to handle successfully a great

TONNAGE RATING FOR ENGINES
(Gross Tonnage of Train Exclusive of Way Car)

		Rate A				Rate B				Rate C			
	Cars	A1	A2	H1	H2	A1	A2	H1	H2	A1	A2	H1	H2
	50	500		665	775			550	645			505	590
	45	515		680	790			565	660			520	605
Murrays	40	530	450	695	805	440		580	680	400		535	620
and	35	545	465	715	825	455	385	595	700	415	350	550	640
Kansas	30	560	480	735	845	470	400	615	720	430	365	565	660
City	25	580	495	755	870	490	415	640	740	445	380	580	680
	20	600	515	780	895	510	435	660	760	465	395	600	700
	15	625	540	805	920	530	455	680	785	485	415	625	720
	10	650	565			555	480	705	810	510	440	650	745
	80	895		1250	1505			845	1020				830
	75	925	760	1290	1545			875	1050				855
	70	955	785	1330	1590			905	1085			740	885
	65	990	815	1370	1635	675		940	1125			765	920
	60	1030	845	1420	1685	700		975	1165	575		795	955
Murrays	55	1070	880	1470	1740	730	595	1015	1210	595		830	995
and	50	1115	920	1520	1795	765	625	1060	1260	620	505	865	1040
Council	45	1165	965	1575	1855	805	655	1105	1315	655	535	905	1085
Bluffs	40	1220	1015	1635	1915	845	695	1155	1370	690	565	955	1135
	35	1275	1065	1695	1975	895	740	1210	1425	735	600	1005	1190
	30	1330	1120	1755		950	790	1270	1485	785	645	1055	1245
	25	1390	1175	1815		1010	840	1330	1545	835	695	1115	1305
	20		1235			1070	895	1390		895	750	1175	1365
	15					1130	950			955	810		
	45			555	620			475	530				495
	40			565	630			485	545			455	505
Bigelow	35	420	360	580	645	365		495	560	340		465	520
and	30	435	370	595	660	375	320	510	575	350	300	475	535
Villisca	25	450	380	610	675	385	330	525	590	360	310	490	550
	20	465	395	625	690	400	340	540	605	375	320	510	565
	15	485	410	645	720	415	355	560	625	390	335	530	595
	10	505	420	670	745	435	375	580	645	410	350	550	605

TONNAGE RATING FOR ENGINES
(Gross Tonnage of Train Exclusive of Way Car)

		Rate A				Rate B				Rate C			
	Cars	A1	A2	H1	H2	A1	A2	H1	H2	A1	A2	H1	H2
Corning and Clarinda		450	410			410	370						

Rating "A" governs under ordinary conditions when temperature is above 30 degrees; "B", when temperature is between 30 degrees and 10 degrees, or when weather is markedly windy or stormy; "C", when temperature is below 10 degrees above or in high head winds or very stormy weather.

Time freight and stock trains will ordinarily use "B" rating, and in unfavorable conditions "C" rating; but in very stormy or very cold weather will reduce tonnage so that approximately schedule time may be made.

Rating over Kansas City Bridge should be carefully considered by Conductors, in asking for helper.

When an engine fails to haul its rating Conductor must telegraph Chief Dispatcher the cause of failure, and at end of trip Engineer must make written report to Master Mechanic.

Fig. 93.

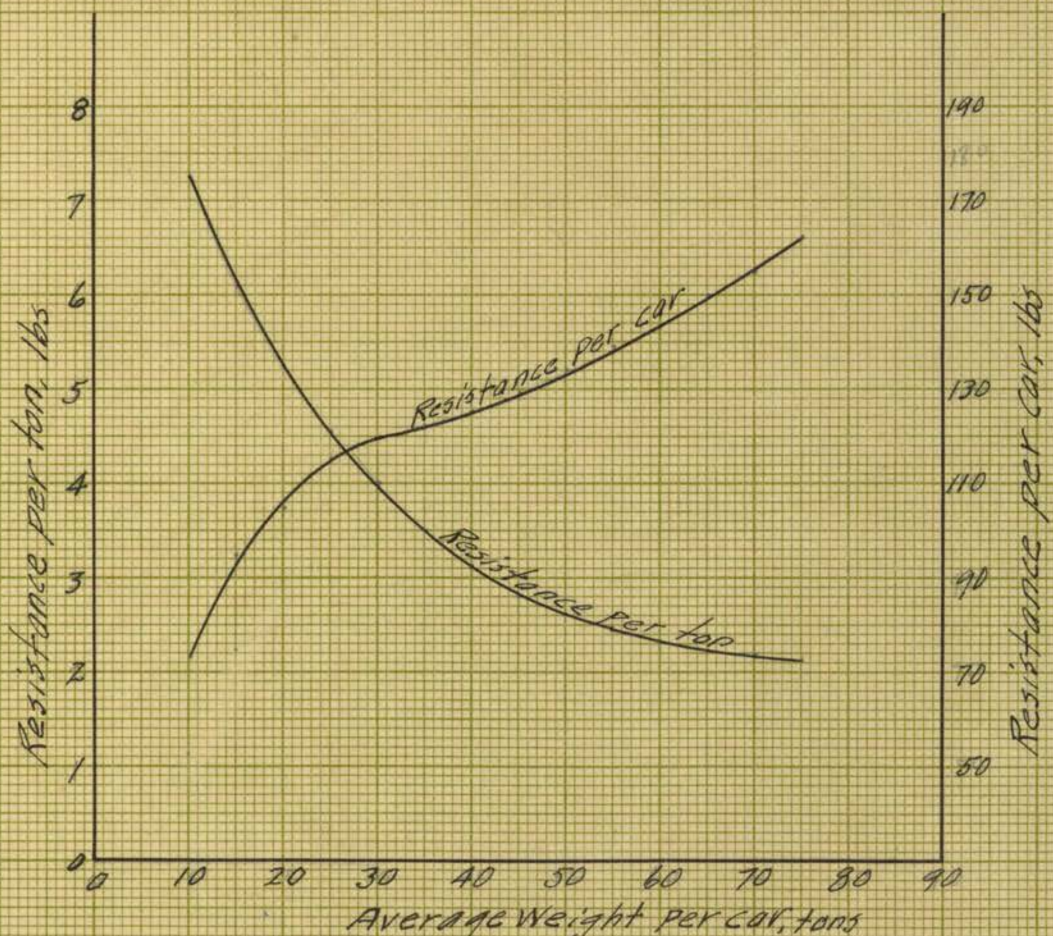
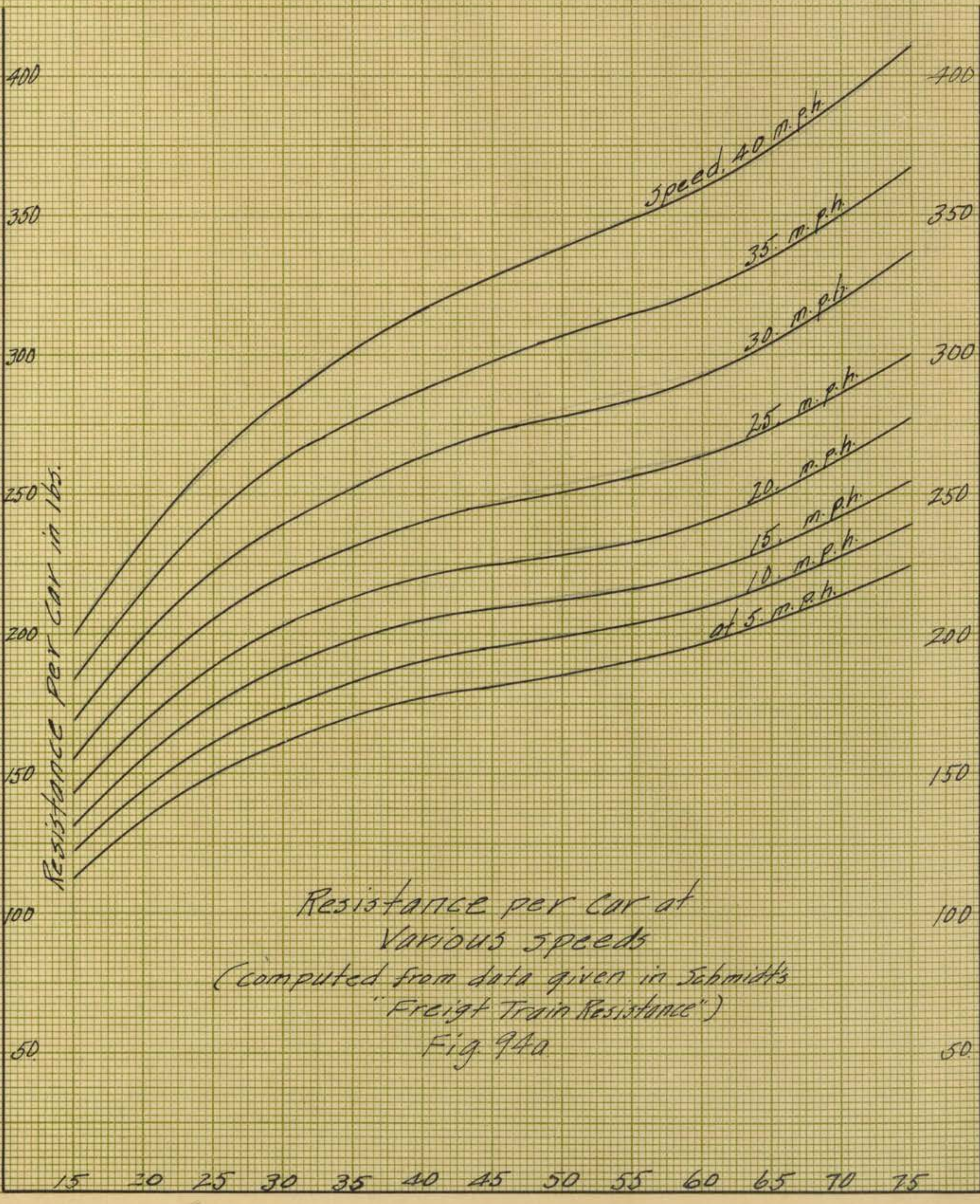

Resistance per ton, lbs
Resistance per car, lbs
Resistance per car
Resistance per ton
Average Weight per car, tons
0 10 20 30 40 50 60 70 80 90
0 1 2 3 4 5 6 7 8
50 70 90 110 130 150 170 190

Fig. 94



Resistance per Car at
Various speeds
(computed from data given in Schmidt's
"Freigt Train Resistance")
Fig. 94a

tide of traffic at its peak with a given track, cars, and motive power. Although the capacity of each element of railway equipment is fixed, he may, by a careful study, find a way to secure their combined maximum capacity.

There are two distinct ways by which this sort of problem can be solved. Ths first is to keep statistics of locomotives, train loads, running time, etc. for a division under consideration and, when sufficient data are accumulated, to analyze them carefully and draw a certain conclusion. The practical difficulty in this method is that the accumulation of this data requires considerable time and labor and that the statistics must have included all ranges of train loads, and furthermore, the result of the analysis applies only to the division from which the statistics were gathered. The second method is to solve the problem by means of established principles of mechanics, with tractive effort and train resistance data. This method is rational and simple, and requires much less time and labor while the result is as accurate as that obtained by the first method.

In 1905, G. R. Henderson published* an interesting article in which he touched upon the second method. The method was then new and it attracted the attention of engineers; but as we have seen in a previous section it contained a serious defect in the assumption made in determining running time. With this exception his method is rational and of great value. Many

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* "Economical train operation", Am. Eng. and Railroad Journal, vol. 78, pp. 371, 411, and vol. 79, p. 11. (1905); see also his "Cost of Locomotive Operation".

articles which relate more or less closely to this subject may be found in American Railway Engineering Association Bulletins and other periodicals. An interesting result of his statistical study of the problem has been published by Mr. Mott. This will be shown later and compared with the results of another method. We have seen in the previous section how the running time of a train can be determined accurately by means of speed-time curves. When the running time of trains with various train loads on a particular division has been found, the tonnage rating for maximum ton-miles per locomotive per day is very simple.

Let t denote the time required for a train with W gross tons back of the tender to pass over the division, and let H denote the hours the locomotive is on road, then the ton-mileage which the locomotive can make every day is

$$M = \frac{W\ m\ H}{t}.$$

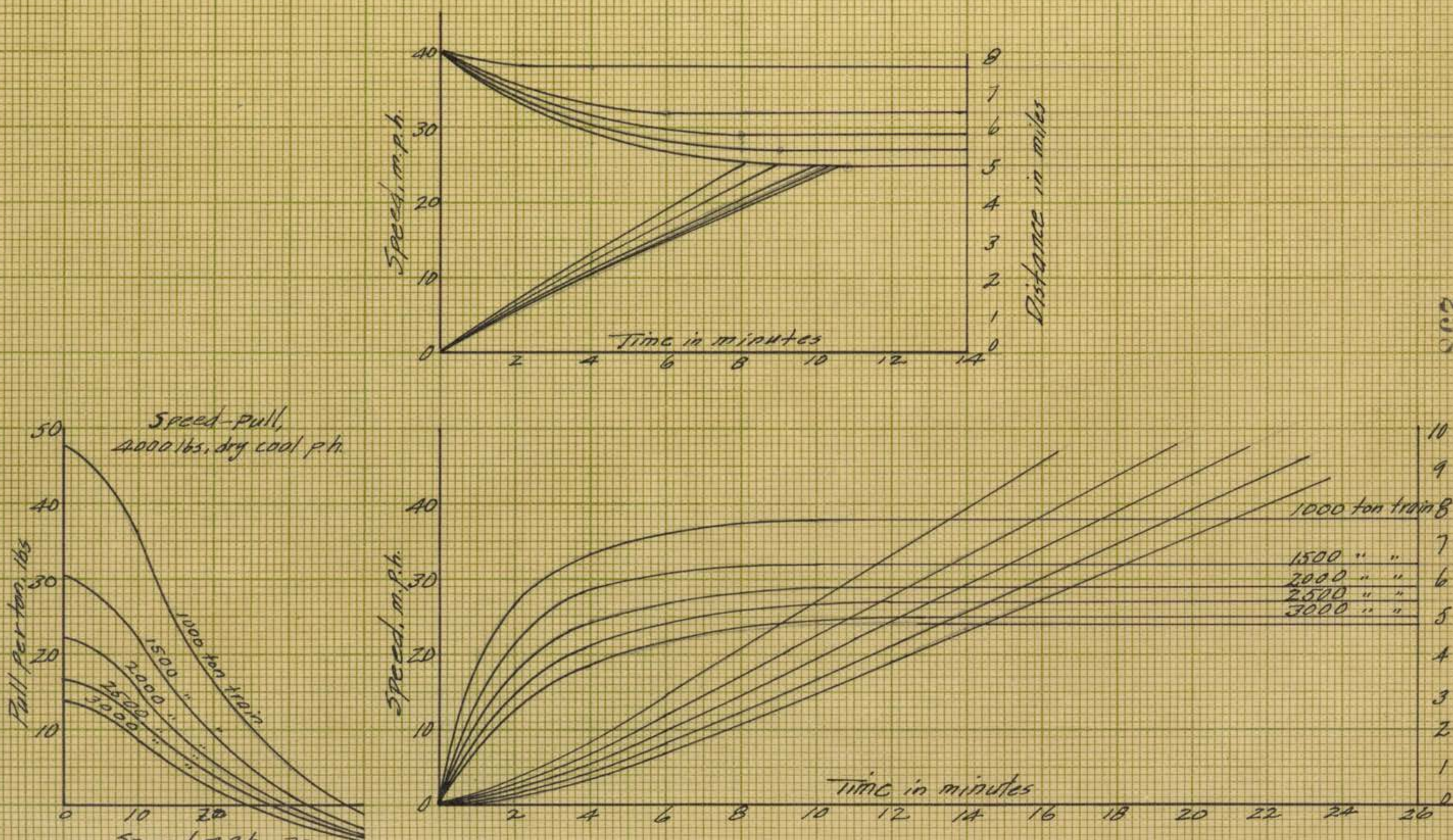
In this equation m is a constant and H should be considered as constant. The value of t varies with W, and it will be determined by means of speed-time curves. Then, with the values of t corresponding to various values of W, by means of the above formula we can find values of M corresponding to different train loads W. Representing then the relation of M and W on a coordinate system the maximum value of M can be easily found.

As an illustration of the method, we will consider a division of 120 miles whose profile is as shown in Fig. 96a, and a locomotive whose speed-pull relations at various firing rates is as shown in Fig. 86. In this example we assume: (1) that the trains stop at Stations A, B, C, D, E, F, and G in order to let other trains pass; (2) that the trains take necessary coal and water during these stops; (3) that the time required by these stops amounts to 20 percent of the actual running time; and (4) that the boiler is fired at the rate of 5000 lbs. per hour on all up-grades, and 4000 lbs. on level track and on down-grades until it reaches the speed of 40 m.p.h., which is the maximum speed allowable.

Under these conditions the run curves shown in Fig. 96 have been produced from the speed curves shown in Figs. 95a, 95b, and 95c, which in turn were produced from the speed-pull relations of the locomotive. The running time and the actual time on the road for trains with 1000, 1500, 2000, 2500, and 3000 gross tons behind the tender are found from the diagrams to be as shown in the following table. With this data and the formula mentioned in the preceding paragraph the values in item 6 are

1. Weight of train, tons, behind tender	1000.	1500.	2000.	2500.	3000
2. Ton-mile per trip, in 1000 ton-mile	120	180	240	300	360
3. Running time, minute.	213.3	250.2	285.5	334.0	409.6
4. Actual time on road, hour	4.26	5.00	5.77	6.68	8.19
5. Number of trips per 24 hours	5.62	4.8	4.16	3.58	2.93
6. Ton-miles per locomotive per 24 hours, in 1000	675	865	1000	1075	1050

Fig. 95a.



Speed, m.p.h.
Distance in miles
Time in minutes
Speed-pull,
4000 lbs. dry coal p.h.
Pull per ton, lbs
Speed, m.p.h.
1000 ton train
1500 " "
2000 " "
2500 " "
3000 " "
Speed. m.p.h.
Time in minutes
Distance in miles
1000 ton train
1500 " "
2000 " "
2500 " "
3000 " "

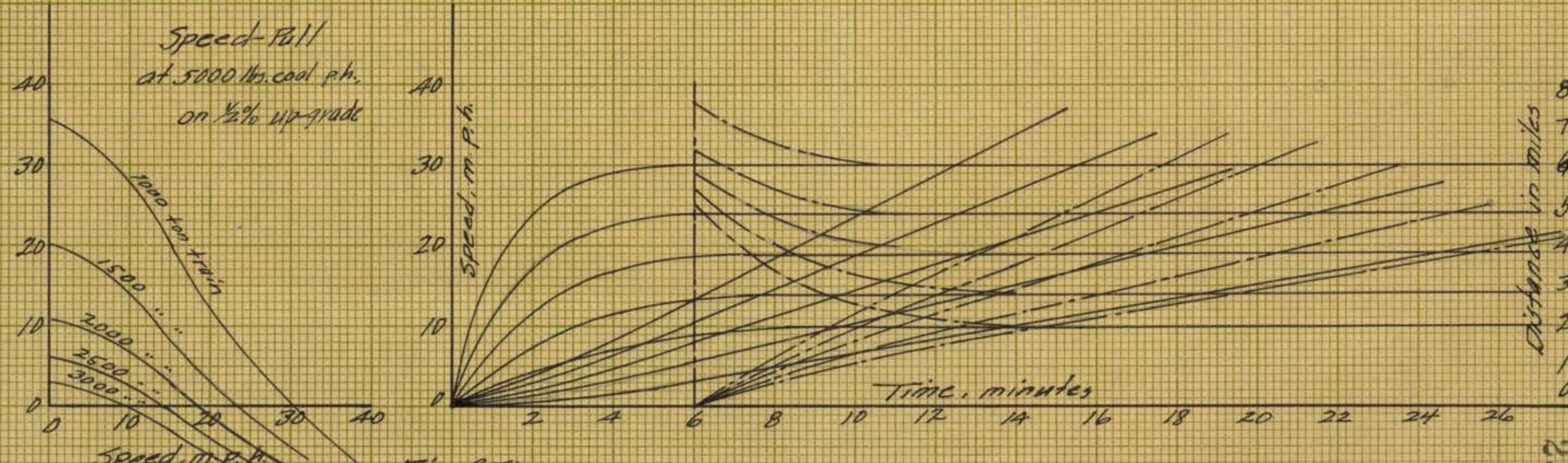


Fig. 95.b.

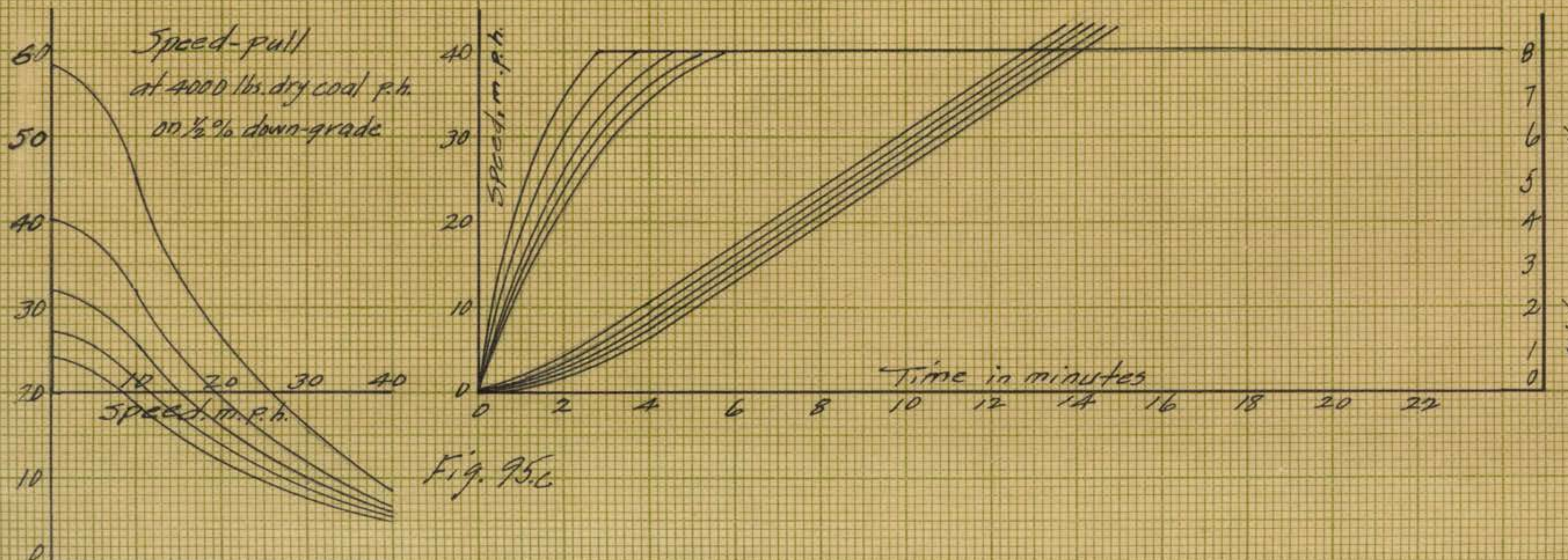
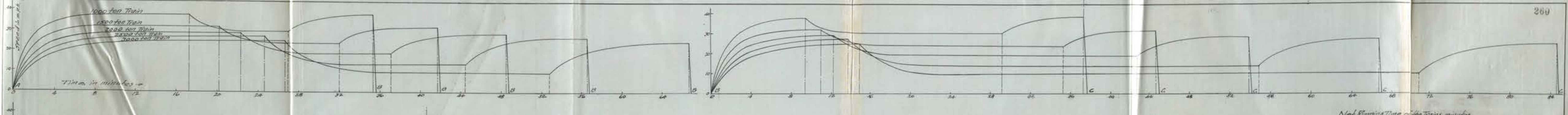
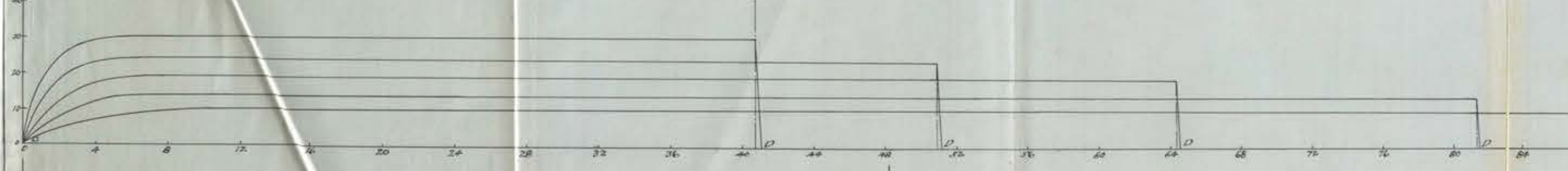


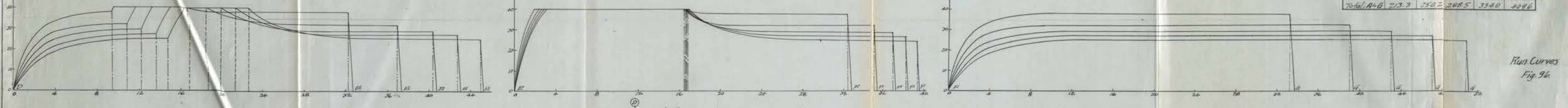
Fig. 95.c



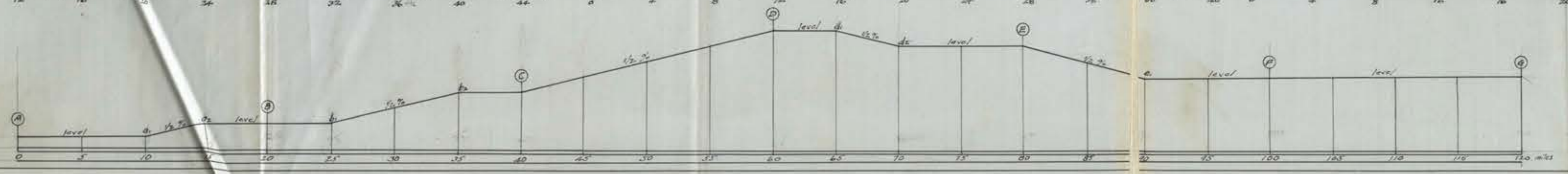


Net Running Time of the Trains, minutes

Run	Gross tons behind tender				
	1000	1500	2000	2500	3000
A to B	35.6	42.0	48.7	56.8	66.8
B to C	37.6	45.0	54.0	66.9	84.8
C to D	41.0	51.0	65.5	81.4	122.1
D to E	36.6	37.3	40.7	43.0	45.3
E to F	32.8	35.5	37.3	38.5	39.6
F to G	33.7	39.4	43.3	47.4	51.0
Total, A to G	213.3	250.2	288.5	334.0	409.6



Run Curves.
Fig. 96.



Profile.
Fig. 96a.

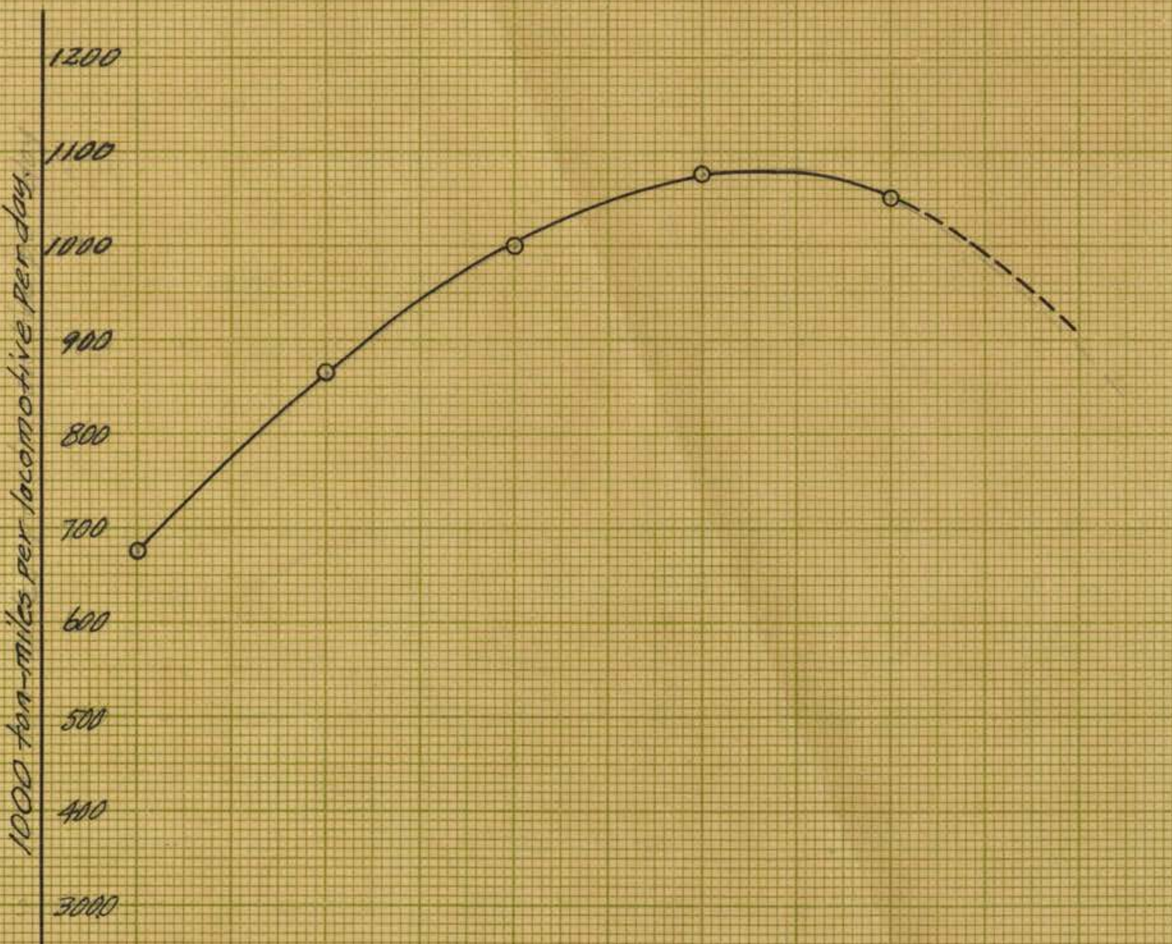

1200
1100
1000
900
800
700
600
500
400
3000
200
100
1000 ton-miles per locomotive per day.
1000.
1500.
2000.
2500.
3000.
4000.
Train load behind tender, tons

Fig. 97.

calculated and plotted against the values in item 1, in Fig. 97. This diagram indicates that in order to transport the maximum amount of traffic over the assumed division with a certain number of locomotives, the trains should weigh between 2500 and 2750 tons each. The speed of these trains on the ruling grade is between 12 and 14 m.p.h. which fully satisfies the conditions defined in the first class of tonnage rating previously described. In this example train resistance per ton for 50 ton cars is used. The tonnage for other trains with other average car weights may be calculated by processes which have already been explained.

4. Tonnage rating for minimum ton-mile cost. - Under present economic conditions and government regulation, no railroad property can yield a fair return on the investment unless the property is operated most effectively and economically. Every year in this country alone over \$220,000,000. is expended for coal and over \$250,000,000 for train crew wages, and this annual expenditure of some \$500,000,000 - a quarter of the total annual operating expense of \$2,000,000,000 - is most sensitively affected by tonnage rating even when the annual ton-mileage is constant. Being of such an important character, the solution of the problem has been attempted by many students of the subject. As in the case of tonnage rating for maximum ton-miles per locomotive per day the tonnage rating for minimum ton-mile cost can be found from statistics if sufficient data have been accumulated. An excellent example of the method will be found in Mr. Mott's article mentioned before. Henderson attacked this problem from an entirely different angle. His method is

described fully in the article referred to in the preceding section. His general plan is thorough and valuable for any investigator who attempts to solve this problem by other than the statistical method; but the determination of running time which is of vital importance in the problem ought not be made as Henderson suggests.

A careful study of railway operating expenses as shown in the following pages convinces us that strictly speaking practically every item of operating expense is more or less affected by train load even when we assume that the total ton-miles made over a division is constant. It is impossible at the present stage of our knowledge to detect and express quantitatively the influence of train load on every item. Fortunately, however, the effect on the most of the items is very slight, and they can be safely assumed to be independent of train load if a definite number of ton-miles is to be made with locomotives of definite power and weight. For instance, item (9), "Bridges, trestles, and culverts" apparently is affected by train load, but it is in reality very little affected because the maximum strain on a bridge is chiefly due to the weight of locomotives and is practically independent of the weight of trains. Another conspicuous item, (25) "Steam locomotive-repairs", for instance, is also little affected by train load. A heavy train runs slowly but requires great tractive effort, while light trains run at higher speed with lower tractive effort; but they develop about the same horse power, that is, the boiler at least works at the same rate in the two cases. The engines of light trains have to

Analysis of Operating Expenses for the Year Ended

June 30, 1914 - Class I Roads.*

I. Maintenance of way and structures:	Percent
1. Superintendence	0.977
2. Ballast	0.332
3. Ties	3.074
4. Rails	0.875
5. Other track material	0.987
6. Roadway and track	6.846
7. Removal of snow, sand, and ice	0.266
8. Tunnels	0.051
9. Bridges, trestles, and culverts	1.685
10. Over and under grade crossing	0.071
11. Grade crossings, fences, cattle guards, and signs	0.329
12. Snow and sand fences and snow sheds	0.021
13. Signals and interlocking plants	0.546
14. Telegraph and telephone lines	0.217
15. Electric power transmission	0.037
16. Buildings, fixtures, and grounds	1.747
17. Docks and wharves	0.152
18. Roadway tools and supplies	0.249
19. Injuries to persons	0.144
20. Stationary and printing	0.037
21. Other expenses	0.011
22. Maintaining joint tracks, yards, and other facilities - Dr.	0.773
23. Maintaining joint tracks, yards, and other facilities - Cr.	-0.561
Total - Maintenance of Way and Struct.	18.866
II. Maintenance of equipment:	
24. Superintendence	0.704
25. Steam locomotives - repairs	8.183
26. " " - renewals	0.132
27. " " - depreciation	1.027
28. Electric locomotives - repairs	0.036
29. " " - renewals	---
30. " " - depreciation	0.012
31. Passenger-train cars - repairs	1.523
32. " " " - renewals	0.050
33. " " " - depreciation	0.298
34. Freight-train cars - repairs	8.510
35. " " " - renewals	0.597

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* Interstate Commerce Commission: "Statistics of Railways in the United States", 1914, p. 59-60.

II. Maintenance of Equipment (continued).	Percent
36. Freight-train cars - depreciation	2.004
37. Electric equipment of cars - repairs	0.014
38. " " " " - renewals	---
39. " " " " - depreciation	0.007
40. Floating equipment - repairs	0.044
41. " " - renewals	0.003
42. " " - depreciation	0.019
43. Work equipment - repairs	0.228
44. " " - renewals	0.038
45. " " - depreciation	0.079
46. Shop machinery and tools	0.546
47. Power plant equipment	0.018
48. Injuries to persons	0.117
49. Stationary and printing	0.054
50. Other expenses	o.025
51. Maintaining joint equipment at terminals, Dr.	0.087
52. Maintaining joint equipment at terminals, Cr.	-0.044
Total - Maintenance of equipment	24.311

III. Traffic expenses:

53. Superintendence	0.765
54. Outside agencies	1.137
55. Advertising	0.379
56. Traffic association	0.071
57. Fast freight lines	0.154
58. Industrial and immigration bureaus	0.082
59. Stationary and printing	0.320
60. Other expenses	0.006
Total - Traffic expenses	2.914

IV. Transportation expenses:

61. Superintendence	1.256
62. Dispatching trains	0.845
63. Station employees	6.886
64. Weighing and car-service association	0.117
65. Coal and ore docks	0.176
66. Station supplies and expenses	0.574
67. Yard masters and other clerks	0.850
68. Yard conductors and brakemen	2.830
69. Yard switch and signal tenders	0.213
70. Yard supplies and expenses	0.079
71. Yard enginemen	1.637
72. Enginehouse expenses - yard	0.524
73. Fuel for yard locomotives	1.570
74. Water for yard locomotives	0.109
75. Lubricants for yard locomotives	0.030

IV. Transportation expenses: (continued)	Percent
76. Other expenses for yard locomotives	0.035
77. Operating joint yard and terminals - Dr...	1.332
78. Operating joint yard and terminals - Cr...	-0.793
79. Motormen	0.048
80. Road enginemen	5.947
81. Enginehouse expenses - road	1.729
82. Fuel for road locomotives	9.423
83. Water for road locomotives	0.617
84. Lubricants for road locomotives	0.175
85. Other expenses for road locomotives	0.191
86. Operating power plants	0.058
87. Purchased power	0.048
88. Road trainmen	6.415
89. Train supplies and expenses	1.843
90. Interlockers and block and other signals - operation	0.532
91. Crossing flagmen and gatemen	0.360
92. Drawbridge operation	0.049
93. Clearing wrecks	0.267
94. Telegraph and telephone-operation	0.296
95. Operating floating equipment	0.145
96. Express service	0.002
97. Stationary and printing	0.425
98. Other expenses	0.129
99. Loss and damages - freight	1.555
100. Loss and damage - baggage	0.014
101. Damage to property	0.204
102. Damage to stock on right of way	0.190
103. Injuries to persons	1.233
104.Operating joint track and facilities - dr.	0.284
105. Operating joint tracks and facilities - Cr.	-0.257
Total - Transportation expenses	50.192

V. General expenses:	
106. Salaries and expenses of general offices	0.481
107. Salaries and expenses of clerks and attendants	1.542
108. General office supplies and expenses	0.182
109. Law expenses	0.564
110. Insurance	0.350
111. Relief department expenses	0.034
112. Pensions	0.174
113. Stationary and printing	0.159
114. Other expenses	0.200
115. General administration joint tracks, yards, and terminals - Dr....	0.015
116. General administration joint tracks, yards, and terminals - Cr....	-0.015
Total - General expenses	3.717
Total operating expenses	100.000

work at higher speed and consequently the wear of cylinders, pistonrings, etc, may be a little greater; but this is likely to be offset by less trouble with main rods, pins, etc. Similar study shows that not every item of operating expense is affected by train load. It does affect radically the following four items:

Item 80.	Road enginemen	5.947%
" 82.	Fuel for road locomotives	9.423%
" 83.	Water for road locomotives ...	0.617%
" 88.	Road trainmen	6.415%

That these four items are very sensitive to the influence of train load will be shown presently. It may be added here that the foregoing analysis indicates that the minimum ton-mile cost is obtained when the sum of these four variable items is a minimum.

The wages for train crews can be easily calculated from the contract between a railroad and its employees, when the exact time of service is known. At present the Illinois Central Railroad Company is paying the train crews of Mikado locomotives at the following rates:

Engineer	.... 64.25	cents per hour
Fireman	.... 36.25	" " "
Conductor	...... 52.25	" " "
Flagman	.... 34.75	" " "
Brakeman	.... 34.75	" " "

Each man of the train crew is entitled to ten hours' pay when he makes 100 miles, even though the actual time spent is less than eight hours. If he makes more than 100 miles he is paid in proportion to the mileage. When he works over eight hours the overtime is paid at the rate of 150 percent of the regular hourly pay.

The expenditure for road locomotive coal, which is the greatest of the 116 items, can be estimated by the method described in the preceding section. The item for water is very small compared with the other three items which vary with train load, and can be assumed to be proportional to the coal consumption.

As an illustration of the new method let us consider that, as in the previous example, a Mikado locomotive is to be run on the division whose profile is as shown in Fig. 96a, and that the train load which will result in minimum ton-mile cost is to be determined. From the run curves in Fig. 96, the time during which coal is consumed at various rates for various train loads is found to be as shown in the following table. Knowing the rate of coal consumption and the time of firing for each stretch of the track, coal consumption on each stretch and also for each trip is easily calculated. The results are shown in Table I. In this table, the actual coal consumption is assumed to be 115 percent of the coal consumed while running. The coal consumed for firing up and that used while on passing tracks is estimated to vary from 10 to 20 percent and 15 percent is regarded as a probable average value. The time consumed in taking coal and water and on passing tracks is assumed to be 20 percent of the time the train is actually running.

TABLE I.

Run	Stretch of Track	Coal Consumption per min. lbs.	1000-Ton Train: Coal Consuming Time in Min.	1000-Ton Train: Coal Consumption lb.	1500-Ton Train: Coal Consuming Time in Min.	1500-Ton Train: Coal Consumption lb.	2000-Ton Train: Coal Consuming Time in Min.	2000-Ton Train: Coal Consumption lb.	2500-Ton Train: Coal Consuming Time in Min.	2500-Ton Train: Coal Consumption lb.	3000-Ton Train: Coal Consuming Time in Min.	3000-Ton Train: Coal Consumption lb.
A	Aa_1	66.7	17.2	1148	20.2	1346	22.2	1480	24.6	1640	26.8	1788
to	a_1a_2	83.3	9.7	810	11.8	985	14.8	1235	19.8	1650	26.0	2170
B	a_2B	66.7	8.3	554	9.6	640	11.3	754	12.1	806	13.8	920
B	Bb_1	66.7	9.4	627	10.9	728	12.2	815	13.7	914	14.9	995
to	b_1b_2	83.3	19.8	1650	24.3	2028	30.6	2550	41.1	3425	56.0	4665
C	b_2C	66.7	8.1	540	9.4	627	11.0	733	11.8	787	13.6	906
C												
to	CD	83.3	40.7	3390	50.8	4240	64.3	5370	81.2	6760	12.2	10180
D												
D	Dd_1	66.7	9.4	627	10.8	728	12.2	815	13.6	913	14.8	995
to	d_1d_2	66.7	0.2	13	0.8	54	1.0	67	1.4	93	1.5	100
E	d_2E	66.7	15.6	1040	18.4	1230	20.8	1365	21.4	1430	22.4	1490
E	Ee_1	66.7	2.0	133	2.2	163	2.6	173	2.8	1870	3.0	200
to	e_1F	66.7	15.7	1080	18.5	1230	20.5	1365	21.5	1430	22.4	1490
F												
F												
to	FG	66.7	33.2	1215	39.0	2600	43.3	2895	47.1	3140	50.7	3380
G												

	1000-Ton Train	1500-Ton Train	2000-Ton Train	2500-Ton Train	3000-Ton Train
Coal for Running, lbs. per trip	12,785	16,589	19,617	23,174	29,279
Actual coal Consumption, lbs. per trip	14,800	19,050	22,550	26,600	33,650
Time for actual Running, minutes	213.3	250.2	288.5	334.0	409.6
Actual Time on Road, hours, per trip	4.26	5.00	5.77	6.68	8.19
Average Speed, m.p.h.	28.2	24.0	20.8	18.0	14.8
Speed on Ruling Grade, m.p.h.	30.0	24.0	19.0	14.0	10.0

With these data, the values in Table II have been calculated. Items 1 and 4 in Table II have been computed from the rates mentioned before. The train wages per trip for the trains with 1000, 1500, 2000, and 2500 tons are the same since the trips are made within eight hours. That for the 3000 ton train is greater because of overtime. The price of coal has been assumed to be \$2.00 per ton. The cost of water is assumed to be 6.5% of the fuel cost. This cost can be more carefully estimated but it would not be profitable to spend much time in making a more accurete estimate since the influence of the cost for water on this problem is very slight.

TABLE II.

Item	1000	1500	2000	2500	3000
1. Road enginemen, per trip	\$12.05	\$12.05	\$12.05	\$12.05	\$12.55
2. Fuel for road locomotives	14.80	19.05	22.55	26.60	33.65
3. Water for road locomotives	0.98	1.25	1.48	1.75	2.22
4. Road trainmen	14.50	14.50	14.50	14.50	15.10
5. Total Cost per trip	42.38	46.85	50.58	54.90	63.62
6. Thousand ton-miles per trip	120	180	240	300	360
7. Cost per 1000 ton-miles	0.353	0.260	0.211	0.183	0.177

Item 5 gives the total cost of coal, water, and train wages per trip for trains of various tonnages. Item 6 has been obtained by simply multiplying the length of the division, 120 miles, into the tonnage. Item 7, shows as the result of the computations the costs per 1000 ton-miles for various trains, which are plotted against the train loads and shown in Fig. 98. The diagram indicates that with the particular locomotive and the profile used in our example, the traffic can be transported most economically or with minimum ton-mile cost when the locomotive is rated at 3000 tons. There is some indication on the curve that still heavier trains can be more economically operated; but this would not be true since the speed of a 3000 ton train on the ruling grade is 10 m.p.h. as shown in the Table I. The example has been solved on the assumption that the average car weight is 50 tons. In actual ratings proper correction should be made for trains with other average car weights.

A comparison of the result of our example and that of Mr. Mott obtained from statistics covering a long period is interesting. As regards coal costs he arrived at the conclusion that the train load that gives an average speed of 22 m.p.h. is the most economical. We find that a speed between 18 and 20 is the best. This difference is slight and is due to the fact that the ruling grade in our example is 1/2 percent while in Mr. Mott's it is slightly lower. He found that 100 percent train loading gave minimum ton-mile costs of coal, water, and train wages. It is not clear what he means by "100 percent loading", but if it means the tonnage which gives a speed of about 10 m.p.h. on ruling grades, our result checks perfectly with his result, obtained from statistics of actual operation under similar conditions.

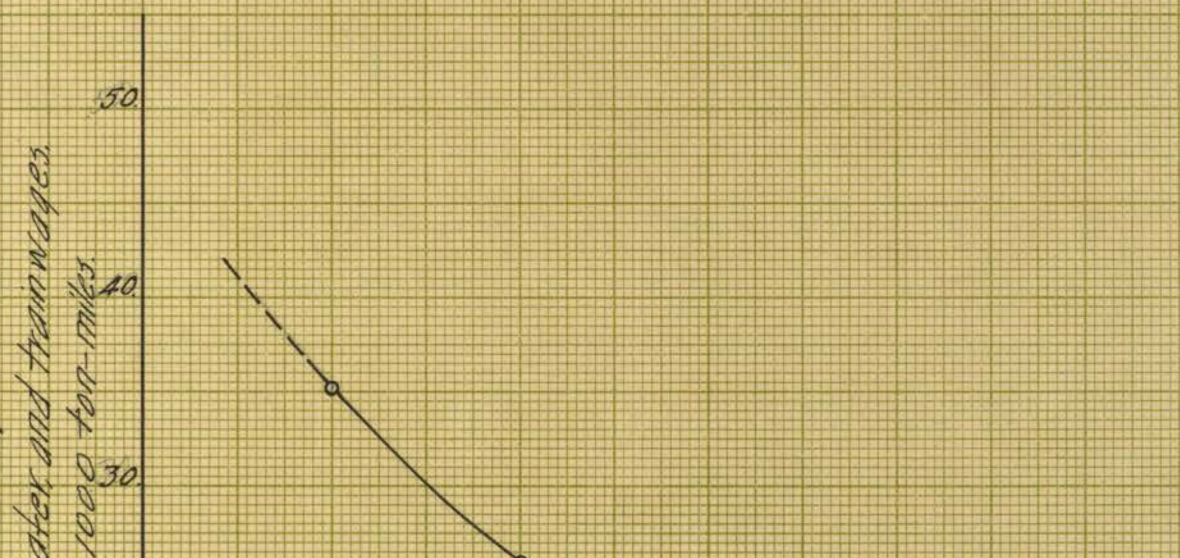
Cost of fuel, water, and train wages.
Cents per 1000 ton-miles.
50.
40.
30.
20.
10.
0.
1000.
1500.
2000.
2500.
3000.
Train load behind tender, gross ton.

Fig. 98

D. Selection of Locomotives for a Particular Service.

In his paper read before the American Society of Mechanical Engineers, Mr. Beyer says, "To make an intelligent selection of motive power for a railroad, it is necessary to study the effect which various types and sizes of locomotives will have on operating expenses and fixed charges." * In the days when there was insufficient data it was not possible to make a careful investigation of this phase of the problem, but it has been the opinion of far-sighted engineers for some time that the influence of different types and designs of locomotives on operating expenses and capital investment should be carefully studied by all possible means before a definite type of locomotive is purchased or assigned to a particular duty on a certain division.

The factors to be considered in a selection of motive power are numerous and every factor related to this problem which has any influence on successful train operation should be considered. Every type and design of locomotive has certain merits and demerits for a particular kind of service. It is not, therefore, difficult to select two or three prospective types for a contemplated duty. For instance, if the locomotive is to haul heavy freight trains on a moderate grade division, only heavy consolidation, Mikado, or some Mallet type will answer; and after some consideration one of the types, say Consolidation, will be decided upon.

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* O.S.Beyer, Jr.: "Factors in the selection of locomotives in relation to the economics of railway operation". Tran. A.S.M.E. vol. page

This is a simple method, but it may not be the most satisfactory method, for the reason that if for instance in the above case, the Mikado type were adopted instead of the Consolidation, it might give more satisfactory results in operation, and perhaps the Mallet type would give still better results than the other two. The operating results of the three types in this particular case cannot be known until they are actually tried separately on the road for a considerable period of time, but can be estimated very closely by the method which is described in the following paragraphs.

As has been stated; in a proper selection of locomotives, the merits and demerits of various types for a particular service must be thoroughly understood. For instance:

1. Large diameter of driving wheels. - Locomotives with large drivers produce high speed and low tractive effort, and are suitable, therefore, for passenger and fast freight trains.
2. Small driving wheels. - Locomotives with small driving wheels generally produce high tractive effort and low speed, therefore they are suitable for freight and switching service.
3. Four-wheel leading truck. - Locomotives with swiveling truck are more safely operated at high speed, and their adhesive tractive effort is relatively low. They are therefore suitable for high speed passenger trains.
4. Two-wheel leading truck. - Locomotives with a 2-wheel leading truck have higher adhesive tractive effort, but they are not safe for very high speed service. They are therefore suitable for freight train service.

5. Locomotives without leading trucks have high adhesive tractive effort, but are not safe for road service; they are, therefore suitable for switching service in yards.
6. Two-wheel trailing trucks make it possible to build locomotives with large fire-box and boiler. Such locomotives have high evaporative capacity and are therefore suitable for through trains in both passenger and freight service.
7. Locomotives without trailing truck on the contrary have low evaporative capacity, and are not suitable for through trains, although they can be used for this service with certain disadvantages.
8. Locomotives having a high percentage of their total weight on the drivers have high adhesive tractive effort.
9. Locomotives with large number of driving wheels or long rigid wheel base are not suitable for sharp curves.
10. Locomotives with large cylinders are capable of producing high rates of acceleration but low maximum velocity.
11. Locomotives with large boilers are capable of producing relatively low rates of acceleration but high maximum velocity.
12. Boiler pressures between 190 and 210 lbs. per sq. in. for locomotives using saturated steam and 170 to 200 lbs. per sq. in. for locomotives using superheated steam are found to be economical and practical.
13. Superheaters reduce fuel and water consumption from 10 to 20 percent and are highly recommended for through train service.
14. Mechanical stokers increase the capacity of locomotives and in a certain sense reduce the expense for the train crew.

Classification of Locomotives

Type of Locomotive		For Passenger Service									For Freight Service						For Pusher Service	For Switching	
		On Low Grade Division			On Heavy Grade Division			On Mountain Grade Div.			Low grade		Heavy grade		Mountain grade				
Name	Symbol	Through High Speed Heavy	Express	Local	Through	Express	Local	Through	Express	Local	Fast	Slow	Fast	Slow	Fast	Slow			
American	4-4-0		B	A			B			B									
Atlantic	4-4-2	B	A	B		A	B		B	A									
Ten wheeler	4-6-0	B	A	B	B	A	C	B	A	B									
Pacific	4-6-2	A	A	C	A	A	B	B	B	B									
Twelve wheeler	4-8-0	B	A	C	B	A	C	A	A	B									
Mountain	4-8-2	C			A	B	C	A	A	B			C		A	C		B	
Mogul	2-6-0								B	B	B	B	B	B	B	B	A	B	
Prarie	2-6-2								B	B	B	B	A	B	A	B	A	B	
Consolidation	2-8-0							B	A	B	A	A	A	A	A	A	A	B	
Mikado	2-8-2							A	B	C	A	A	A	A	A	A	A	B	
Decapod	2-10-0							B	C	C	C	B	C	B	C	A	A	B	
Santa Fè	2-10-2							A	B	C	C	C	B	B	B	B	A	B	
Mallet								A	B	C			B	A	B	A.	A		
Erie Triplex	2-8-8-8-2																A		
Six-wheel Switcher	0-6-0																A	A	
Eight-wheel Switcher	0-8-0																A	A	
Ten-wheel Switcher	0-10-0																A	A	

Note: "A" indicates type best suited for the service.
"B" indicates type not best suited for the service but can be employed fairly satisfactorily.
"C" indicates type not well fit for the service but can be used if necessary.

15. Brick arches improve combustion of long flame coals and show an economy of about 5 percent.
16. Combustion chambers and automatic fire-doors, etc. have certain merits.

With definite knowledge of the items outlined above and numerous other details, a table like the following may be produced. From this table two or three types of locomotives can be selected for further comparative investigation for a particular service.

Let us suppose as an example, that the profile of the division to which the locomotives under consideration are to be assigned is as shown in Fig. 69a, and that after a careful consideration three prospective types, viz: a heavy consolidation, a Mikado, and a certain Mallet, whose speed-pull relations are as shown in Fig. 101, are selected for further consideration. The next step, then, is to study the effect of each of the three types on operating expenses. In this problem, unlike the tonnage rating problem there is a radical difference in the weight of locomotives, train loads, etc., and consequently, many items of operating expense are affected. Not only the operating expense but also additional capital investment may be necessary. For instance, if a Mallet type is finally adopted it may be necessary to rebuild bridges, to increase the length of yard tracks and passing sidings, to remodel repair shops and equip them with larger tools suitable for economical maintenance of the Mallet, etc. Assuming that the fixed charges and taxes on the investment are correctly estimated, and that track facilities and equipment are adequate for safe operation of any locomotives,

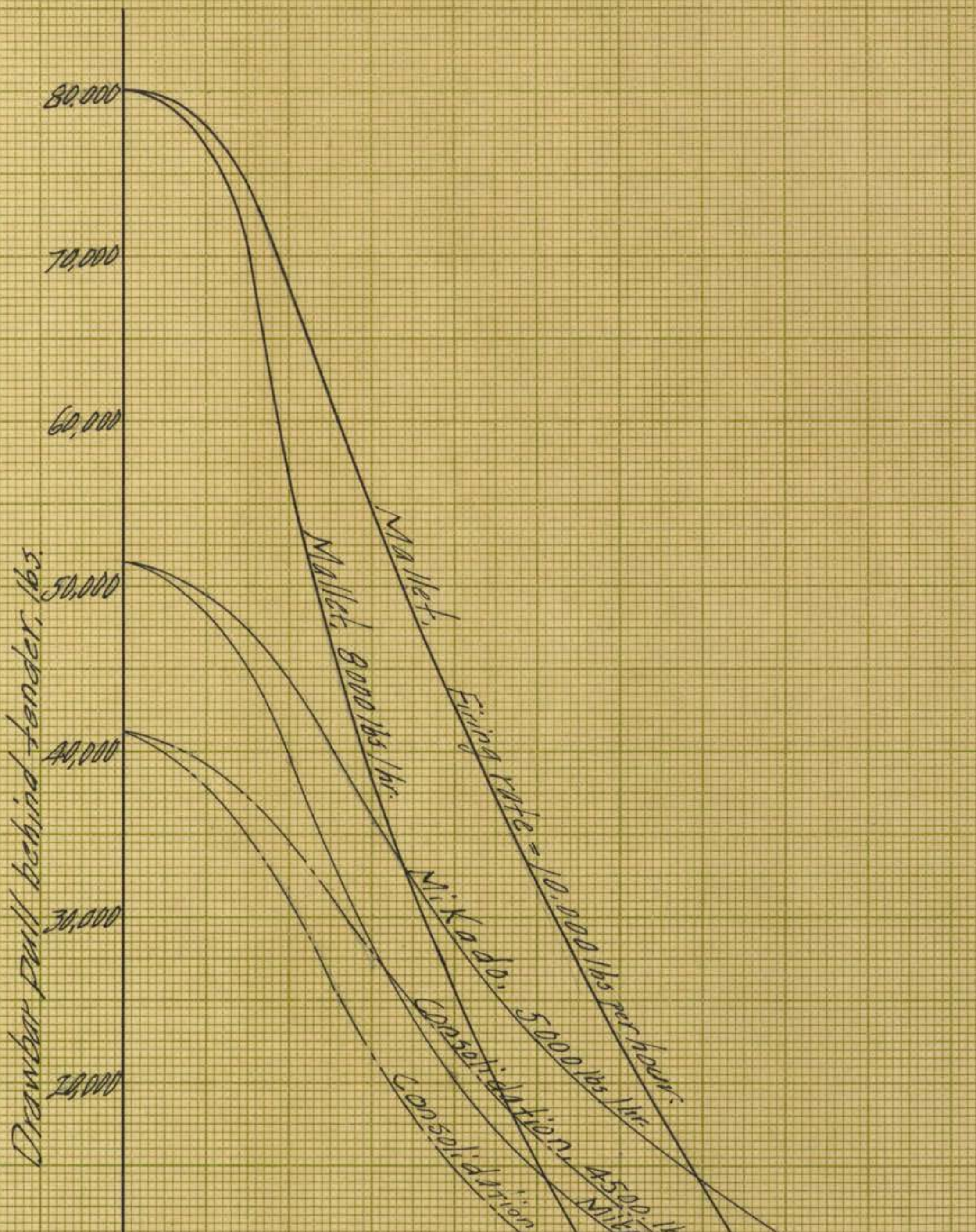
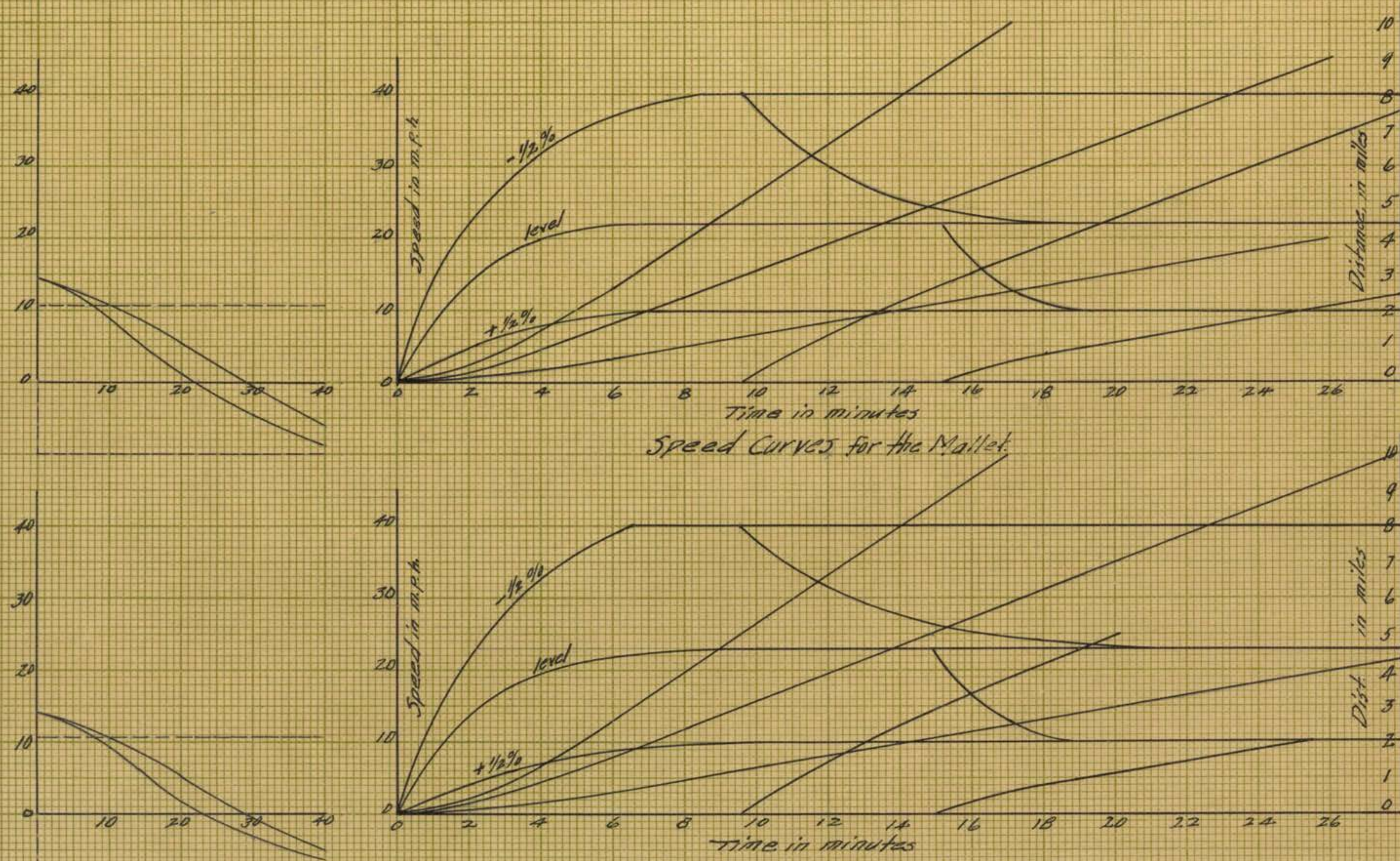
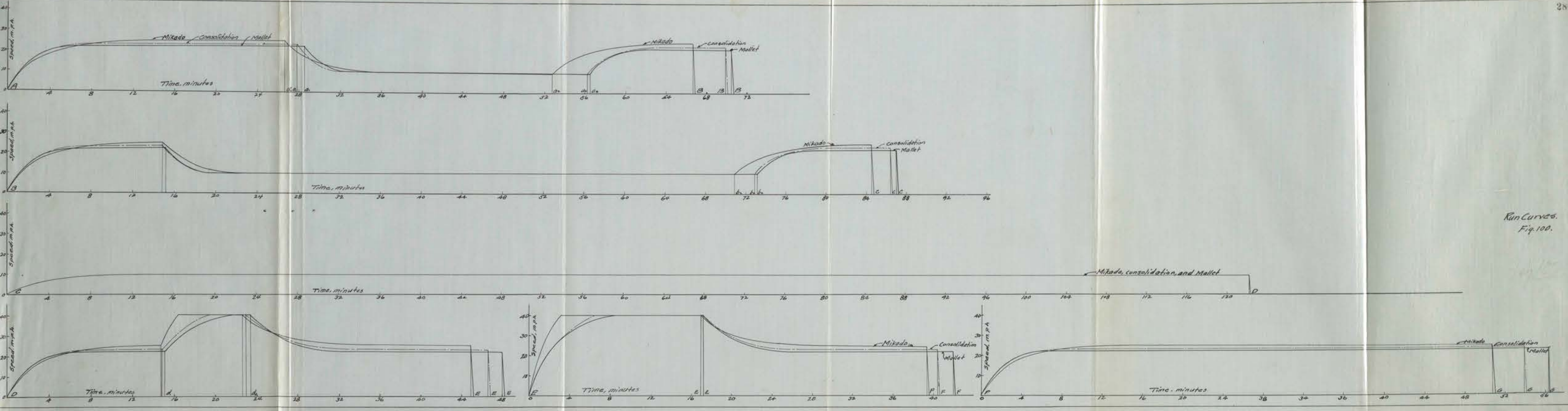

80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
Drawbar pull behind tender, lbs.
0
10
20
30
40
50
Speed, m.p.h.
Mallet, Firing rate = 10,000 lbs per hour.
Mallet, 8000 lbs/hr.
Mikado, 5000 lbs/hr.
Consolidation, 4500 lbs/hr.
Mikado, 4000 lbs/hr
Consolidation, 3300 lbs/hr.

Fig. 101



Speed in m.p.h.
-1/2%
level
+1/2%
Distance in miles
Time in minutes
Speed Curves for the Mallet.
Speed in m.p.h.
-1/2%
level
+1/2%
Dist in miles
Time in minutes
Speed curves for the Consolidation

Fig. 102.



Run Curves.
Fig. 100.

we will consider the effect of trains with the different types of locomotives on the following four items:

Item 80.	Road enginemen	5.947%
Item 82.	Fuel for road locomotives	9.423%
Item 83.	Water for road locomotives	0.617%
Item 88.	Road trainmen	6.415%

In order to estimate the expense of fuel, water, and train wages, which are the governing factors of total operating expense in our particular problem, the run curves, which are shown in Fig. 100 were produced as in the problem of the preceding section, assuming that the most economical train load for each type of locomotive is one which gives a minimum speed of 10 m.p.h. on the ruling grade.* From these curves the running time for each locomotive is found as shown in Item 2, Table I.

TABLE I.

	Consolidation	Mikado	Mallet
1. Train load behind the tender, tons	2350	3000	4600
2. Running time, minutes:			
From A to B,	69.8	66.8	70.4
B to C,	86.6	84.8	87.2
C to D,	122.1	122.1	122.1
D to E,	46.9	45.3	48.3
E to F,	40.6	39.6	42.2
F to G.	54.1	51.0	56.4
Total from A to G	420.1	409.6	426.6
3. Time on the road per trip, Hours	8.40	8.19	8.53
4. Train wages	\$28.42	\$27.65	\$35.98

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* In an actual problem this point must be carefully investigated by the method described in the preceding section. To avoid repetition here it is simply assumed that the train load which will give a minimum speed of 10 m.p.h. on the ruling grade is the most economical tonnage.

The time on the road given in Item 3 on the table is assumed to be 120 percent of the time required in actual running. The train wages in Item 4 are calculated from the values in Item 3 and the following rate:

	Consolidation	Mikado	Mallet
Engineer	1 @ 64.00¢	1 @ 64.25¢	1 @ 75.00¢
Fireman	1 @ 36.00¢	1 @ 36.25¢	2 @ 36.25¢
Conductor	1 @ 52.75¢	1 @ 52.75¢	1 @ 52.75¢
Flagman	1 @ 34.75¢	1 @ 34.75¢	1 @ 34.75¢
Brakeman	1 @ 34.75¢	1 @ 34.75¢	1 @ 34.75¢

From the run curves in Fig. 100 the time during which the locomotives consume coal for actual running is found as shown in columns 3, 6 and 9, Table II. The values in columns 4, 7 and 10 are the rates of coal consumption per minute in pounds.* The values in columns 5, 8, and 11, are obtained by simply multiplying the values in columns 3 and 4, 6 and 7, and 9 and 10 respectively. Item 2, at the bottom of Table II is the summation of the values in columns 5, 9 and 11. Item 3 is obtained by assuming that the total coal consumption per trip is 115 percent of the coal used for actual running. Item 4 is computed from the values in Item 3 assuming that the price of coal is $2.00 per ton on the tender. Item 5, the cost of water per trip, is obtained by the assumption that it is 6.5 percent of the cost for fuel.

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* The run curves in Fig. 100 have been produced under the following condition of coal consumption:

	Coal Consumption, pounds per hour.		
	Consolidation	Mikado	Mallet
On up-grades	4500	5000	10000
On level	3500	4000	8000
On down-grades (until it reaches 40 m.p.h.)	3500	4000	8000

TABLE II.

		CONSOLIDATION			MIKADO			MALLET		
		Time during which coal is consumed for running. Min.	Coal Consumption per Min. Lb.	Total Coal Consumption Lb.	Time during which coal is consumed for running. Min.	Coal Consumption per Min. Lb.	Total Coal Consumption Lb.	Time during which coal is consumed for running. Min.	Coal Consumption per Min. Lb.	Total Coal Consumption Lb.
1	2	3	4	5	6	7	8	9	10	11
	Aa_1	27.9	58.3	1615	26.8	66.7	1788	28.6	133	3805
AB	$a_1 a_2$	28.4	75.0	2130	26.0	83.3	2170	28.0	167	4670
	$a_2 B$	13.7	58.3	800	13.8	66.7	920	13.9	133	1850
	Bb_1	15.0	58.3	874	14.9	66.7	995	15.2	133	2020
BC	$b_1 b_2$	58.0	75.0	4350	56.0	83.3	4665	58.	167	9660
	$b_2 c$	13.5	58.3	788	13.6	66.7	906	13.9	133	1850
CD	CD	122.0	75.0	9150	122.0	83.3	10,180	122.0	167	20,350
	Dd_1	14.9	58.3	868	14.8	66.7	995	152.	133	2023
DE	$d_1 d_2$	4.2	58.3	245	1.5	66.7	100	6.6	133	880
	$d_2 E$	27.4	58.3	1595	22.4	66.7	1490	26.4	133	3520
	Ee_1	6.5	58.3	379	3.0	66.7	200	8.4	133	1120
EF	$e_1 F$	23.4	58.3	1363	22.4	66.7	1490	25.0	133	3330
FG	FG	53.9	58.3	3140	40.7	66.7	3380	56.4	133	7500

	Consolidation	Mikado	Mallet
2. Coal consumption for running, lbs.	27,297	29,279	62,578
3. Actual coal consumption lbs. per trip, lb.	31,400	33,650	72,000
4. Cost of coal per trip			
5. Cost of water per trip			

From the values in items 1 and 4, Table I, and items 4 and 5, Table II, the values in the following table are obtained.

TABLE III.

	Consolidation	Mikado	Mallet
1. Train wages per trip	\$28.32	\$27.65	\$35.98
2. Cost of fuel and water per trip	33.40	35.89	76.50
3. Cost of fuel, water and train wages per trip	61.72	63.52	112.48
4. 1000-ton-miles per trip	282	360	552
5. Cost of fuel, water, and train wages per 1000 ton-miles	0.219	0.177	0.202

From the result obtained in this table it is clear that, in this example, the Mikado type requires the least expense for fuel, water, and train wages per ton-mile. As mentioned before, there are some other items of operating expense which are more or less affected by the operation of different types of locomotives, but the effect of their variation is not large enough to alter the result attained by studying the cost of fuel, water, and train wages. Therefore, it may be said that the least operating expense will be secured by the adoption of the Mikado type.

The next and final step in a proper selection of locomotives is to find out what amount of capital investment must be made for the necessary change in equipment, structures, track, etc. When the amount of annual fixed charge is estimated and the total ton-mileage per year to be made over the division is known, the fixed charge per ton-mile can be readily computed. Adding the fixed charge per ton-mile thus computed for each of the three types, to the cost of fuel, water, and train wages found above, we can readily find which of the three types of locomotives will make a unit transportation at minimum expense or at maximum economy, which leads to the final decision.

E. Railway Location and Relocation Problems.

It is needless to state here what influence gradient, distance, rise-and-fall, and curvature have on the economical operation of railroads. The enormous amount of capital that has been expended in this country in recent years on grade reductions, elimination of distance, etc., is conclusive evidence that such work brings about economies in operation.* Mechanically speaking, however, grade reduction is unnecessary work because any amount of traffic can be transported over any ordinary grade without reduction if sufficient motive power is available. But it is not a problem in mechanics. If the reduction in annual expenses for motive power is less than the annual interest on the expenditure for grade reduction, it is unwise to reduce the grade. If, however, the final expense for a unit transportation can be lowered by grade reduction, it would benefit the railroad and the public as well to make this reduction. Whether a particular grade reduction for instance, is a profitable undertaking or not, can not be determined unless a careful detailed study of the proposition is made.

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* "The Nicholson relocation of the Lackawanna Railroad", Eng. Record, vol. 67, page 461. (1913).
"A railroad through Great Salt Lake", Railway Age, March 15, 1901, page 208.
"The railroad through Salt Lake", Railway Age, Aug. 15, 1902, page 161.

Several methods for the investigation of such problems have been described by several prominent engineers and writers.* They differ in details but the fundamental principles are the same.

The solution of a grade reduction problem is essentially an estimation of the reduction in the number of trains per year or the train-mileage, and the consequent saving in operating expenses, from which a justifiable amount of capital expenditure for the reduction of the grade may be estimated. As we have seen in the foregoing sections, the speed curves are a very effective means for the determination of ton-mileages, train-mileages and an estimation of the cost of fuel, water, and train wages. These are very important items in problems of grade reduction.

The determination of train-mileage by means of speed curves, is easy and the result is accurate. It may not be neccessary to repeat here the description of the method.

It may be pointed out here that according to the estimation of A. M. Wellington** 47.8 percent of the operating cost is affected by train-mileage and about 21. percent of this

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* J. B. Berry: "Reduction of gradient, and elemination of distance, curvature, and rise and fall on Union Pacific Railroad", Proc. Am. Ry. E. & M.W.A., vol. 5, p. 689-719, 1905.
A. K. Shurtleff: "Time as an element in considering grade reduction", Proc. A.R.E. & M.W.A., vol. 9, p. 775-798, 1909.
C. P. Howard: "Grade Reduction Problems ", Bull. Am. Ry. Eng. Assoc., No. 138, August 1913.

** A. M. Wellington: "Economic Theory of Railway Location", page 571.

47.8 percent is due to the cost of fuel, water and train wages. A similar estimation by Mr. Ray* shows that 33.67 percent of the annual operating expenses varies directly with train-mileage and that 22.73 percent of this 33.67 percent is due to the cost of fuel, water and train wages. These estimations indicate clearly the importance of these items in the study of grade reduction problems. As we have seen, the cost of fuel, water, and train wages per ton-mile or per train-mile can be accurately estimated by means of speed curves. When this cost is accurately determined, the percentage of operating expense which is affected by train-mileage can be more accurately ascertained.

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* G.J.Ray: "Relocation of a portion of the Delaware, Lackawanna and Western Railroad main line", a thesis for C.E., University of Illinois, 1910.

APPENDIX.

TABLE I-A.

Drawbar Pull behind Tender of Freight Locomotives using Superheated Steam.

Speed m.p.h.	L1s	L1s*	H9s	"H8sb"	H8sb	H6sb
0.0	61,400	59,100	49,500	51,500	45,750	41,250
2.5	61,400	59,100	49,200	51,500	44,200	20,500
5.0	61,400	59,100	48,250	51,500	43,600	39,300
7.5	61,200	58,800	46,700	50,000	42,750	37,300
10.0	59,000	57,500	47,600	46,000	40,300	34,300
12.5	56,000	55,300	41,800	41,200	36,800	30,700
15.0	52,520	52,500	38,200	37,600	33,400	27,300
17.5	48,750	49,200	34,750	34,200	30,200	24,400
20.0	45,300	45,500	31,400	30,600	27,400	21,800
25.0	38,000	38,000	25,400	24,600	22,400	17,500
30.0	32,400	32,300		20,000		
35.0	27,800			16,500		

L1s - Pennsylvania Railroad Co. Test Dept. Bulletin 26, p. 36

L1s* - Penn. Railroad Co. Test Dept. Bulletin 28, p. 79.

H9s - Penn. Railroad.Co. Test Dept. Bulletin 26, p. 36.

"H8sb"- Penn. Railroad Co. Test Dept. Bulletin 10, p. 93.

H8sb - Penn. Railroad Co. Test Dept. Bulletin 26, p. 36.

H6sb - Penn. Railroad Co. Test Dept. Bulletin 26, p. 36.

TABLE I-B.

Drawbar Pull behind Tender of Passenger Locomotives using Superheated Steam.

Speed m.p.h.	K4s	K29s	K2sa	E6s-51	E6s-89	E3sd
0.0	45,500	44,800	33,400	31,000	28,100	28,100
5.	45,500	44,700	33,400	31,000	28,100	27,100
10.	45,500	43,000	28,900	31,000	27,600	25,200
15.	45,300	38,700	27,800	30,700	26,500	23,200
20.	43,800	34,400	26,600	29,800	25,000	21,200
25.	38,000	30,200	25,300	27,800	22,700	19,200
30.	33,700	26,400	23,800	24,300	20,300	17,500
35.	30,200	23,000	21,800	21,800	18,300	15,800
40.	27,400	20,200	19,600	20,000	16,500	14,300
50.	22,600	16,200	16,200	16,500	14,000	11,400
60.	18,500	13,200	13,500	13,700	12,000	9,000
70.	14,700	10,600	11,500	11,200	10,200	7,200
80.	11,000	8,200	9,600	9,000	8,400	
85.	9,200	7,000	8,800	7,950	7,600	

K4s - Penn. R.R. Co. Test Department Bulletin 29, p. 70

K29s - " " " " " " 19, p. 120

K2sa - " " " " " " 18, p. 122

E6s-51 - Pa. R.R. Co. Test Department Bulletin 27, p. 78

E6s-89 - " " " " " " 21, p. 134

E3sd - " " " " " " 11, p. 106

TABLE I-C.

Drawbar Pull behind Tender of Freight Locomotives using Saturated Steam.

Speed m.p.h.	H8b	H6b	H3
0.0	45,750	41,250	23,600
2.5	43,700	40,500	23,300
5.0	42,800	38,600	22,500
7.5	40,300	35,400	21,150
10.0	37,300	31,100	19,150
12.5	33,200	27,000	16,800
15.0	30,000	23,400	14,600
17.5	26,700	20,300	12,500
20.0	24,000	17,750	
22.5	19,200	13,400	
25.0		10,000	
30.0			
35.0			

H8b - Pa. RR.Co. Test Dept. Bulletin 26, p. 36

H6b - " " " " " " 26, p. 36

H3 - " " " " " " 26, p. 36

TABLE I-D.

Drawbar Pull behind Tender of Passenger Locomotives using Saturated Steam.

Speed m.p.h.	K2	E6	E2d
0	33,400	28,100	24,300
5	33,400	28,100	24,300
10	33,100	27,200	24,400
15	31,200	25,200	21,400
20	28,000	23,000	18,900
25	24,800	20,600	16,600
30	22,000	18,500	14,500
35	19,350	16,400	12,600
40	17,000	14,300	10,900
50		10,400	8,400
60			
70			
80			
85			

K2 - Pa. R.R. Co. Test Dept. Bulletin 18, p. 122

E6 - " " " " " " 21, p. 138

E2d - " " " " " " 11, p. 109

TABLE II-A.

Data of the Diagrams in Figs. 27 & 29.

	L1s	H9s or "H8sb"	H8sb	H6sb
1. Boiler Pres. lb/sq.in.	205	205	205	205
2. Stroke, in l	30	28	28	28
3. Diameter of cylinder, d	27	25	24	22
4. Dia. of Drivers, in. D	62	62	62	56
5. Speed in m.p.h. of 1000 f.p.m. Piston Speed	36.9	39.5	39.5	35.7
6. To = pld^2/D	72,300	57,800	53,400	49,500
7. .85To = Tao	61,450	49,100	45,100	42,100
8. .8To = Tx	57,900	46,250	42,650	39,500
9.* Tbo	89,000	68,000	60,500	54,000
10. Tbo/To	1.24	1.175	1.135	1.09
11. dl	810	700	672	611
12.* Tbooo	25,200	14,000		
13. Tbooo/Tbo	.2815	.206		
14.** Eq. Evap. lb/hr.,E	76,000	45,000		
15. Pl^2d^2 in 1,000,000	134.2	100.0		
16. Pl^2d^2/E in 1,000	1.77	2.22		

Note:- To is the "Theoretical cylinder tractive effort", pld^2/D; Tbo is the "Theoretical boiler drawbar pull at zero speed; and Tbooo drawbar pull at 1000 f.p.m. piston speed, See Fig. 26.

* The values of Tbo and Tbooo are taken from the diagrams in Figs. 9 - 18.

** The values of E are taken from Tables III-A and III-B.

TABLE II-A (continued).

Data of the Diagrams in Figs. 27 & 29.

Item	K4s	K29	K2sa	E6s-51	E6s-89	E3sd
1.	205	200	205	205	205	205
2.	28	28	26	26	26	26
3.	27	27	24	23½	22	22
4.	80	80	80	80	80	80
5.	51.08		55.0	55.0	55.0	55.0
6.	52,300	51,000	38,400	36,800	32,200	32,200
7.	44,450	43,350	32,640	31,300	27,400	27,400
8.	41,800	40,800	30,703	29,450	25,800	25,800
9.	63,000	60,500	42,000	40,500	34,500	34,000
10.	1.200	1.187	1.095	1.10	1.071	1.055
11.	755	755	625	610	570	570
12.	22,000	16,500	15,300	14,700	12,800	10,100
13.	.349	.273	.365	.362	.372	.315
14.	84,000	66,000	63,500	57,500	53,000	44,000
15.	117.0	114.1	79.8	76.3	66.9	66.9
16.	1.395	1.78	1.258	1.33	1.26	1.52

TABLE II-B.

Data of the Diagrams in Figs. 28, 30, and 31.

	H8b	H6b	H3	K2	E6	E2d
1. Boiler Press. lb. per sq. in.	205	205	140	205	205	205
2. Stroke, in. l	28	28	24	26	26	26
3. Diameter of cylinder in. d	24	22	20	24	22	$20\frac{1}{2}$
4. Diameter drivers, in.D	62	56	50	80	80	80
5. Speed in m.p.h. of 1000 f.p.m. piston speed	39.5	35.7	37.2	55.0	55.0	55.0
6. pld^2/D = To	53400	49500	26900	38350	32200	28000
7. Tao = .85To	45100	42100	23870	32600	27350	23700
8. Tx = .8To	42650	39550	21500	30600	25750	22400
9.* Tbo	58000	55500	33000	43000	37500	33000
10. Tbo/To	1.087	1.123	1.225	1.122	1.165	1.185
11. dl	672	611	480	625	371	533
12.* Tbooo	10300	7200		11800	10000	7300
13. Tbooo/Tbo	0.178	0.130		0.274	0.264	0.224
14.** Equiv. Evap. lb. per hr., E	48500	35600		65000	53000	41500
15. Pl^2d^2/E in 1000	1.89	2.18		1.215	1.263	1.40
16. Pd^2/E	2.415	2.78		1.80	1.87	2.07

* The values of Tbo and Tbooo are taken from the diagram in Figure 19 to 24 inclusive.

** The values of E are taken from Tables III-A and III-B.

TABLE III-A.

Data of the Item 14 of the Tables II-A and II-B.

Loco-mo-tive.	Test No.	Test Designa-tion	Speed, miles per hour.	Equiv. Evap. lb. per hr. Observ-ed.	Drawbar Pull lb. Observ-ed.	Drawbar Pull lb. on Curve	Equiv. Evap. lb. corrected for Draw-bar Pull on Curve.
L1s *	3969	100-80-F	17.9	74,669	47,471	48,600	76,800
	3931	120-75-F	22.0	75,988	42,151	42,151	75,988
	3930	140-70-F	25.6	77,017	37,036	37,036	77,017
	3927	160-65-F	29.3	76,461	32,877	32,877	76,461
	3928	170-60-F	31.1	72,940	30,347	31,200	75,000
						Average	76,000
"H8sb" ** or H9s	3247	40-88-F	7.19	39,050	49,872	50,300	39,400
	3244	60-86-F	10.78	44,456	44,284	44,000	44,500
	3241	80-63-F	14.44	45,256	37,502	39,000	47,100
	3217	120-50-F	21.56	43,306	37,184	28,750	46,100
	3235	160-40-F	28.75	43,086	19,924	20,800	45,100
	3224	170-35-F	30.50	40,581	17,835	19,500	45,100
						Average	45,000
H8b #			17.97	44,826	24,620	26,240	48,500
			21.56	46,100	21,408	22,400	48,500
						Average	48,500
H6b ##			16.71	37,213	23,085	21,500	35,600
						Average	35,600

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* Mikado type locomotive, Pa. R.R.Co. Test. Dept. Bulletin 28, page 41, 72.

** Consolidation locomotive, Pa. R.R.Co. Test Dept. Bulletin 10, page 38, 89.

Consolidation locomotive, Pa. R.R.Co. Test Dept. Bulletin 10, page 125, 129; Bulletin 26, page 36.

Consolidation locomotive, Pa. R.R. Co. Test Dept. Bulletin 12, page 23; Bulletin 26, page 36.

H8sb, H6sb, and H3, Consolidation locomotives. No data available for equivalent evaporation corresponding to the speed-pull curves.

TABLE III-B.

Data of the Item 14 of the Tables II-A and II-B.

Locomotive.	Test No.	Test Designation	Speed, miles per hour.	Equiv. Evap. lb. per hr. Observed.	Drawbar Pull lb. Observed.	Drawbar Pull lb. on Curve	Equiv. Evap. lb. corrected for Drawbar Pull on Curve
K4s*	4068	120-75-F	28.38	75,388	33,173	35,000	79,600
	4062	160-70-F	37.84	83,882	28,500	28,500	83,882
	4054	200-64-F	47.30	87,414	23,900	23,900	87,414
	4025	240-55-F	56.76	81,589	18,200	20,000	89,700
	4049	280-50-F	66.22	80,235	15,734	16,100	82,000
	4069	320-45-F	75.34	77,027	12,478	12,478	77,027
						Average	84,000
K29**	2460	180-45-F	42.66	64,204	18,876	18,900	64,204
	2448	240-45-F	56.86	72,229	12,855	14,100	79,300
	2440	280-35-F	66.25	62,250	10,794	11,500	66,300
	2445	320-35-F	76.71	65,088	9,136	65,088	65,088
	2451	360-25-F	85.32	66,187	6,146	7,100	77,600
						Average	66,000
K2sa#	3029	160-50-F	37.31	53,726	18,805	21,000	60,000
	3017	200-50-F	46.52	62,702	16,843	17,200	63,700
	3020	240-50-F	55.82	64,711	13,978	14,500	66,000
	3015	280-35-F	65.13	54,100	11,192	12,500	60,500
	3022	320-30-F	74.43	54,665	9,123	10,600	63,600
	3030	360-30-F	82.74	58,795	7,535	9,000	70,400
						Average	63,500
E6s-51ø	3849	160-50-F	37.5	52,711	19,410	20,800	56,600
	3848	200-50-F	46.9	58,611	17,293	17,293	58,611
	3810	240-45-F	56.4	55,231	13,691	14,700	59,200
	3811	280-40-F	65.8	52,659	11,695	12,350	55,700
	3838	320-35-F	75.0	55,561	10,129	10,129	55,561
						Average	57,500

* Pacific Type, Pa. R.R. Co. Test Dept. Bulletin No. 29, p. 34 and 69.

** Pacific Type, Pa. R.R. Co. Test Dept. Bulletin 19, p. 42, 121.

\# Pacific Type, Pa. R.R. Co. Test Dept. Bulletin 18, p. 44, 124.

ø Atlantic Type, Pa. R.R.Co. Test Dept. Bulletin 27, p. 35, 71.

TABLE III-B (continued)

Data of the Item 14 of the Tables II-A and II-B.

Loco-mo-tive.	Test No.	Test Designa-tion	Speed, miles per hour.	Equiv. Evap. lb. per hr. Observ-ed.	Drawbar Pull lb. Observ-ed	Drawbar Pull lb. on Curve	Equiv. Evap. lb. corrected for Draw-bar pull on Curve
E6s-89*	2808	160-50-F	37.7	48,997	17,013	17,013	48,991
	2809	200-50-F	47.1	51,095	14,685	14,685	51,095
	2825	240-38-F	56.5	49,433	10,614	12,400	57,600
	2820	280-35-F	65.7	51,963	10,231	11,000	35,700
	2823	320-25-F	75.0	46,863	7,356	8,700	55,400
	2839	360-30-F	84.4	51,241	7,685	7,700	51,241
						Average	53,000
E3sd**	3133	160-50-F	37.34	41,080	15,114	15,114	41,080
	3124	200-45-F	46.68	44,275	12,357	12,357	44,275
	3139	240-45-F	56.02	46,078	10,536	10,506	46,078
	3125	280-35-F	65.35	42,502	8,034	9,100	42,900
	3142	360-25-F	84.00	37,549	5,676	7,500	49,500
						Average	44,000
K2 #	2307	180-40-F	42.0	64,995	17,500	17,500	65,000
						Average	65,000
E6 ∅	2004	220-30-F	52.4	52,804	9,437	9,437	52,804
	2005	240-30-F	57.2	51,010	7,612	7,800	53,000
						Average	53,000
E2d ∅∅	1120	160-32-F	37.19	39,000	11,674	11,800	39,400
	1117	200-30-F	46.52	40,800	8,700	9,200	43,200
						Average	41,500

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* Atlantic Type, Pa. R.R. Co. Test Dept. Bulletin 21, p. 34, 41.

** Atlantic Type, Pa. R.R. Co. Test Dept. Bulletin 11, p. 40, 97.

Pacific Type, Pa. R.R. Co. Test Dept. Bulletin 18, p. 148, 152.

∅ Atlantic Type, Pa. R.R. Co. Test Dept. Bulletin 21, p. 157, 161.

∅∅ Atlantic Type, Pa. R.R. Co. Test Dept. Bulletin 11, p. 139, 140.

Equiv. Evaporation per hour in Locomotive Boilers at Different Speeds.
Fig. 44.

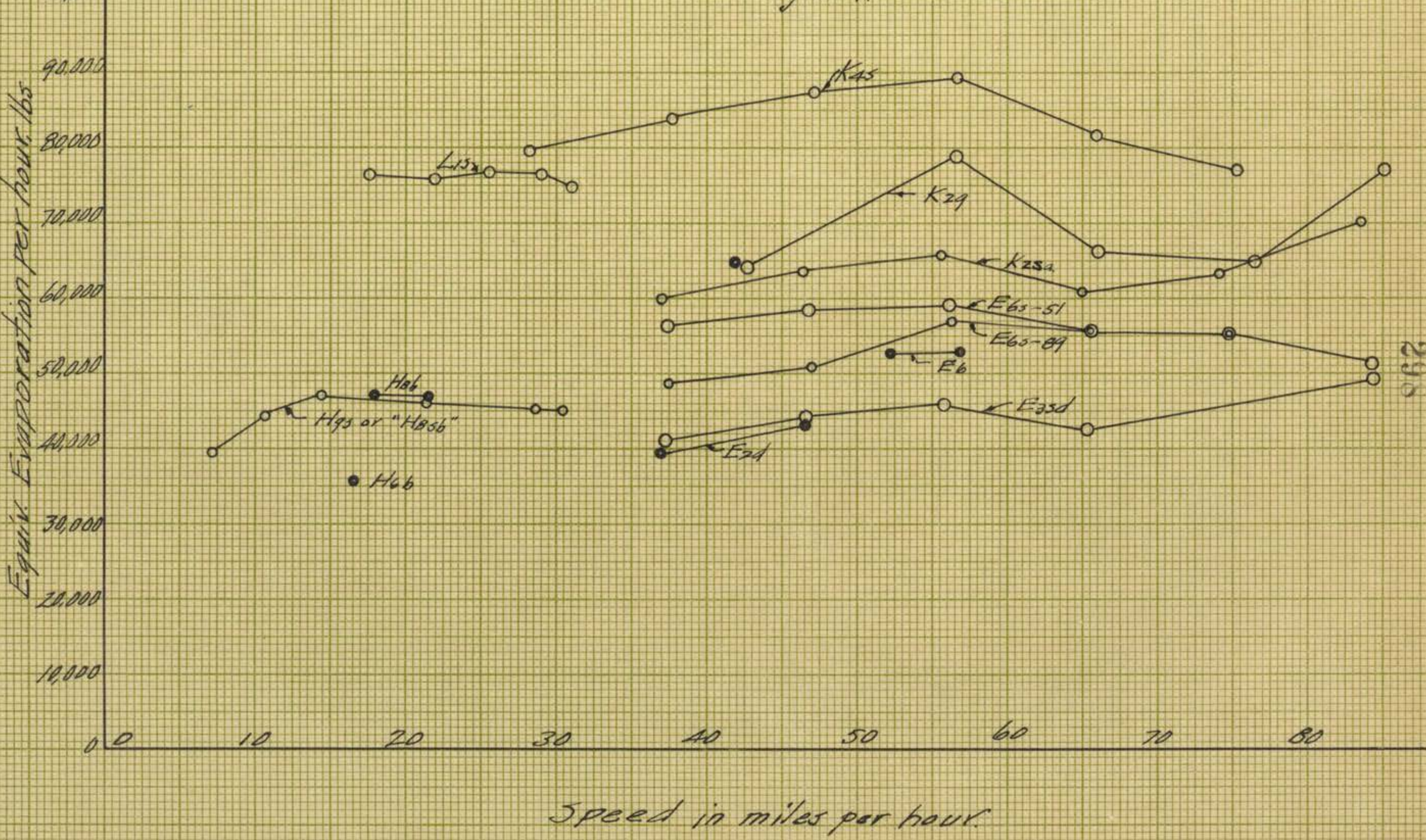


TABLE IV-A

U.S. Geological Survey Fuel Tests.
(U.S.G.S. Professional Paper 48, Part II, p. 946)

Name of State where coals were mined.	Number of Tests Made	Average Thermal Value per lb. of Dry Coal, B.t.u.	Average Equiv. Evap. per lb. Dry Coal, lbs.
Alabama	3	12,865	8.60
Arkansas	9	13,725	8.97
Colorado	1	12,577	7.21
Illinois	6	12,293	7.55
Indiana	3	13,155	8.40
Indian Territory	4	12,820	8.19
Iowa	6	11,800	7.23
Kansas	8	12,983	8.23
Kentucky	5	13,296	8.35
Missouri	8	12,169	7.60
New Mexico	3	12,093	7.00
North Dakota	1	10,402	5.40
Pennsylvania	4	14,298	9.17
Texas	1	10,886	4.69
West Virginia	14	14,466	9.72
Wyoming	2	11,607	6.46

TABLE IV-B.

Data from Bement's Fuel Test.
(Proc. Western Railway Club, vol. 21, 1908-1909, p. 227).

Percent of Ashes	Efficiency of Boiler percent	Thermal Value of Mixture B.t.u.	Equiv. Evap. per Pound of Mixture lb.
0	64	13,000	8.31
10	60	17,700	7.01
20	53.5	10,400	5.55
30	43	9,100	3.91
35	30	8,450	2.53
40	0	7,800	0

Note: The calorific value of No. 4 coal from Williamson County, Illinois, is 12,500 B.t.u. in average, and No. 4 washed coal, which Bement used, is assumed to have had 13,000 B.t.u., although definite data is not available. It is also assumed that the ashes mixed had no combustible in it.

TABLE IV-C.

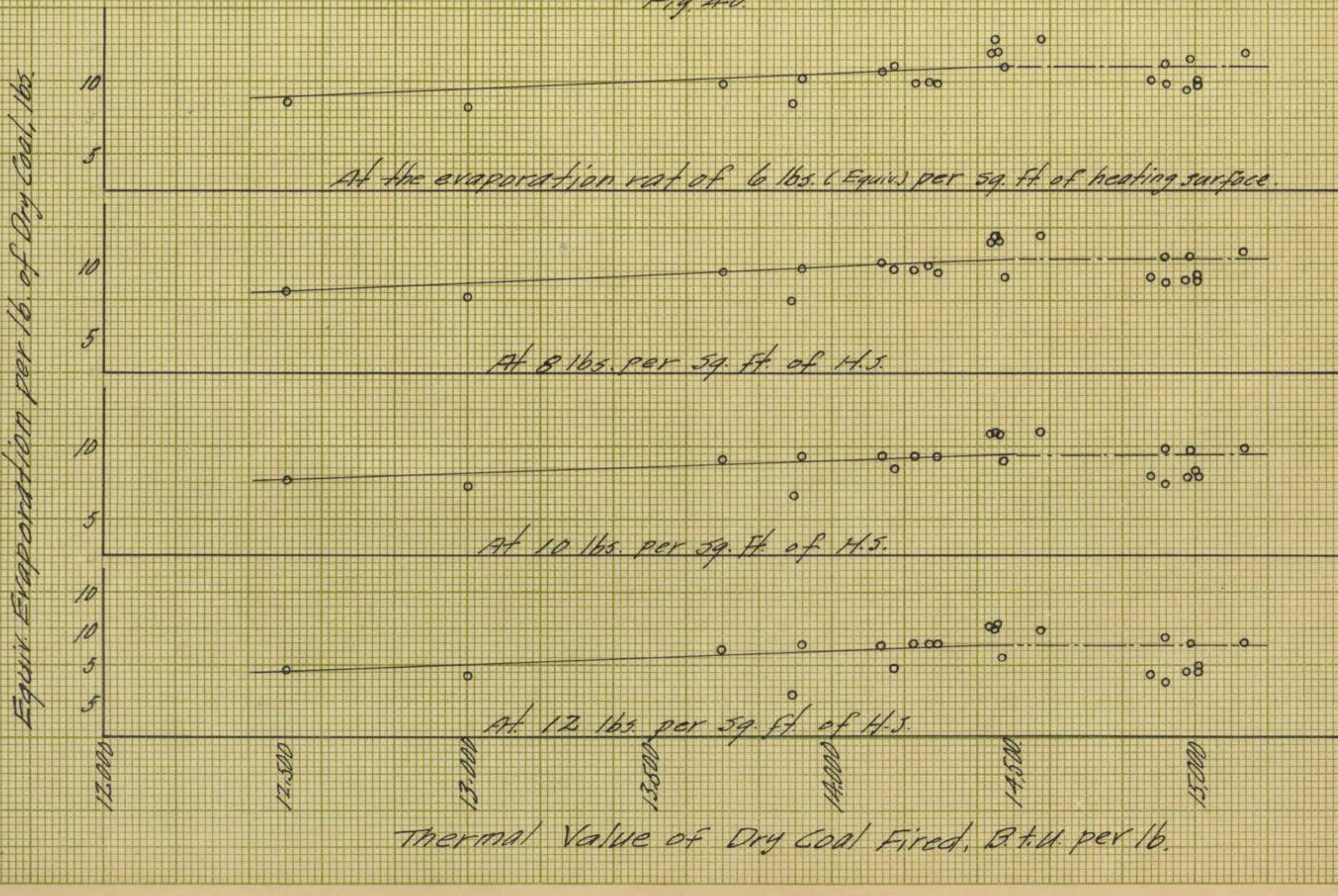
Data from Meier's Fuel Test.

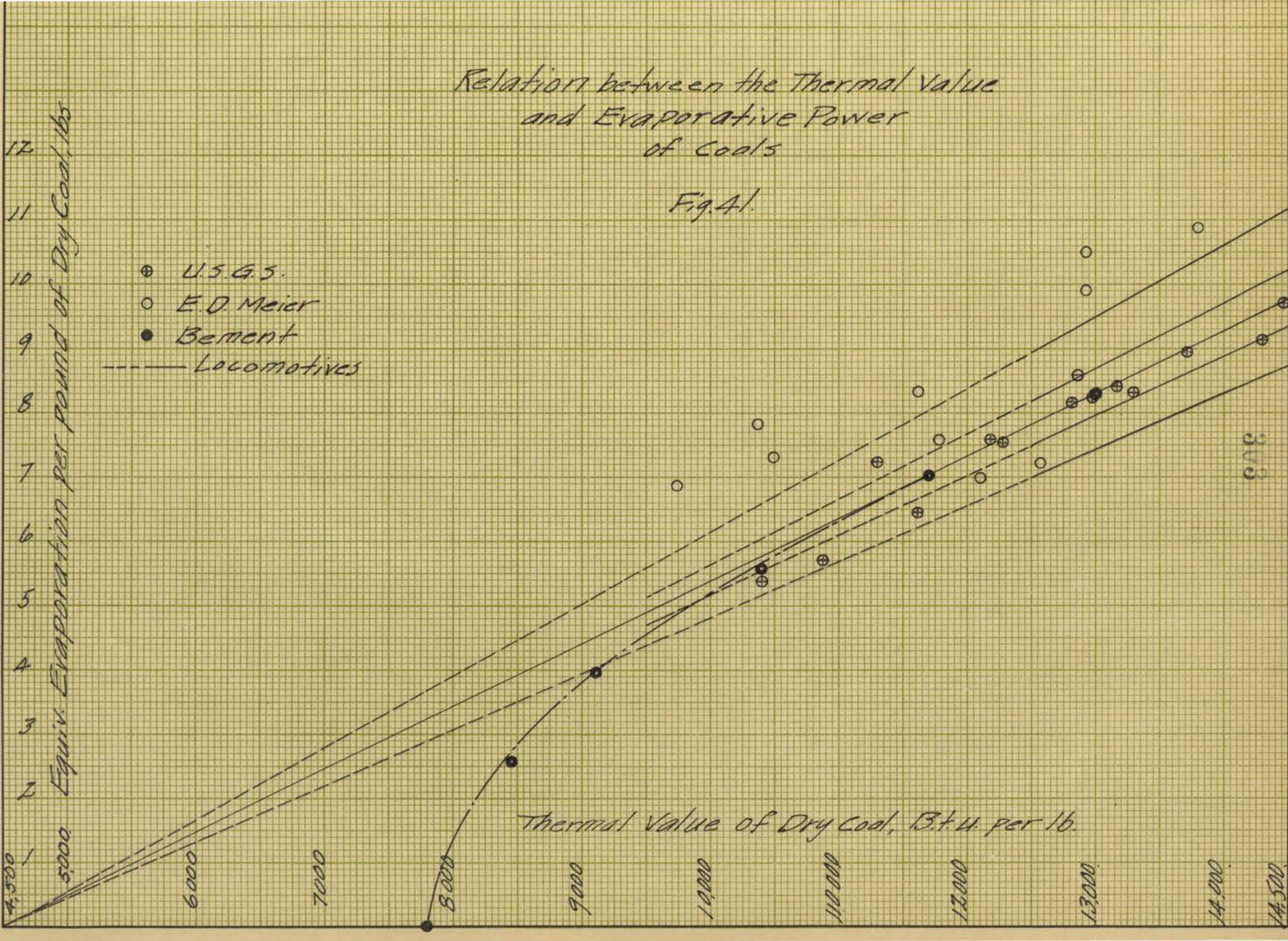
(Kent's Mechanical Engineers' Pocket Book, 1907 ed. p. 688.)

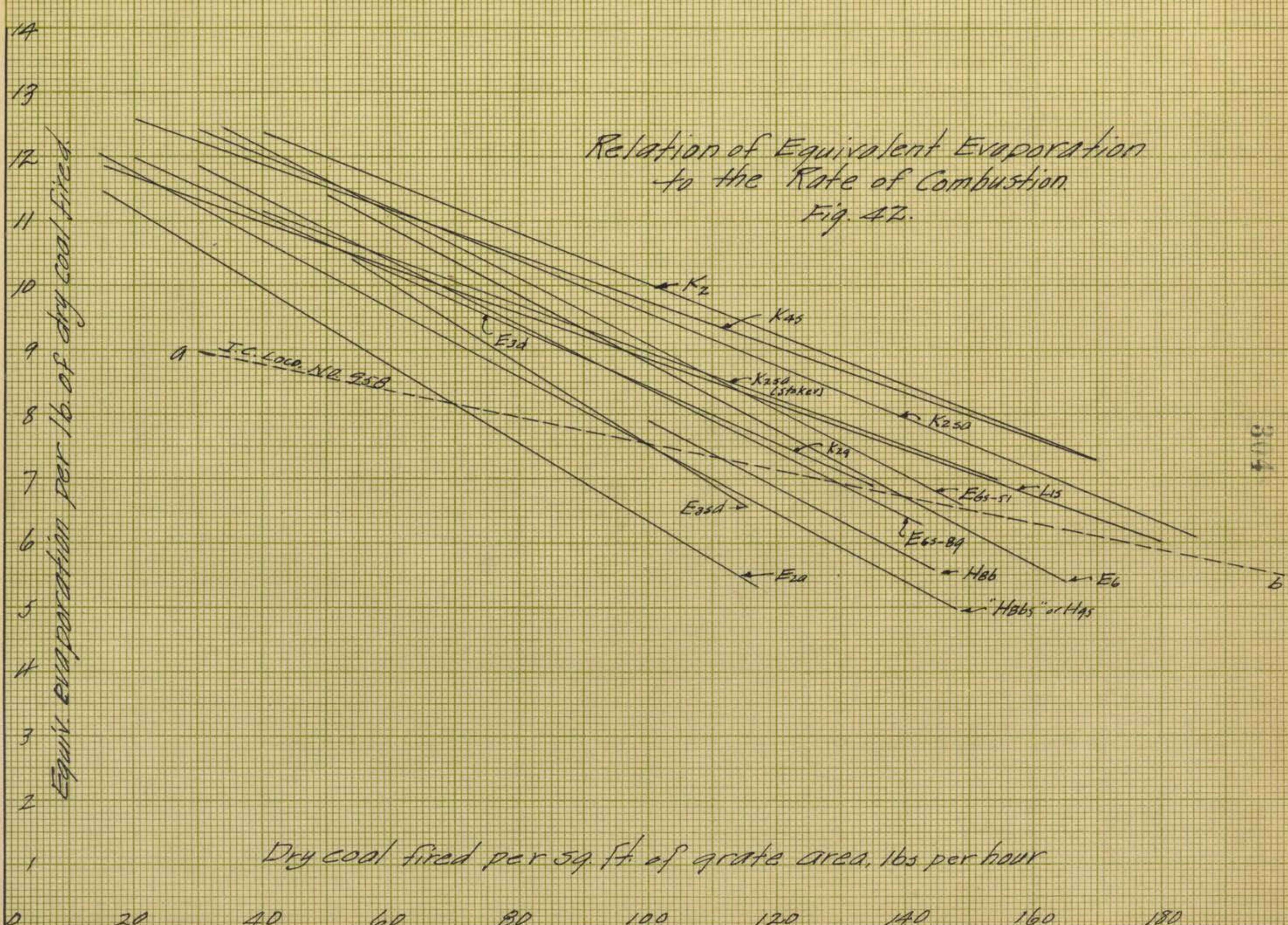
Coal	Equiv. Evap. per lb. of coal lb.	Calorific Value of Fuel
Cumberland Semi-bitum.	10.91	13,800
Second Pool Younghiogheny	9.94	12,936
Younghiogheny	10.51	12,936
Turkey Hill, Ill.	7.31	10,487
Carbon Hill Wash.	7.59	11,785
Hocking Valley Ohio	8.33	11,610
Gillespie Lump, Ill.	7.36	9,739
Collinsville Ill.	7.81	10,359

Relation of Thermal Value of Coal to Evaporation in Locomotive Boilers.

Fig. 40.




Relation between the Thermal Value
and Evaporative Power
of Coals
Fig. 41.
U.S.G.S.
E.D. Meier
Bement
Locomotives
Equiv. Evaporation per pound of Dry Coal, lbs
Thermal Value of Dry Coal, B.t.u. per lb.
1
2
3
4
5
6
7
8
9
10
11
12
4,500
5000
6000
7000
8000
9000
10,000
11,000
12,000
13,000
14,000
14,500


Relation of Equivalent Evaporation
to the Rate of Combustion.
Fig. 42.
Equiv. evaporation per lb. of dry coal fired
Dry coal fired per sq. ft. of grate area, lbs per hour
a
I.C. Loco. No. 958
b
K2
K4s
K2sa (stoker)
K2sa
K2a
E3d
E2sd
E2a
E6s-51
L1s
E6s-89
H8b
E6
"H8bs" or H4s
0
20
40
60
80
100
120
140
160
180
1
2
3
4
5
6
7
8
9
10
11
12
13
14

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Relation of Equiv. Evaporation to the Firing Rate.
Fig. 43.

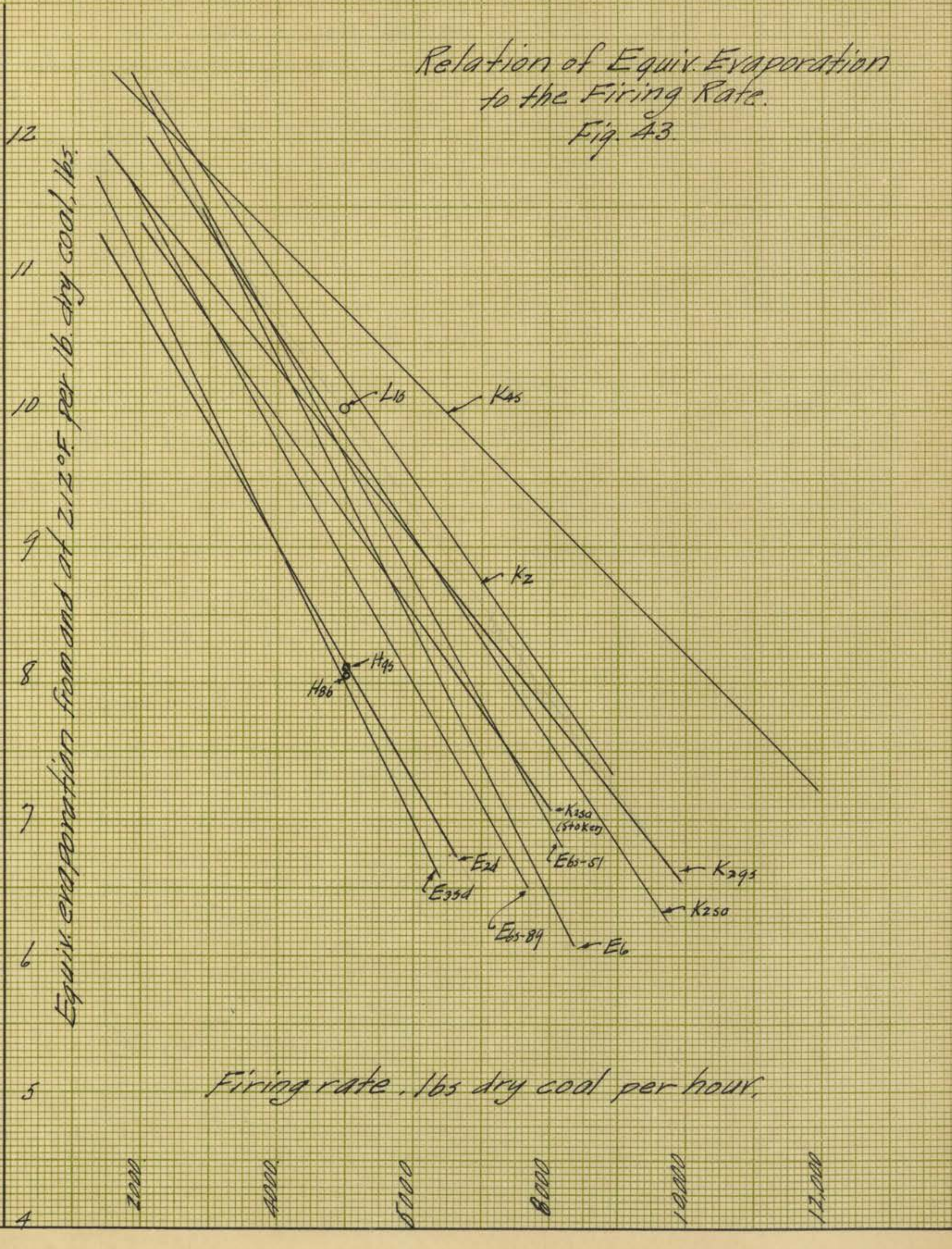

TABLE V.

Data of the Diagrams in Figs. 36 & 37.

	Item	L1s	K4s	K29	"H8sb" or H 9s
	Heating surfaces, sq. ft.:-				
1.	Firebox, including arch tubes	301.5	207.0	233.	190.
2.	Tubes, (water side)	3716.	3729.	2733.	2840.
3.	Superheater	1172.	1172.	1302.	809.
4.	Total heating surface, sq.ft.				
5.	Adjusted heating surface	1162.	1170.	1192.	839.
6.	Grate area, sq. ft.	70.	69.3	58.	55.3
7.	Thermal value of dry coal fired, B.t.u. per lb.	14,40	14467	14427	14140
8.	Coal factor, f	.964	.9967	.9927	.964
9.	Equivalent Evaporation per lb. dry coal at the firing rate of 5000 lb. dry coal per hr., E	9.85	10.75	9.70	8.00
10.	E corrected to 14500 B.t.u. coal = E/f	10.22	10.8	9.78	8.30
11.	Equiv. evap. per lb. dry coal of the rate of combustion, 50 lb. per sq. ft. of grate area = E (50)	10.65	11.55	10.66	10.18
12.	Equiv. evap. per lb. dry coal of the rate of combustion, 100 lb. per sq. ft. of grate area = E(100)	8.95	9.78	8.45	7.52
13.	Change in equiv. evap. per lb. dry coal, per 1 lb. per sq. ft. of grate combustion rate = [E(50)-E(100)] /50	.034	.0354	.0442	.0532
14.	Item (13) corrected to 14500 B.t.u. coal				

TABLE V (continued).

Data of the Diagrams in Figs. 36 & 37.

Item	H8b	K2sa No. 877	K2sa No. 7166	K2	E6s No. 51
1.	188.34	208.	213.4	211.5	232.7
2.	3674.0	3436	3457	4439	2625
3.	0	989	989	0	811
4.	3862.3				
5.	923.14	944	1031	1099	842
6.	53.9	53.2		55.4	55.8
7.	13369	14530	14530	14530	14470
8.	.887	1.0	1.0	1.0	.993
9.	8.05	9.90	9.25	10.27	9.65
10.	9.09	9.90	9.25	10.27	9.66
11.	10.2	11.65	10.80	12.0	11.66
12.	7.9	9.59	8.98	10.02	9.11
13.	0.46	.0412	.0364	.0396	.0570

TABLE V (concluded).

Data of the Diagrams in Figs. 36 & 37.

Item	E6s No. 89	E6	E3sd	E2d	E2a
1.	254.5	245.9	179.5	162.5	156.9
2.	2405	3373	1836	2484	2471
3.	689	0	561.0	0	0
4.					
5.	804	919	603	659	651
6.	55.2	54.7	54.7	55.3	55.5
7.	14470	14490	14390	14390	15150
8.	.993	.999	.981	.981	1.0
9.	8.92	9.40	8.00	8.15	8.20
10.	9.00	9.40	8.15	8.30	8.20
11.	10.90	11.45	10.30	10.73	9.40
12.	8.42	8.93	7.49	7.58	6.35
13.	.0496	.0524	.0562	.0632	.0610

VITA.

The writer, SENTARO SEKINE, was born at Miyauchi, Japan, 1888. He received his early education at Kodama. He entered in 1902 the Iwakura Railway School, where he had instruction in railway civil engineering and graduated with honor in 1905. After graduation he served for two years as junior engineer with the Keifu Railway Company. In 1907 he came to the United States, and after working for a short time as a track laborer on the Great Northern Railway, he entered the Columbia School at Seattle, Wash. in order to learn English. In 1908 he was admitted to the College of Applied Science at the University of Iowa, where he pursued for one year the course in civil engineering. In 1909 he entered the University of Illinois where in 1913 he received the degree of B.S. in Mechanical Engineering, and in 1914 the degree of A.B. He was admitted to the Graduate School of the University of Illinois in 1914 where, under the general direction of Professor Edward C. Schmidt, he has studied railway engineering, taking work in this field under Professor Schmidt, Dr. W.F.M.Goss, Professor J. M. Snodgrass, Professor A. M. Buck, and Mr. A. F. Comstock. He has also done work in mechanics under Professor A. N. Talbot, in mechanical engineering under Professor G. A. Goodenough, and in transportation under Professor E. R. Dewsnup. During the summer of 1914 he worked in the Chicago shops of the Chicago and Northwestern Railway. In 1911, there were issued to him United States Letters Patent #991657. In Bulletin No. 90 of the Engineering Experiment Station, University of Illinois, and in an article on "A New Method for Determining Railway Motor Speeds with Varying Voltage", in the Electric Railway Journal for September 18, 1915, both written by Professor A. M. Buck, the writer is credited with some of the processes there set forth.